On Liveness and Boundedness in Time Petri Nets

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Abstract. This paper deals with necessary and sufficient conditions for defining classes of Time Petri Nets (in sense of Merlin) which have the same boundedness and liveness behaviour as the corr. timeless Petri Nets (the skeletons).

1. Introduction

The Petri nets are meanwhile well known and widely used for describing concurrent systems. The reason for this is, that they are actually convenient for using: graphically presentable, formally defined and therefore well supported by software tools.

Several time extensions of Petri nets already can be found (comp. Starke [12]). Two main directions in using the time are recognized: considering the time as a duration (first intriduced by Ramchandani[10]) and considering time intervals (comp. Merlin[5], Popova[6], Berthomieu/Diaz[2], Boucheneb/Berthelot[4]). Combinations of high-level nets and time constrains are desirable, e.g. for modelling industrial control systems, but difficult to analyze. For the verification of such systems simulation can be used. (comp. Schöf/Sonnenschein/Wieting[13]).

Time Petri Nets, in sense of Merlin, are classical Petri Nets where to each transition t a time interval $[a_t, b_t]$ is associated. The times a_t and b_t are relative to the moment at which t was last enabled. Assuming that t was last enabled at time c_t, then t may fire only during the interval $[c_t + a_t, c_t + b_t]$ and must fire at the time $c_t + b_t$ at the latest, unless it is disabled before by the firing of another transition. Firing a transition takes no time. Thus, the philosophy of this kind of time dependent Petri net is : when a transition becomes enabled it may not fire at once (in general) but during a certain time interval and at the end of the interval there is a force for firing. If the end of the interval is infinite then the second characteristic of this kind of net, the force for firing, is lost. That is way, we consider only time intervals, which upper bounds are finite numbers ($b_t \neq \infty$ for each transition t). The Time Petri Nets are well applicable e.g. for design and analysing of communication protocols and further problems with recoverability aspects.

For analysing Time Petri Nets there are defined two different methods: one is defined by Berthomieu/Diaz/Menasche and the other is defined by Popova. The both consist on the computation of a reachability graph of the Time Petri Net but they have different backgrounds.

By Berthomieu/Diaz/Menasche the notion "state" is used for describing (infinitely) many situations of the Time Petri Nets and it is defined by a marking and time intervals for the enabled transitions (comp. Berthomieu/Menasche[3], Berthomieu/Diaz[2]) while in Popova[6] a state is a description for a snapshot of the net. A state here is given by marking and the momentary local time for the enabled transitions and the empty sign for the disabled transitions. Therefore we develop completely different method for building a reachability graph for a Time Petri Net: a node of the Berthomieu/Diaz/Menasche's reachability graph (a state class (comp. Berthomieu/ Menasche[3], Berthomieu/Diaz[2])) include an infinite number of possible situations of the Time Petri Net and is given as a solution of a system of inequalities.The nodes of our reachability graph (comp. Popova[6]) are only the "essential" states, these are the integer-states, where the momentary local time for all enabled transitions are integers. For example $z = ((1,0,3), \begin{pmatrix} 3 \\ \sharp \\ 0 \\ \sharp \end{pmatrix})$ can be a state in a Time Petri Net with 3 places and 4 transitions. The first and the third transitions are enabled. The first transition is enabled at least since 3 time units, and the third transition is just enabled at least (since 0 time units). The second and the fourth transitions are disabled. Obviously, this reachability graph includs only a discrete part of all the situations, which are possible for a given Time Petri Net. But the knowledge of the net behaviour in the integer-states is sufficient to know the whole behaviour of the net (comp. Popova [6]). The computing of the integer-states is easy.

However, the computation of the reachability graph with the both methods is more time and space expensive as the computation of the reachability graph of the corresponding classical Petri Net.

In general, there is just no comparison between the liveness behaviour of a given Time Petri Net and the corr. Petri Net without time restrictions (comp. Popova[6],[7]). Of course, a Time Petri Net is bounded when the corr. Petri net (the "skeleton" of the TPN) is bounded. It is possible, that a bounded Time Petri Net has an unbounded corr. classical Petri Net (corr. Popova[6],[7]).

In Popova[9] two classes of TPN are given, which have the same behaviour as the corr. Petri Net.

In this paper we present two further net classes, defined by structural conditions. The Time Petri Nets in these classes have the same laveness and boundedness behaviour as the corr. classical Petri Nets (the skeletons).

