On Time Invariance in Time Petri Nets

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Abstract: In this paper we consider as well structural as dynamic conditions which are sufficient, that a Time Petri Net (in sence of Merlin definition) has the same liveness behaviour as the considered net without time restrictions.

1. Introduction

One of the suitable kinds of discribing and analysing as well parallel as concurent systems is given with the net theory and particularly with the Petri net theory. Almost, the clasical Petri Nets are not sufficient for studying some special properties as random properties, time dependances, unfinite domains etc.

Certain local time dependances between some processes/actions in a parallel or concurent system are often given resp. claimed. For studying such systems a large variety of time dependent Petri Nets are defined and well searched and meanwhile widely applicated.

All these nets have the same origin: the classical Petri Nets. Then, in different ways the time is associated. In this paper we dealing with Time Petri Nets, introduced 1972 by Merlin and further investigated by Berthomieu/Menasche[4], Popova[7],Popova[8], Berthomieu/Diaz[3], Boucheneb/Berthelot[5], Berthelot/Boucheneb[2] etc.

Time Petri Nets are classical Petri Nets where to each transition t a time interval $[a_t, b_t]$ is associated. The times a_t and b_t are relative to the moment at which t was last enabled. Assuming that t was last enabled at time c_t, then t may fire only during the interval $[c_t + a_t, c_t + b_t]$ and must fire at the time $c_t + b_t$ at the latest, unless it is disabled before by the firing of another transition. Firing a transition takes no time. Thus, the philosophy of this kind of time dependent Petri net is : when a transition become enabled it may not fire at once (in general) but during a certain time interval and at the end of the interval there is a force for firing. If the end of the interval is infinite then the second characteristic of this kind of net, the force for firing, is lost. That is way, we consider only time intervals, which upper bounds are finite numbers ($b_t \neq \infty$ for each transition t). The Time Petri Nets are well applicable e.g. for design and analysing of communication protocols and further problems with recoverability aspects.

For analysing Time Petri Nets there are defined two different methods: one is defined by Berthomieu/Diaz/Menasche and the other is defined by Popova. The both consist on the computation of a reachability graph of the Time Petri Net but they have different backgrounds.

By Berthomieu/Diaz/Menasche the notion "state" is used for describing (infinitely) many situations of the Time Petri Nets and it is defined by a marking and time intervals for the enabled transitions (comp. Berthomieu/Menasche[4], Berthomieu/Diaz[3]) while in Popova[7] a state is a description for a snapshot of the net. A state here is given by marking and the momentary local time for the enabled transitions and the empty sign for the disabled transitions. Therefore we develop completely different method for building a reachability graph for a Time Petri Net: a node of the Berthomieu/Diaz/Menasche's reachability graph (a state class (comp. Berthomieu/ Menasche[4], Berthomieu/Diaz[3])) include an infinite number of possible situations of the Time Petri Net and is given as a solution of a system of inequalities.The nodes of our reachability graph (comp. Popova[7]) are only the "essential" states, these are the integer-states, where the momentary local time for all enabled transitions are integers. For example $z = ((1,0,3), \begin{pmatrix} 3 \\ \sharp \\ 0 \\ \sharp \end{pmatrix})$ can be a state in a Time Petri Net with 3 places and 4 transitions. The first and the third transitions are enabled. The first transition is enabled at least since 3 time units, and the third transition is just enabled at least (since 0 time units). The second and the fourth transitions are disabled. Obviously, this reachability graph includs only a discret part of all the situations, which are possible for a given Time Petri Net. But the knowledge of the net behaviour in the integer-states is sufficient to know the whole behaviour of the net (comp. Popova [7]). The computing of the integer-states is easy.

The reachability graph defined by Boucheneb/Berthelot is rather similar to the reachability graph defined by Berthomieu/Diaz/Menasche, especially refer to the computing of the graph nodes there is to solve a system of inequalities.

However, the computation of the reachability graph with the both methods is more time and space expensive as the computation of the reachability graph of the corresponding classical Petri Net.

In general, there is just no comparison between the liveness behaviour of a given Time Petri Net and the corr. Petri Net without time restrictions.:

Example 1.1. :

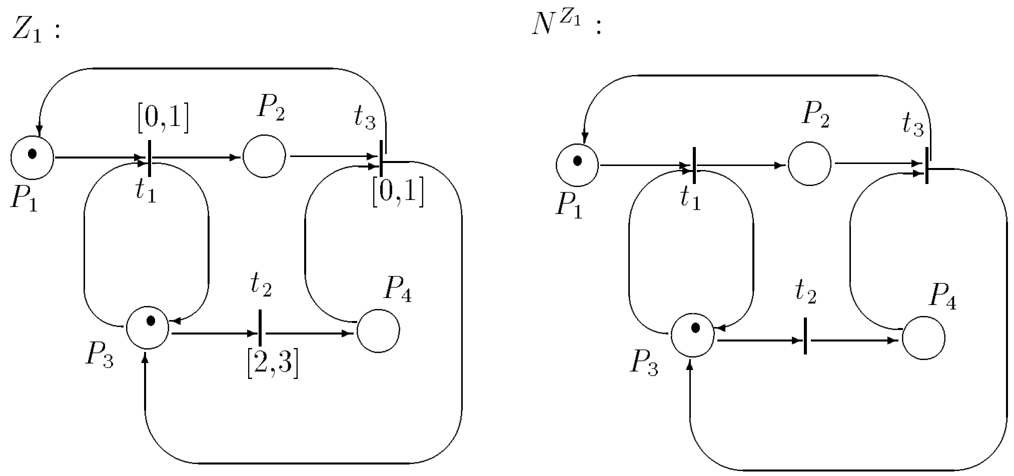

Fig. 1.

