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On Integration of Qualitative and Quantitative Analysis of Manufacturing Systems Using Petri Nets

1 Motivation

Among those methods, which aim at the improvement of the dependability of any system, different kinds of Petri net based validation techniques to avoid faults during the development phase have attracted a lot of attention in the last two decades. Within this general framework, various Petri net based methodologies of dependability engineering have been proposed. At the beginning, only qualitative (i. e. time-less) properties have been discussed. But because of the crucial impact of performance on parallel or distributed (shortly called concurrent) systems, special emphasis has been laid more and more on incorporation of quantitative (i. e. time-dependent) criteria as part of the system development cycle [Heiner 94].

Therefore, maybe the most important advantage of the Petri net approach to dependability engineering is its ability to combine qualitative analysis, monitoring and testing as well as quantitative analysis (in terms of time-dependent safety and worst-case analysis, performance/reliability prediction) on the basis of a common Petri net-based intermediate representation of the concurrent system under development. In [Heiner 95], a method is demonstrated how to develop at first qualitative models as place/transition nets suitable for analysis of general and special qualitative properties. Afterwards, the validated qualitative models are transformed step-by-step by quantitative expansion (e. g. time consumption of atomic system activities, branching probabilities) and property-preserving structural compression into (different kinds of) quantitative models for time-dependent analyses.

However, it is well-known in Petri net theory that there is generally no strong relation between the properties of qualitative and corresponding quantitative Petri net models due to the possible influence of time on the total net behaviour. That holds for any type of time-dependent Petri net imposing additional time constraints on any conflict decisions. Therefore, in order to sequentialize qualitative and quantitative analysis one after the other, an important feature of a related systematic methodology is the ability of precise preservation of any analysis results gained during qualitative validation steps.

Let’s consider an example [Heiner 97a]. For practical reasons, it does make sense to split safety analysis of manufacturing systems into two steps. At first, safety properties expressible in terms of “legal” local states are analyzed on the basis of qualitative Petri nets. Afterwards, safety properties in terms of explicit error states and explicit error transitions into them have to be discussed. E.g. we want to show that a machine motion is stopped fast enough, before the machine has been moved beyond an upper limit. Obviously, we need now a notion of time in order to describe that something happens fast enough. To describe and analyze the unreachability of such explicit error states (or equally - that the explicit error transitions are facts, i. e. dead at the initial marking), we need the preemptive firing principle (race model) - as e.g. applied in interval Petri nets (usually called time nets [Merlin 74], [Popova 91]). But of course, time-dependent safety proofs should preserve any time-independent safety proofs done before.

For that purpose, we are going to summarize and explain available results dealing with the influence of the race models’ interval time on general qualitative properties of Petri nets, boundedness as well as liveness. This discussion covers implicitly all time-dependent net types, which can be simulated by interval Petri nets, e. g. duration Petri nets (usually called timed nets [Ramchandani 74], [Starke 95]), and duration interval Petri nets [Heiner 97b].

The rest of the paper is organized as follows. In the next section the prerequisites are recalled. Thereafter, we are ready to discuss reasonably available results. Finally, we give some suggestions to further improvements.

2 Mathematical Background

We will use the following notations. $\boldsymbol{N}$ denotes the set of natural numbers, and $\boldsymbol{Q_0}^+$ is the set of nonnegative rational numbers.

Definition 1:

The structure $PN = (P, T, F, m_0)$ is called **Petri Net**, where:

(1) P, T are finite sets, and F is a mapping
F: $(P \times T) \cup (T \times P) \to \boldsymbol{N}$, indicating arc weights.
We define $X := P \cup T$, and we assume
$P \cap T = \emptyset$, $P \cup T \neq \emptyset$, and
$\forall x \in X : \exists y \in X : F(x, y) \neq 0 \vee F(y, x) \neq 0$

(2) m_0: $P \to \boldsymbol{N}$ (initial marking)

The pre- and postsets of a transition t resp. of a place p are given by
$Ft := \{p | p \in P \wedge F(p, t) \neq 0\}$ and $tF := \{p | p \in P \wedge F(t, p) \neq 0\}$ resp.
$Fp := \{t | t \in T \wedge F(t, p) \neq 0\}$ and $pF := \{t | t \in T \wedge F(p, t) \neq 0\}$.
Each transition $t \in T$ induces the marking t^- and t^+, defined as $t^-(p) := F(p, t)$ and $t^+(p) := F(t, p)$. By Δt we denote $t^+ - t^-$. A transition $t \in T$ is enabled (may fire) at a marking m iff $t^- \leq m$ (i.e. $t^-(p) \leq m(p)$ for each place $p \in P$). When an enabled transition t at a marking m fires, a new marking m' given by $m'(p) := m(p) + \Delta t(p)$ is reached. The set of markings reachable from a given marking m of PN is denoted by $\boldsymbol{R}_N(m)$.