2. Notions and basic definitions

We will use the folowing notations. $\mathcal{N}$ denotes the set of natural numbers, $\mathcal{N}^+$ is $\mathcal{N} \setminus \{ 0\}$. $\mathcal{Q}o$ is the set of nonnegative rational numbers. Let g be a given function from A to B. Then, the symbol $\#$ defines a special value from the set $g(A)$. $\|a\|$ is the maximal integer, nongreater then a, and $\ll a \gg$ is defined as $a - \|a\|$. T^* denotes the language of all words over the alphabet T, including the empty word e.

Definition 2.1.: The structure $N = (P, T, F, V, m_o)$ is called Petri net (PN), iff
(1) P, T, F are finite sets with

$$P \cap T = \emptyset, \qquad P \cup T \neq \emptyset, \qquad F \subseteq (P \times T) \cup (T \times P) \text{ and}$$

$$dom(F) \cup cod(F) = P \cup T \text{ (net)}$$

(2) $V : F \longrightarrow \mathcal{N}^+$ (weight of the arcs)

(3) $m_o : P \longrightarrow \mathcal{N}$ (initial marking)

A marking of a PN is a function $m : P \longrightarrow \mathcal{N}$, where $m(p)$ denotes the number of tokens in place p. The pre- and postsets of a transition t resp. of a place p are given by $Ft := \{p \mid p \in P \wedge pFt\}$ and $tF := \{p \mid p \in P \wedge tFp\}$ resp. Fp and pF. Each transition $t \in T$ induces the marking t^- und t^+, defined as follows:

$$t^- = \begin{cases} V(p,t) & \text{,iff} \quad (p,t) \in F \\ 0 & \text{,iff} \quad (p,t) \notin F \end{cases}$$

$$t^+ = \begin{cases} V(t,p) & \text{,iff} \quad (t,p) \in F \\ 0 & \text{,iff} \quad (t,p) \notin F \end{cases}$$

By Δt we denote t^+- t^-. A transition $t \in T$ is enabled (may fire) by a marking m iff $t^- \leq m$ (i.g. $t^-(p) \leq m(p)$ for each place $p \in P$). When an enabled transition t by a marking m fires, there yields a new marking m' given by

$m'(p) := m'(p) + \Delta t(p)$, denotes by $m \xrightarrow{t} m'$. A PN is homogeneous iff for each transition t its input arcs have the same weight. An extended free choice (EFC) net is a PN with the structural restriction: if two places have at least one common posttransition then these places have the same postset (formal: $\forall p_1 \forall p_2 (p_1 \in P \wedge p_2 \in P \wedge p_1 F \cap p_2 F \neq \emptyset \longrightarrow p_1 F = p_2 F)$.

Definition 2.2.: The structure $Z = (P, T, F, V, m_o, I)$ is called a Time Petri Net (TPN) iff:
(1) (P, T, F, V, m_o) is a PN (sleleton of Z).
(2) $I : T \longrightarrow \mathcal{Q}_0 \times (\mathcal{Q}_0 \cup \{\infty\}$ and for each $t \in T$ $I_1(t) \leq I_2(t)$, where $I(t) = (I_1(t), I_2(t))$.

The skeleton of Z we denote by $S(Z)$. I is called the time function of Z, $I_1(t)$ and $I_2(t)$ earlest firing time of t ($eft(t)$) and latest firing time of t ($lft(t)$), respectively. It is not difficult to see (comp. Popova[6]) that considerations of TPN with $I : T \longrightarrow \mathcal{N} \times (\mathcal{N} \cup \{\infty\})$ is not a restriction in general. In the following we always consider such time functions I. Futhermore, we use here the strong meaning of conflict: two transitions t_1 and t_2 are in conflict iff $Ft_1 \cap Ft_2 \neq \emptyset$.

As already mentioned in the introduction, we will use the notion "state" in order to define a reachability graph for a TPN. Thus, now we define "state" and "change from a state into another state" by firing a transition and by time duration.
Definition 2.3.: Let $Z = (P, T, F, V, m_o, I)$ be a TPN and $J : T \longrightarrow \mathcal{Q}o \cup \{\#\}$. $z = (m, J)$ is called a state in Z iff:
(1) m is a reachable marking in N^Z.
(2) $\forall t (t \in T \wedge t^- \leq m \longrightarrow J(t) \leq lft(t))$.
(3) $\forall t (t \in T \wedge t^- \not\leq m \longrightarrow J(t) = \#)$.