Example 1.2. :

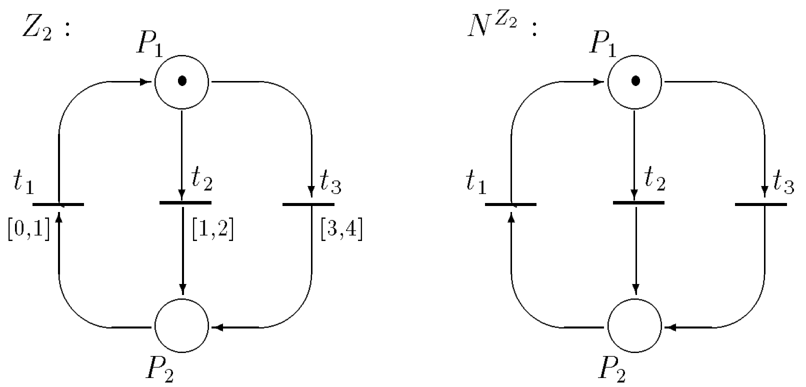

Fig. 2.

The Time Petri Net Z_1 is live, the corr. Petri Net N^{Z_1} is not live. On the other hand the Time Petri Net Z_2 is not live (because of t_3) while the corr. Petri Net N^{Z_2} is live.

In this paper we concider two classes of Time Petri Nets which have the same liveness behaviour as the corr. Petri Net without time restrictions. The conditions of the one class are structural and therefore easy to be found out. The conditions of the second class are dynamicly.

This paper is organized as follows. In section 2 basic notions and some basic definitions are recalled. A background for Time Petri Net considerations are given here, too. The section 3 is dealing with the introduction of a structural defined class of Time Petri Nets which liveness behaviour is time independent while in section 4 we consider a dynamic defined class of Time Petri Nets with time independent liveness behaviour.

2. Notions and basic definitions

We will use the folowing notations. $\mathcal{N}$ denotes the set of natural numbers, $\mathcal{N}^+$ is $\mathcal{N} \setminus \{\ 0\}$. $\mathcal{Q}o$ is the set of nonnegative rational numbers. Let g be a given function from A to B. Then, the symbol $\#$ defines a special value from the set $g(A)$. $\|a\|$ is the maximal integer, nongreater then a, and $\ll a \gg$ is defined as $a- \ll a- \gg$. T^* denotes the language of all words over the alphabet T, including the empty word e.

Definition 2.1.: The structure $N = (P, T, F, V, m_o)$ is called Petri net (PN), iff
(1) P, T, F are finite sets with

$P \cap T = \emptyset, \qquad P \cup T \neq \emptyset, \qquad F \subseteq (P \times T) \cup (T \times P)$ and

$dom(F) \cup cod(F) = P \cup T$ (net)

(2) $V : F \longrightarrow \mathcal{N}^+$ (weight of the arcs)

(3) $m_o : P \longrightarrow \mathcal{N}$ (initial marking)

A marking of a PN is a function $m : P \longrightarrow \mathcal{N}$, where $m(p)$ denotes the number of tokens in place p. The pre- and postsets of a transition t resp. of a place p are given by $Ft := \{p \mid p \in P \wedge pFt\}$ and $tF := \{p \mid p \in P \wedge tFp\}$ resp. Fp and pF. Each transition $t \in T$ induces the marking t^- und t^+, defined as follows:

$$t^- = \begin{cases} V(p,t) & ,\text{iff} \quad (p,t) \in F \\ 0 & ,\text{iff} \quad (p,t) \notin F \end{cases}$$

$$t^+ = \begin{cases} V(t,p) & ,\text{iff} \quad (t,p) \in F \\ 0 & ,\text{iff} \quad (t,p) \notin F \end{cases}$$

By Δt we denote t^+- t^-. A transition $t \in T$ is enabled (may fire) by a marking m iff $t^- \leq m$ (i.g. $t^-(p) \leq m(p)$ for each place $p \in P$). When an enabled transition t by a marking m fires, there yields a new marking m' given by $m'(p) := m'(p) + \Delta t(p)$, denotes by $m \xrightarrow{t} m'$. A PN is homogeneous iff for each transition t its input arcs have the same weight. An extended free choice (EFC) net is a PN with the structural restriction: if two places have at least one common posttransition then these places have the same postset (formal: $\forall p_1 \forall p_2 (p_1 \in P \wedge p_2 \in P \wedge p_1F \cap p_2F \neq \emptyset \longrightarrow p_1F = p_2F)$.

Definition 2.2.: The structure $Z = (P, T, F, V, m_o, I)$ is called a Time Petri Net (TPN) iff:
(1) (P, T, F, V, m_o) is a PN.
(2) $I : T \longrightarrow \mathcal{Q}_0 \times (\mathcal{Q}_0 \cup \{\infty\}$ and for each $t \in T$ $I_1(t) \leq I_2(t)$, where $I(t) = (I_1(t), I_2(t))$.

I is called the time function of Z, $I_1(t)$ and $I_2(t)$ earlest firing time of t ($eft(t)$) and latest firing time of t ($lft(t)$), respectively. It is not difficult to see (comp. Popova[6]) that considerations of TPN with $I : T \longrightarrow \mathcal{N} \times (\mathcal{N} \cup \{\infty\})$ is not a restriction in general. In the following we always consider such time functions I. Futhermore, we use here the strong meaning of conflict: two transitions t_1 and t_2 are in conflict iff $Ft_1 \cap Ft_2 \neq \emptyset$.

As already mentioned in the introduction, we will use the notion "state" in order to define a reachability graph for a TPN. Thus, now we define "state" and "change from a state into another state" by firing a transition and by time duration.