Two transitions t_1, t_2 are in a **static conflict**, if they share preplaces, i.e. $Ft_1 \cap Ft_2 \neq \emptyset$. Two transitions are in a **dynamic conflict** at the marking m, if both transitions are enabled at m, but the firing of one transition disables the other one.

Definition 2:

The structure $IPN = (P, T, F, m_0, I)$ is called **Interval Petri Net** where:

(1) $S(IPN) = (P, T, F, m_0)$ is a Petri net (Skeleton of IPN),

(2) I: $T \to \boldsymbol{Q_0}^+ \times (\boldsymbol{Q_0}^+ \cup \{\infty\})$ and for each $t \in T$ holds $eft(t) \leq lft(t)$, where
$I(t) = (eft(t), lft(t))$.

(The lower interval bound is called earliest firing time, and the upper interval bound is called latest firing time.)

The behaviour of an interval Petri net can be shortly described as follows: If a transition t is enabled at time τ, then it must not fire before time $\tau + eft(t)$. Afterwards, the transition may fire at any time point in the time interval $[\tau + eft(t), \tau + lft(t)]$, provided it is still enabled (race model). At latest at time $\tau + lft(t)$ the transition has to fire. The firing itself does not consume any time.

Definition 3:

A Petri net is called an **homogeneous net**, if all outgoing arcs of each place have identical arc weights, i. e. it holds for each place p of P: $t_1, t_2 \in pF \Rightarrow F(p, t_1) = F(p, t_2)$.

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Definition 4:

An interval Petri net is called a **timely homogeneous net**, if any transitions in static conflict have overlapping time windows, i. e. it holds for each place p of P:
$t_1, t_2 \in pF \Rightarrow eft(t_1) \leq eft(t_2) \leq lft(t_1) \vee eft(t_2) \leq eft(t_1) \leq lft(t_2)$.

Definition 5:

A Petri net is called an **extended free choice net** (EFC net), if the posttransitions of shared places have the same sets of preplaces, i. e. it holds for each place p of P:
$t_1, t_2 \in pF \Rightarrow Ft_1 = Ft_2$.

Definition 6:

A Petri net is called a **behaviourally free choice net** (BFC net), if each two transitions which have at least one place in common are at each reachable marking either both enabled or both disabled, i. e.:
$\forall m \forall t_1 \forall t_2 (m \in \boldsymbol{R}_N(m_0) \wedge t_1 \in T \wedge t_2 \in T \wedge Ft_1 \cap Ft_2 \neq \emptyset \Rightarrow (t^-_1 \leq m \Leftrightarrow t^-_2 \leq m))$.

3 Results

As a rule, there is no strong relation between the general behavioural properties - such as (un-) boundedness and (non-) liveness - of a given interval Petri net *ipn* and its skeleton *pn*, i. e.

$$prop(pn) \not\Leftrightarrow prop(ipn) .$$

Therefore, there is regularly no justification to transfer the qualitative analysis results to quantitative models. The reason why these properties may change with or without time assumptions consists basically in the fact that time imposes additional constraints on the net behaviour. Conflicts which are realizable under time-less conditions may disappear by the influence of time, resulting into less reachable markings, i. e. it holds

$$\boldsymbol{R}_N(pn) \supseteq \boldsymbol{R}_N(ipn) .$$

Actually, we are interested only in conclusions from left to right, i. e. of the type

$$prop(pn) \Rightarrow prop(ipn) ,$$

according our application background. Due to the subset relation of reachable markings, there are also some time-insensitive qualitative properties. Evidently, boundedness (*BND*), nonexistence of dead states ($\neg DSt$), and existence of dead transitions (*DTr*) are preserved under any timing. So, it holds

(a) $BND(pn) \Rightarrow BND(ipn)$,
(b) $\neg DSt(pn) \Rightarrow \neg DSt(ipn)$, and
(c) $DTr(pn) \Rightarrow DTr(ipn)$.