An interpretation of the notion "state" is as follows: in the net each transition t has a watch. The watch does not work ($J(t) = \#$) at the marking m, if t is disabled at m. If t is enabled at m, the watch of t shows the time ($J(t)$), elapsed since t was enabled last.

The state $z_o := (m_o, J_o)$ with $J_o(t) := \begin{cases} 0 & ,\text{iff} \quad t \leq m \\ \# & ,\text{iff} \quad t \not\leq m \end{cases}$ is set to be the initial state of the TPN Z. A transition t is ready to fire in the state $z = (m, J)$, denote by $z \xrightarrow{t}$, iff $t^- \leq m$ and $eft(t) \leq J(t)$. A transition t , being ready to fire in the state $z = (m, J)$ may fire yielding a new state $z' = (m', J')$ defined by

$$m' = m + \Delta t \quad \text{and } J'(t) =: \begin{cases} \# & ,\text{iff} \quad t^- \not\leq m \\ J(t) & ,\text{iff} \quad t^- \leq m \ \wedge \ t^- \leq m' \ \wedge \ Ft \cap Ft' = \emptyset \\ 0 & ,\text{iff} \quad \text{otherwise} \end{cases} .$$

The state $z = (m, J)$ changes into the state $z' = (m', J')$ by the time duration $\tau \in \mathcal{Q}$, denoded by $z \xrightarrow{\tau} z'$ iff $m' = m$ and the time duration τ is possible

(formal: $\forall t(t \in T \wedge J(t) \neq \# \longrightarrow J(t) + \tau \leq lft(t)$ and
$J'(t) := \begin{cases} J(t)+\tau & ,\text{iff} \quad t \leq m \\ \# & ,\text{iff} \quad t \not\leq m \end{cases}$).
$z = (m, J)$ is called an integer-state iff for each enabled transition t in m $J(t)$ is an integer. The graph $RG_Z(z_o)$ is called the reachability graph of the TPN Z iff its nodes are the reachable integer-states and its arcs are defined by the triples (z, τ, z') respectively (z, t, z'), where $z \xrightarrow{t} z'$, respectively $z \xrightarrow{\tau} z'$.Let $R_Z(z)$ be the set of all reachable states in Z started at z and let $R_{S(Z)}(m)$ be the set of all reachable markings in N started at the marking m. A transition t is called live in z iff $\forall z'(z' \in R_Z(z) \longrightarrow \exists z''(z'' \in R_Z(z') \quad \wedge \quad t$ is ready to fire in $z'')$. The state z is live in the TPN Z iff all transitions $t \in T$ are live in z and Z is live iff z_o is live in Z.

3. Criterions

The class, defined below, contains all TPNs, which transitions can (but do not have to) fire immediately. This property is a structural one and therefore easy to be proved.

Proposition 1:

Let $Z = (P, T, F, V, m_o, I)$ be a TPN with $\forall t(t \in T \longrightarrow eft(t) = 0)$ and let $S(Z)$ be the skeleton of Z. Than it holds:
(1) $S(Z)$ is unbounded if and only if Z is unbounded, and
(2) $S(Z)$ is live if and only if Z is live.

Proof:

ad (1). ($\longrightarrow$) Let $S(Z)$ be unbounded. We will show, that the TPN Z is unbounded, too.

For this reason we have to prove, that for each natural number k there is, at least, a place p which includes more than k tokens in a marking m, where m is a marking belonging to an in Z reachable state $z = (m, J)$, i.e. we have to show:

$$\forall k(k \in \mathcal{N} \longrightarrow \exists z \exists p(z \in R_Z(z_o) \wedge p \in P \wedge z = (m, J) \wedge m(p) > k)).$$

Because the PN $S(Z)$ is unbounded, for each natural number k there exists a reachable marking m^k ($m^k \in R_{S(Z)}(m_o)$) and it is true, that $m^k(p) > k$ in $S(Z)$.

Since the marking m^k is reachable in $S(Z)$ there exists a path

$$m_o \xrightarrow{t_1} m_1 \xrightarrow{t_2} ... \xrightarrow{t_n} m_n = m^k \qquad \text{(in } S(Z)).$$

We consider the states $z_i := (m_i, J_i)$ with $J_i(t) := \begin{cases} 0 & \text{,iff} \quad t \leq m_i \\ \# & \text{,iff} \quad t \not\leq m_i \end{cases}$.