Definition 2.3.: Let $Z = (P, T, F, V, m_o, I)$ be a TPN and $J : T \longrightarrow \mathcal{Q}_0 \cup \{\#\}$. $z = (m, J)$ is called a state in Z iff:
(1) m is a reachable marking in N^Z.
(2) $\forall t (t \in T \wedge t^- \leq m \longrightarrow J(t) \leq lft(t))$.
(3) $\forall t (t \in T \wedge t^- \not\leq m \longrightarrow J(t) = \#)$.

An interpretation of the notion "state" is as follow: in the net each transition t has a watch. The watch does not work ($J(t) = \#$) at the marking m if t is disabled at m. If t is enabled at m, the watch of t shows the time ($J(t)$), elapsed since t was enabled last.

The state $z_o := (m_o, J_o)$ with $J_o(t) := \begin{cases} 0 & \text{,iff} \quad t \leq m \\ \# & \text{,iff} \quad t \not\leq m \end{cases}$ is set to be the initial state of the TPN Z. A transition t is ready to fire in the state $z = (m, J)$, denote by $z \xrightarrow{t}$, iff $t^- \leq m$ and $eft(t) \leq J(t)$. A transition t , being ready to fire in the state $z = (m, J)$ may fire yielding a new state $z' = (m', J')$ defined by $m' = m + \Delta t$ and
$J'(t) =: \begin{cases} \# & \text{,iff} \quad t^- \not\leq m \\ J(t) & \text{,iff} \quad t^- \leq m \quad \wedge \quad t^- \leq m' \quad \wedge \quad Ft \cap Ft' = \emptyset \\ 0 & \text{,iff} \quad \text{otherwise} \end{cases}$.

The state $z = (m, J)$ changes into the state $z' = (m', J')$ by the time duration $\tau \in \mathcal{Q}$, denoded by $z \xrightarrow{\tau} z'$ iff $m' = m$ and the time duration τ is possible (formal: $\forall t (t \in T \wedge J(t) \neq \# \longrightarrow J(t) + \tau \leq lft(t)$ and $J'(t) := \begin{cases} J(t) + \tau & \text{,iff} \quad t \leq m \\ \# & \text{,iff} \quad t \not\leq m \end{cases}$).
$z = (m, J)$ is called an integer-state iff for each enabled transition t in m $J(t)$ is an integer. The graph $RG_Z(z_o)$ is called the reachability graph of the TPN Z iff its nodes are the reachable integer-states and its arcs are defined by the triples (z, τ, z') respectively (z, t, z'), where $z \xrightarrow{t} z'$, respectively $z \xrightarrow{\tau} z'$.Let $R_Z(z)$ be the set of all reachable states in Z started at z and let $R_N(m)$ be the set of all reachable markings in N started at the marking m. A transition t is called live in z iff

$\forall z'(z' \in R_Z(z) \longrightarrow \exists z''(z'' \in R_Z(z') \quad \wedge \quad t$ is ready to fire in z''). The state z is live in the TPN Z iff all transitions $t \in T$ are live in z and Z is live iff z_o is live in Z.

3. Structural properties for time independence

In this part we will show that each TPN Z, which corr. PN N^Z belong to a certain class of EFC PN, has the same liveness behaviour as N^Z. Following, for such nets it is sufficient to stady the liveness in the timeless net - computing the reachability graph or using structural methods as invariants etc. (comp. e.g. BEST[1], Desel[6], Starke[9] etc.).

Let $Min(p)$ and $Max(p)$ be the largest elt of all posttransitions of p resp. the smallest lft of all posttransitionen (formal : $Min(p) := max\{eft(t) \mid t \in pF\}$ and $Max(p) := min\{lft(t) \mid t \in pF\}$).

Proposition 1: Let $Z = (P, T, F, V, m_o, I)$ be a TPN with
(1) N^Z is a EFC net,
(2) N^Z is homogeneous,
(3) $Min(p) \leq Max(p)$ for each place $p \in P$,
(4) $\forall t(t \in T \longrightarrow lft(t) \neq 0)$.
Then it holds: if N^z is live, then Z is live, too.

Proof: The **idea** of the proof is: an enabled transition can lose its ability to fire only when at least a token on an input place is fired by another transition. Because of the both properties of the TPN - EFC and the homogeneity - all the enabled transitions with a common place have the same possibility to fire as in the PN during the interval $[Min(p), Max(p)]$. In this interval (eventualy a timepoint only) there are no priorities.

And now, the **proof**. Let t be an arbitrary transition, $z = (m_z, J_z) \in R_Z(z_o)$ be an arbitrary reachable state in Z and the net N^Z be live.
We consider the marking m_z in the net N^Z. Because the net N^Z is live there exist reachable markings $m_1, m_2, ..., m_n \in R_{N^Z}(m_o)$ such that

$$m_z \xrightarrow{t_1} m_1 \xrightarrow{t_2} \ldots \xrightarrow{t_n} m_n \xrightarrow{t=t_{n+1}} (\text{in } N^Z). \qquad (5)$$

We can assume that $\forall_1^{n-1} i(n \geq 2) \quad \forall t(t \in T \longrightarrow (t^- \not\leq m_i \wedge t \not\leq m_z))$.

Now we will show that the sequence of transitions $K_o := (t_1, t_2, ..., t_{n+1})$ can fire in Z, too. Exactly, we try to fire the transition t_1 in the state z, yield the state $z_1 = (m_1, J_1)$; then we try to fire the transition t_2 in the state z_1, yield the state $z_2 = (m_2, J_2)$ etc. until we can fire the transition t_{n+1} in the state z_n.
Eventually, we reach during these firings a state $z_{g-1} = (m_{g-1}, J_{g-1})$, where $z_{g-1} = z$ or $1 < g \leq n+1$ and t_g cannot fire at the state z_{g-1}. Because t_g can fire at the marking

m_{g-1} (in N^Z) it is clear that the "age" $J_{g-1}(t_g)$ of t_g in the state z_{g-1} is not large enough, i.e. $J_{g-1}(t_g) < eft(t_g)$.

When the change from the state z_{g-1} into the state z'_{g-1} by the time duration $\tau = eft(t_g) - J_{g-1}(t_g)$ is possible in Z and therefore t_g becomes "old" enough in order to fire, then we can get a state in Z and t_g can fire in Z and we will see, that the rest of the sequence K_o can still fire in N^Z, too.

When the change from the state z_{g-1} to the state z'_{g-1} by the time duration τ is not possible in Z we can try to change from the state z_{g-1} into the state z''_{g-1} by a time duration $0 < \tau_1 < \tau$. If this change is also not possible, then, in the net Z in the state z_{g-1} there is at least one transition, which has reched its lft. Then we will choose special transitions for firing, in dependence of the case, in order t_g to become "old" enough and may fire, and that the rest of the sequence K_o keeps the ability to fire in N^Z.

However, we will show that every change from a state into another one brings

either	**firing of the "actual" transition in K_o** (i.g. the transition t_g)
or	**the "actual" transition in K_o become "older"**
or	**the number of transitions in a state, which prevent the "growing older" of the "actual" transition in K_o decreases.**

By all these chaneges of states the rest of the not yet fired transitions in K_o is an enabled sequence in the net N^Z (in the marking which belong to the last reched state in Z).

Sketch of a **basic step** in order to fire the transition t_{g-1} in Z:

$$z_{g-1} \xrightarrow{\hat{\sigma}_1} \xrightarrow{\tau_1} \xrightarrow{\hat{\sigma}_2} \xrightarrow{\tau_2} \dots \xrightarrow{\hat{\sigma}_r} \xrightarrow{\tau_r} \hat{z}_{g-1} \xrightarrow{t_g} \quad \text{in } Z,$$

where τ_i are time durations, $0 \leq \tau_i \leq \tau$,
$\hat{\sigma}_i$ are (eventuell the empty) sequence of transitions and
r dependes on the index $g-1$.

The exactly and formal proof is rather difficult and it is given in the appendix.

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Further, we will show that omiting one of the conditions (1), (2), (3) or (4) the proposition becomes false.

Example 3.1.: (omiting condition (1))

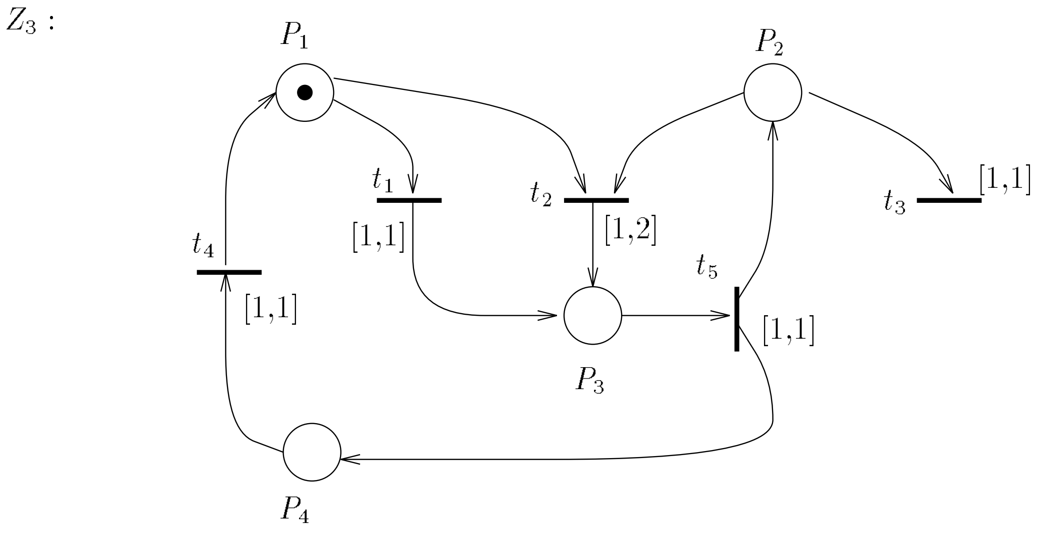

Fig. 3.

Example 3.2.: (omiting condition (2))

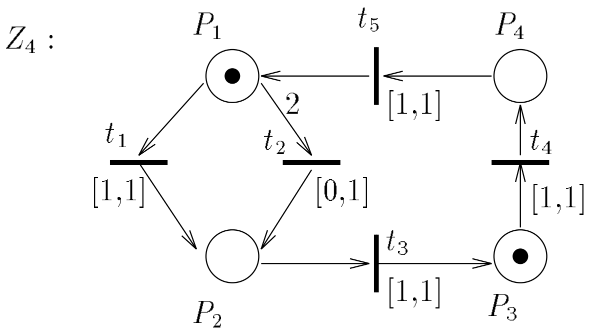

Fig. 4.

Example 3.3.: (omiting condition (3))

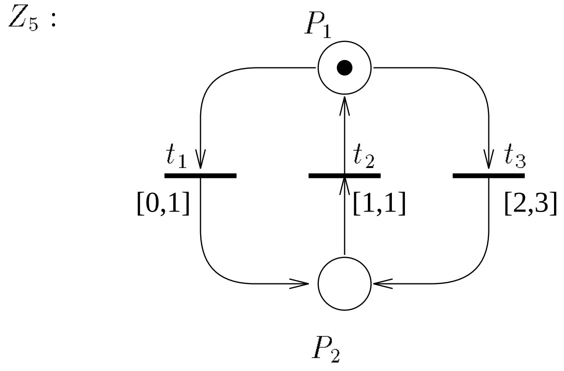

Fig. 5.