It is easy to see that the sequence

$$z_o \xrightarrow{t_1} z_1 \xrightarrow{t_2} ... \xrightarrow{t_n} z_n = z^k \qquad \text{(in } Z).$$

is enable in Z, too. Therefore, the state $z_n = z^k$ is a reachable one in Z and z^k satisfies the inequality $m^k(p) > k$.

($\longleftarrow$) This direction is obvisious .

ad (2). ($\longrightarrow$) Let the PN $S(Z)$ be live, i.e. $\forall t \forall m(t \in T \wedge m \in R_{S(Z)}(m_o) \longrightarrow \exists m'(m' \in R_{N^Z}(m) \wedge t^- \leq m'))$.

We will show that the TPN Z is live, too.

Let $z^* = (m^*, J^*)$ be an arbitrary reachable state in Z ($z^* \in R_Z(z_o)$) and t^* be an arbitrary transition in Z. It is enough to show, that

$$\exists z'(z' = (m', J') \in R_Z(z^*) \wedge t^{*-} \leq m' \wedge eft(t^{*-}) \leq J'(t^*)). \tag{3}$$

The last inequality $eft(t^{*-}) \leq J'(t^*)$ is always true, because of the "eft" condition of this proposition. Thus, we are looking for a state z', reachable in Z from the state z^* , with the property $t^{*-} \leq m'$.

Since z^* is a reachable state in Z, it is easy to see, that m^* is a reachable marking in $S(Z)$. Following (and because N is live), there exists a marking m' with $m' \in R_{S(Z)}(m^*)$ and $t^- \leq m'$. This means, there exists a sequence $t'_1 ... t'_r$ with $m^* =: m'_o \xrightarrow{t'_1} m'_1 \xrightarrow{t'_2} ... \xrightarrow{t'_r} m'_r = m'$ in $S(Z)$ and $t^{*-} \leq m'$.

Now, we consider the state sequence $z'_o, ..., z'_r$ with $z'_i := (m'_i, J'_i)$ and

$$J'_o(t) := J^*(t)$$

$$J'_{i+1}(t) =: \begin{cases} \# & \text{,iff} \quad t^- \not\leq m_{i+1} \\ J'_i(t) & \text{,iff} \quad t^- \leq m_{i+1} \ \wedge \ t^- \leq m_i \ \wedge \ Ft \cap Ft'_i = \emptyset \\ 0 & \text{,iff} \quad \text{otherwise} \end{cases}$$

for each $i = 0, ..., r-1$.
Obviously, this last sequence can fire in Z (from z^*) and therefore the state $z' = z'_r$ is a state, satisfying (3).

($\longleftarrow$) Let the TPN Z be live , i.e. for each transition $t \in T$ and each state z, reachable in $Z(z_o)$, there exists a further state z', reachable in $Z(z)$ and t can fire in z. We will show, that the skeleton of Z $S(Z)$ is live, too.

Let m^* be an arbitrary marking in $S(Z)$, reachable from m_o and t^* be an arbitrary transition in $S(Z)$. We have to show: $\exists m'(m' \in R_{S(Z)}(m^*) \wedge t^{*-} \leq m')$.

Because $m^* \in R_{S(Z)}(m_o)$, there exist the sequences $m_o, ..., m_n = m^*$ and $t_1, ..., t_n$ and

$$m_o \xrightarrow{t_1} m_1 \xrightarrow{t_2} ... \xrightarrow{t_n} m_n = m^* \qquad \text{in } S(Z). \tag{4}$$

Now, considering the state sequence $z_o, z_1, ..., z_n$ with $z_i := (m_i, J_i)$ and

$$J_i(t) := \begin{cases} 0 & \text{,iff} \quad t \leq m_i \\ \# & \text{,iff} \quad t \not\leq m_i \end{cases}$$

it is easy to see, that this sequence can fire in Z, because of (4) and the "eft" condition of this proposition. Therefore $z^* = z_n$ is a reachable state in Z ($z^* \in R_Z(z_o)$).