Example 3.4.: (omiting condition (4))

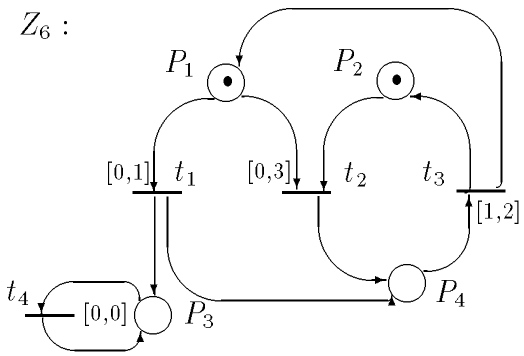

Fig. 6.

4. Dynamic properties for time independence

In the following we discover another net class. In opposite to the one described above, the conditions of this class are not structural but dynamic. The net class from proposition 1 satisfies these conditions, too. Not all nets, which belong to the new class satisfy proposition 1. Nevertheless, we think that this consideration is necessary because of the kind of the conditions and therefore - the different kinds of checking them. The structural properties can be proved on the net directely. In order to proof some dynamic properties of a net we have to compute the reachability graph, in general. Thus, this class separation has to be done.

Definition 4.1.: Let N be a PN. N is called a behaviourally free choice net (BFC net) iff each two transitions which have at least one common place are in each reachable marking either both enabled or both disabled (formal: $\forall m \forall t_1 \forall t_2 (m \in R_N(m_o) \wedge t_1 \in T \wedge t_2 \in T \wedge Ft_1 \cap Ft_2 \neq \emptyset \longrightarrow (t_1^- \leq m \longleftrightarrow t_2^- \leq m)))$.

It is easy to see, that all EFC nets belong to this class, but not each BFC net is a EFC net, e.g. the net N_7 is a BFC net but, obviously, not a EFC:

Example 4.1.:

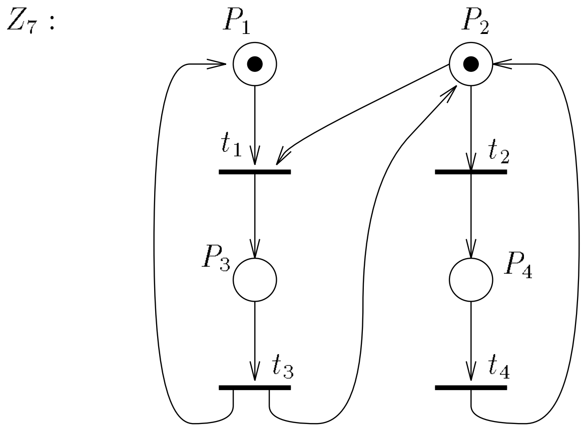

Fig. 7.

For more characterisations about BFC nets comp. BEST[1], STARKE[8].

Here we want to use the follow property of the BFC nets: each BFC net N can be transform to a EFC net N' and N is live if and only if N' is live (comp. BEST[1], STARKE[9], pp.157). For example, N_7 can be transformed to the EFC net N_7':

Example 4.2.:

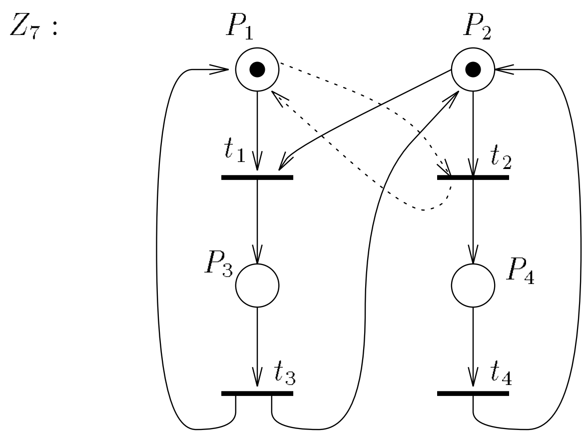

Fig. 8.

Proposition 2: Let $Z = (P, T, F, V, m_o, I)$ be a TPN with
(1) N^Z is a BFC net,
(2) N^Z is homogeneous,
(3) $Min(p) \leq Max(p)$ for each place $p \in P$,
(4) $\forall t(t \in T \longrightarrow lft(t) \neq 0)$.
Then it holds: if N^z is live, then Z is live, too.

Proof: We will proof this assertion in two steps. First we define two new nets: an EFC net N' and a TPN Z'. We will see that N^Z and N' are either both live or both not live. In the second step we show that the net Z and Z' have the same liveness behaviour.

The scheme of the proof is the follows:

Step 1. The net N' arise from the given BFC net N^Z by adding the following arcs: for each common posttransition of the places p and q we add to N a loop with the weight $V(p,t)$ between p and each transition from the set $qF \setminus pF$ and q and each transition from the set $pF \setminus qF$, respectively. It is easy to see that the new net is a EFC net and that the both nets have the same liveness behaviour.

The TPN Z' is defined by the PN N' and the time intervals, given with the net Z. Obviously, Z' differ from Z only in same arcs (because N^Z and N' differ from each other only in same arcs). The TPN Z' satisfies the conditions of the proposition 1. Following, Z' and N' are either both live or both not live. Therefore, Z', N' and N^Z have the same liveness behaviour .

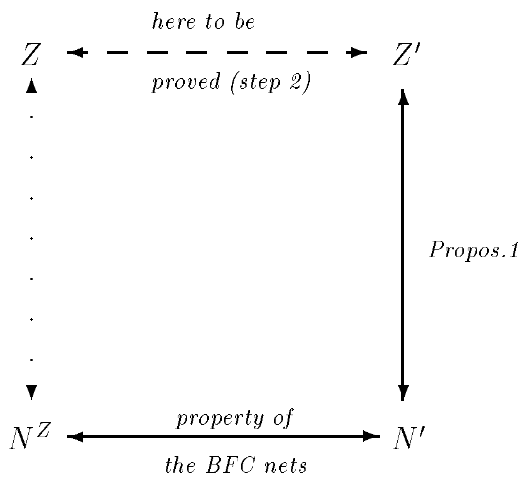

Fig. 9.

Step 2. Z is live if and only if Z' is live.

The **idea** of the proof of this subassertion is the fact that the two nets Z and Z' have the same reachability graph. This is a conclusion, drawn from the property: the two nets have the same incidence matrix and the same time intervals at the corr. transitions, because of the construction of Z'.

And now, we will **prove** by induction the fact, that Z and Z' have the same reachability graph.

($\longrightarrow$) Let $z_o \xrightarrow{\tau_1}\xrightarrow{t_1}\xrightarrow{\tau_2}\xrightarrow{t_2} \quad ... \quad \xrightarrow{\tau_n}\xrightarrow{t_n} z_n$ ($t_i \in T$ and $\tau_i \in \mathcal{Q}o$ for each $i \in \{1,...,n\}$) be a path in the reachability graph of Z with initial state z_o, short $z_o \xrightarrow[\theta]{\sigma} z_n \in RG_Z(z_o)$, where $\sigma = (t_1, ..., t_n)$ and $\theta = (\tau_1, ..., \tau_n)$.

We will show, that this path is a path in the reachability graph of Z', too.

Basis: $ln(\sigma) = 0$. In this case the proof is obvious because of the costructions (z_o is the initial state as well in Z as in Z').

Step: We assume that the assertion is true for $ln(\sigma) = n$ and show that it is true for $n + 1$, too.

Let $z_o \xrightarrow[\theta]{\sigma} z_n \quad \xrightarrow{\tau_{n+1}} \quad z_n' \quad \xrightarrow{t_{n+1}} \quad z_{n+1}$ be a path in $RG_Z(z_o)$ and $ln(\sigma) = n$.

Hence, by the induction hypothesis it holts that $z_o \xrightarrow[\theta]{\sigma} z_n$ is a path in $RG_Z'(z_o)$, too.

Futhermore, when $z_n \xrightarrow{\tau_{n+1}} z'_n$ is possible in Z, then it is

$$J_n(t) + \tau_{n+1} \leq \min_{t^- \leq m_n} lft(t) \qquad (*)$$

in Z.

Because of (1) and (2) (*) in Z if and only if (*) in Z'.

Thus, the state change $z_n \xrightarrow{\tau_{n+1}} z'_n$ is possible in Z', too.

At last we consider the state change $z'_n \xrightarrow{t_{n+1}} z_{n+1}$ in Z.

Because of the construction of Z', the homogeneity of Z and because of N^Z is a BFC net it is easy to see, that

from $\qquad t^-_{n+1} \leq m_n (= m'_n)$ in Z

it follows

$$t^-_{n+1} \leq m_n (= m'_n) \qquad (**)$$

in Z', too.

Since (*) and (**) are true, t_{n+1} may fire in state z'_n in Z'. And, evidently, the new reached state after this firing is z_{n+1}. That means,

$$z_o \underset{\theta}{\overset{\sigma}{\longrightarrow}} z_n \underset{\tau_{n+1}}{\overset{t_{n+1}}{\longrightarrow}} z_n \quad \text{is a path in } RG_{Z'}(z_o), \text{ too.}$$

($\longleftarrow$) This proof can be carried out similarly.

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Conclusions

The TPNs are a convenient and powerful formalism for description time dependent systems. The analysis of TPNs, whatever algorithm we use, is more complicate as checking classical PNs. Furthermore, for computing the reachability graph of a PN (and checking a PN in generale) there is a large variety of different tools, running on different kinds of computers.

Thus, looking for net classes the structural or dynamic properties of which supply indicating the liveness property of a PN to the corr. TPN is very important because of the practical meaning. Using the results shown here check the liveness of same TPNs can be simplify essentially.

Further research in this area is directed to extend the classes given in this paper and to find new classes. We look for structural conditions, above all, because of the simplicity of which. Consideration of certain clusters, which are a generalization of EFC nets bring as to the next class, probably.

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Appendix

Formal proof of the *proposition 1*

Let t be an arbitrary transition, $z = (m_z, J_z) \in R_Z(z_o)$ be an arbitrary reachable state in Z and the net N^Z be live.

We consider the marking m_z in the net N^Z. Because the net N^Z is live there exist reachable markings $m_1, m_2, ..., m_n \in R_{N^Z}(m_o)$ such that

$$m_z \xrightarrow{t_1} m_1 \xrightarrow{t_2} ... \xrightarrow{t_n} m_n \xrightarrow{t=t_{n+1}} (\text{in } N^Z). \qquad (5)$$

We assume that $\forall_1^{n-1} i(n \geq 2)\forall t(t \in T \longrightarrow (t^- \not\leq m_i \wedge t \not\leq m_z))$.

We will show that the sequence of transitions $(t_1, t_2, ..., t_{n+1})$ can fire in Z, too.