Because Z is live, we can find a further state z' with the property:

$$z' \in R_Z(z^*) \wedge t^{*-} \leq m' \wedge eft(t^*) \leq J'(t^*).$$

Let $z^* = z'_o \xrightarrow{\tau'_o} \xrightarrow{t'_1} \xrightarrow{\tau'_1} z'_1 \xrightarrow{t'_2} \xrightarrow{\tau'_1} ... \xrightarrow{t'_k} \xrightarrow{\tau'_k} z_k = z'$ be.

We obey $m^* = m'_o \xrightarrow{t'_1} m'_1 \xrightarrow{t'_2} ... \xrightarrow{t'_k} m'_k = m'$ is enable and $t^{*-} \leq m'$.

■

A further class of TPNs with the same liveness and boundedness behaviour as the class of the corr.(skeletons) PNs is defined by the constrain: the lft of all transitions in a given TPN are infinite. Thus, this is a class of TPN, where each transition has to wait for a certain time in order to fire when it becomes enabled but there is no force to fire for any transition in any time in the net:

Proposition 2:

Let $Z = (P, T, F, V, m_o, I)$ be a TPN with $\forall t(t \in T \longrightarrow lft(t) = \infty)$ and let

$S(Z)$ be the skeleton of Z. Than it holds:
(1) $S(Z)$ is unbounded if and only if Z is unbounded, and
(2) $S(Z)$ is live if and only if Z is live.

Proof:

ad (1). ($\longrightarrow$) Let the PN $S(Z)$ be unbounded. We want to show, that the TPN Z is unbounded, too.

Let k be an arbitrary natural number. Then, because of the unboundedness of $S(Z)$, there is a place $p \in P$ and a reachable marking m^* in the skeleton net $S(Z)$, i.e.

$$m^* \in R_{S(Z)}(m_o) \tag{5}$$

with $m^*(p) > k$. Because of (5) there exists a sequence

$$m_o \xrightarrow{t_1} m_1 \xrightarrow{t_2} ... \xrightarrow{t_n} m_n = m^* \qquad \text{(in } S(Z)\text{)}.$$

We consider now the state sequence

$$z_o \xrightarrow{\tau_o} z_o' \xrightarrow{t_1} z_1 \xrightarrow{\tau_1} z_1' \xrightarrow{t_2} z_2 \xrightarrow{\tau_2} ... \xrightarrow{t_n} z_n \xrightarrow{\tau_n} z_n' \tag{6}$$

with $z_i := (m_i, J_i)$ resp. $z_i' := (m_i', J_i')$ and $\tau_i; = \max(\{eft(t) \mid t^- \leq m_i)$ for each i=0, ... ,n.

The sequence (6) can fire in Z, because the lft of all transitions in Z are infinite. Thus, the state $z_n = (m_n = m^*, J_n)$ is a reachable one in Z and therefore Z is an unbounded TPN.

($\longleftarrow$) This direction is obvisious .

ad (2). ($\longrightarrow$)It is easy to proof this direction, using the same constraction as in the proof of the proposition 2, (10), ($\longrightarrow$).

($\longleftarrow$) Let the TPN Z be live , i.e. for each transition $t \in T$ and each state z, reachable in $Z(z_o)$, there exists a further state z', rechable in $Z(z)$ and t can fire in z. We want to show, that the skeleton of Z, the classical Petri Net $S(Z)$ is live, too.

Let m^* be an arbitrary marking in $S(Z)$, reachable from m_o and t^* be an arbitrary transition in $S(Z)$. Because of $m^* \in R_{S(Z)}(m_o)$, there exist the sequences

$$m_o, ..., m_n = m^* \text{ and } t_1, ..., t_n \text{ and } m_o \xrightarrow{t_1} m_1 \xrightarrow{t_2} ... \xrightarrow{t_n} m_n = m^* \quad \text{in } S(Z).$$

Analogously to the constraction in the proof of the proposition 2, (1), ($\longrightarrow$) we obtain a state $z^* = (m^*, J^*) \in R_Z(z_o)$. Now, we can find a path in the TPN Z, starting with the state z^* and ending with a state, in which the transition t^* can fire (analogously to the proof in proposition 1, (2), ($\longrightarrow$)).

$\blacksquare$

4. Conclusions

This paper is a continuation of the work presented at CS&P94 (Popova[8]) and further developed in Popova[9]. Here, we have defined two new classes of TPNs with the same liveness and boundedness behaviour as the corr. timeless PNs - the skeletons of the TPNs. The constrains, defining the two new TPN classes, are structural and therefore easy and fast to check.

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