And now, we construct exactly and formal the basic step for the different cases:

Let K_o be the sequence of transitions $(t_1, t_2, ..., t_{n+1})$ and assume, that sequence of transitions $(t_1, ..., t_{g-1})$ can fire in Z in the state z:

$$z \xrightarrow{t_1} z_1 \xrightarrow{t_2} ... \xrightarrow{t_{g-1}} z_{g-1}$$

but t_g cannot. It holds for the marking m_{g-1}:

$$m_{g-1} = m_z + \Delta t_1 + ... + \Delta t_{g-1}.$$

Since the transition t_g is enabled in the PN N^Z by m_{g-1}, it holds :

$$t_g^- \leq m_{g-1}.$$

Following, t_g cannot fire in Z now, because t_g is not old enough in the net Z in the state z_{g-1}, i.e.

$$eft(t_g) > J_{g-1}(t_g).$$

Let $\tau = eft(t_g) - J_{g-1}(t_g)$ be and $K_{g-1} := K_o \setminus (t_1, ..., t_{g-1}) = (t_g, ..., t_{n+1})$ be.

Case 1.: $z_{g-1} \xrightarrow{\tau} z'_{g-1}$ *in* Z *is possible.*

Obviously, in the state $z'_{g-1} = (m_{g-1}, J'_{g-1})$ the transition t_g is enabled and it has reached its eft, i.e. t_g may fire in z'_{g-1} in Z. Then, it follows for the sequence of transitions $(t_{g+1}, ..., t_{n+1})$:

$$m'_g = m_g \xrightarrow{t_{g+1}\cdots t_{n+1}} \text{ in } N^Z \text{ is possible.}$$

Case 2.: $z_{g-1} \xrightarrow{\tau}$ *in* Z *is not possible.*

Let τ_1 be a time duration, smaller then τ, i.e. $0 \leq \tau_1 < \tau$, and we consider a possible state change by the maximal large time duration τ_1 with:

$$z_{g-1} \xrightarrow{\tau_1} \text{ in } Z \text{ is possible.}$$

Case 2.1.: $\tau_1 = 0$.

This means that a change from the state z_{g-1} into another one by time duration in Z is not possible. Following, at least one transition has reached its lft in the state z_{g-1}. Let

$$M_g^{(1)} := \{t \mid t \in T \wedge t^- \leq m_{g-1} \wedge J_{g-1}(t) = lft(t)\}$$

be the set of all transitions which have reached its lft in the state z_{g-1}. It is clear that $t_g \notin M_g^{(1)}$ (otherwise t_g would be able to fire in z_{g-1}).

Furthermore we consider the common inputplaces of all transitions in $M_g^{(1)}$ (the set of all inputplaces of $M_g^{(1)}$ is $FM_g^{(1)}$) and the inputplaces of all transitions belonging to the sequence $K_g^{(1)} := K_g = (t_{g+1}, ..., t_{n+1})$ (the set of all inputplaces of $K_g^{(1)}$ is $FK_g^{(1)}$): $FM_g^{(1)} \cap FK_g^{(1)}$.

Case 2.1.1.: $FM_g^{(1)} \cap FK_g^{(1)} = \emptyset$.

Then let $\hat{t}_1 \in M_g^{(1)}$ be an arbitrary transition which has reached its lft in the state z_{g-1}, i.e. it holds:

$$J_{g-1}(\hat{t}_1) = lft(\hat{t}_1).$$

Thus, $\hat{t}_1$ is the first transition in the sequence $\hat{\sigma}_1$. We have:

$$z_{g-1} \xrightarrow{\hat{t}_1} \hat{z}_{g-1}^{(1)} \text{ in } Z \text{ is possible}$$

and it holds for the state $\hat{z}_{g-1}^{(1)} := (\hat{m}_{g-1}^{(1)}, \hat{J}_{g-1}^{(1)}) \qquad \hat{m}_{g-1}^{(1)} = m_{g-1} + \Delta\hat{t}_1$. Because of the fact (in this case) that $FM_g^{(1)}) \cap FK_g^{(1)} = \emptyset$ and conditions (1),(2) and (3), by the firing of the transition $\hat{t}_1$ the number of tokens in the inputplaces of all transitions in $K_g^{(1)}$ is not decreased, i.e.

$$m_{g-1}(FK_g^{(1)}) \leq \hat{m}_{g-1}^{(1)}(FK_g^{(1)}). \tag{6}$$

Futhermore, t_g and $\hat{t}_1$ have not got a common inputplace, then assuming the opposite, t_g should be abled to fire in the state z_{g-1} because of the conditions (1),(2) and

(3). Following, it is true:

$$m_{g-1}(Ft_g) \leq \hat{m}^{(1)}_{g-1}(Ft_g). \qquad (7)$$

.

(5),(6) and (7) lead to the fact:

$$\hat{m}^{(1)}_{g-1} \xrightarrow{t_g K_g^{(1)}} \text{ in } N^Z \text{ is possible.}$$

(formal: $\forall_n^g i \forall p (p \in Ft_{i+1} \longrightarrow t_{i+1}^{-}(p) \leq m_{i-1}(p) = m_{g-1}(p) + \Delta t_g(p) + ... + \Delta t_i(p) \leq \hat{m}^{(1)}_{g-1}(p) + \Delta t_g(p) + ... + \Delta t_i(p))$).

Thus, we have reached a state $z_{\hat{g-1}}$ in which the number of transitions peventing the firing of t_g is smaller as in the state z_{g-1} and it is true:

$$\hat{m}_{g-1} \xrightarrow{t_g K_g^{(1)}} \text{ in } N^Z.$$

Case 2.1.2.: $FM_g^{(1)} \cap FK_g^{(1)} \neq \emptyset$.

Now let $\hat{t}_1$ be the transition in $K_g^{(1)}$, which has at least one common inputplace with at least one transition from $M_g^{(1)}$ and its index in the sequence of transitions $K_g^{(1)}$ is minimal:

$$\hat{t}_1 := t_j \text{ and } j := \min\{r \mid t_r \in K_g^{(1)} \quad \wedge \quad FM_g^{(1)} \cap Ft_r \neq \emptyset\}$$

.

Because of the transitions (1), (2) and (3) it is easy to see, that

$$z_{g-1} \xrightarrow{\hat{t}_1} \hat{z}^{(1)}_{g-1} \text{ in } Z \text{ is possible.}$$

Obviously, for all transitions $t_x \in K_g^{(1)}$ with $g < x < j$ (8)

it holds:

$$F\hat{t}_1 \cap Ft_x = \emptyset$$

because of the choice of $\hat{t}_1$. Furthermore, it is true

$$F\hat{t}_1 \cap Ft_g = \emptyset$$

because of (1), (2) and (3). This means, all the transitions between t_g and t_j can fire in N^Z after t_j, too (similar considerations as in case 2.1.1.).

What is the situation with the transitions t_s in the sequence $K_g^{(1)}$ with $s > j$?

Case 2.1.2.1.: $F\hat{t}_1 \cap Ft_s = \emptyset$

Similar considerations as in the previous case (see (8)) can be done here.

Case 2.1.2.2.: $F\hat{t}_1 \cap Ft_s \neq \emptyset$

After firing of $\hat{t_1}$=t_j the transition t_s becomes disabled, because of the conditions (1) and (2). (If in the inputplaces of t_s there are enough tokens, so that t_s is still enabled after firing of t_j, we have the same situation as in the case 2.1.2.1.) Since we have the facts $t_j, t_s \in K_g^{(1)}$ and (5), t_s can become enabled in N^Z. Then the marking reached after firing $t_j, t_g, ..., t_{j-1}$ is the same as the marking reached after firing $t_g, ..., t_j$ started in the marking m_{g-1} (in the net N^Z). This is the marking m_j.

(formal: Since (5), $\hat{m}_{g-1}^{(1)} = m_{g-1} + \Delta\hat{t}_1 = m_{g-1} + \Delta t_j$ and $m_j \xrightarrow{t_{j+1}...t_s}$ in N^Z are true , it follows:

$$\hat{m}_{g-1}^{(1)} \xrightarrow{t_g...t_{j-1}t_{j+1}...t_s} \text{ in } N^Z$$

is true, too.)

Thus, we have decrease the number of transitions preventing the firing of t_g because of its lft and the sequence of transitions $K_g^{(2)} := K_g^{(1)} \setminus (\hat{t}_1)$ can fire in N^Z.

Let $M_g^{(2)} := \{t \mid t^- \leq \hat{m}_{g-1}^{(1)} \wedge \hat{J}_{g-1}^{(1)}(t) = lft(t)\}$ be. Because of (4) and the fact, that firing a transition takes no time it is true

$$M_g^{(1)} \supset M_g^{(2)}.$$

Therefore, after finite number of firings (exactly, after $ln(\hat{\sigma}_1)$ firings) we reach the state $z_{g-1}^{(ln(\hat{\sigma}_1))}$ in which no transition has reached its lft (i.e. we have

$$M_g^{(1)} \supset M_g^{(2)} ... \supset M_g^{(ln(\hat{\sigma}_1)-1)} \text{ and } M_g^{(ln(\hat{\sigma}_1))} = \emptyset.)$$

It is clear, that in the state $z_{g-1}^{(ln(\hat{\sigma}_1))}$ the transition , t_g is enabled, but still not "old enough". Now we can change from this state into another one by a time duration $\tau_1 > 0$ (e.g. $\tau_1 = \min\{lft(t) - J_{g-1}^{(ln(\hat{\sigma}_1))}(t) \mid t^- \leq m_{g-1}^{(ln(\hat{\sigma}_1))}\}$). If $\tau_1 = \tau$, so in the last reached state t_g can fire. If $\tau_1 < \tau$ we can repeat all the considerations starting in the state, reached after the state change by the time duration τ_1. Thus, after ν such repeats we have the situation:

$$z_{g-1} \xrightarrow{\hat{\sigma}_1\tau_1...\hat{\sigma}_\nu\tau_\nu} \text{ in } Z$$

and each τ_i is not 0. Therefore, there exist a finite number r of time durations τ_i, so that

$$\tau_1 + ... + \tau_r = \tau.$$

It is clear that after the last change of states by the time duration τ_r the transition t_g can fire in Z. Let the yielded state be $\overline{z}_g = (\overline{m}_g, \overline{J}_g)$ (obvisiously, $\overline{m}_g \in R_{NZ}(m_z)$).

With those we have finished the constraction of a basic step and we have the follow situation: let $K_{g+1}^{(o)} := K_g^{ln(\hat{\sigma}_1 ... \hat{\sigma}_r)} \backslash$ (last trans. in the sequence $\hat{\sigma}_r$) be. The last sequence of transisions includes all transitions of $K_g^{(1)}$ exept these, which have already fired, i.e. which belong at least to one of $\sigma_i, 1 \leq i \leq r$. Let $\tilde{t}$ be the first transition in the sequence of transitions $K_{g+1}^{(o)}$. Now we define a new sequence of transitions $K_{g+1}^{(1)}$ implicit : $K_{g+1}^{(o)} = \tilde{t}K_{g+1}^{(1)}$. It is true:

$$\overline{m}_g \xrightarrow{\tilde{t}K_{g+1}^{(1)}} \text{ in } N^Z \text{ is possible}$$

and the number of not yet fired transitions in the sequence K_o is decreased (comp. $K_{g-1}^{(o)}$ and $K_g^{(1)}$).

■