In opposite to that, there are the following time-sensitive qualitative properties:

(d) an unbounded Petri net may become bounded,
(e) dead states of a Petri net may disappear, and
(f) a Petri net without dead transitions may pick up dead transitions

by the influence of time (for examples see [Popova 94]). Situations (d) and (e) happen if the time constraints cut all those branches of the reachable behaviour resulting into unboundedness or dead states, respectively. Because it is not advisable to rely on suitable timing relations to get positive properties, qualitative analysis should prove them generally. Afterwards, we take advantage of (a) and (b).

Altogether, only situation (f) contradicts to a systematic stepwise analysis procedure starting with qualitative analysis as a necessary precondition for going ahead to quantitative analysis. A Petri net proven to be live may lose its liveness if it is considered as interval Petri net. Therefore the very interesting question arises if there are net classes which remain live under any timing, i. e. which are **time-independently live**. Because of the general considerations above, it is straightforward that persistent (dynamic conflict free) Petri nets are such a class. Moreover, it should be possible to generalize this insight to net classes allowing only some standardized conflict sceneries.

In particular, we can characterize four classes of interval Petri nets preserving liveness (and unboundedness). Three of these classes are defined by structural restrictions, and therefore they are easy and less expensive to be checked. For two classes we can give necessary and sufficient conditions. The other two classes serve only as sufficient conditions, but this is enough for our purpose to get justification for reusing qualitative analysis results during quantitative analysis.

The first class contains all interval Petri nets where all transitions have earliest firing times of zero, i. e. transitions may (but do not have to) fire immediately.

Property 1:

Let *IPN* be an interval Petri net with $\forall t\,(t \in T \Rightarrow eft\,(t) = 0)$, and let *S*(*IPN*) be the skeleton of *IPN*. Then it holds:

(1) *IPN* is unbounded if and only if *S*(*IPN*) is unbounded, and

(2) *IPN* is live if and only if *S*(*IPN*) is live.

The second class contains all interval Petri nets where all transitions have infinite latest firing times, i. e. transitions are not forced to fire in finite time.

Property 2:

Let *IPN* be an interval Petri net with $\forall t\,(t \in T \Rightarrow lft\,(t) = \infty)$, and let *S*(*IPN*) be the skeleton of *IPN*. Then it holds:

(1) *IPN* is unbounded if and only if *S*(*IPN*) is unbounded, and

(2) *IPN* is live if and only if *S*(*IPN*) is live.

The proofs of property 1 and property 2 are outlined in [Popova 95].

The third class contains all interval Petri nets where any transitions in static conflict are equally restricted. In opposite to the upper both properties, this property provides (only) a sufficient condition for time-independent liveness.

Property 3:

Let *IPN* be an interval Petri net with

(1) *S*(*IPN*) is an EFC net,

(2) *S*(*IPN*) is homogeneous,

(3) *S*(*IPN*) is timely homogeneous,

(4) $\forall t\,(t \in T \Rightarrow lft\,(t) \neq 0)$.

Then it holds: if *S*(*IPN*) is live, then *IPN* is live, too.

Condition (4) means that there are no transitions which can only fire immediately (shortly called purely immediate transitions).

While the first three classes have been characterized by structural conditions, the last class is given by a dynamic condition (compare [Best 87], [Starke 90]). It contains all interval Petri nets where any transitions in dynamic conflict are equally restricted. This property provides again a sufficient condition for time-independent liveness.

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Property 4:

Let *IPN* be an interval Petri net with

(1) *S*(*IPN*) is a BFC net,

(2) *S*(*IPN*) is homogeneous,

(3) *S*(*IPN*) is timely homogeneous,

(4) $\forall t\,(t \in T \Rightarrow lft\,(t) \neq 0)$.

Then it holds: if *S*(*IPN*) is live, then *IPN* is live, too.

The proofs of property 3 and property 4 are outlined in [Popova 94].

Obviously, each net satisfying property 3 satisfies also property 4, but not vice versa. So, property 4 is less restrictive, but harder to check. While structural properties (like properties 1 - 3) can be proven directly on the net, dynamic properties (like property 4, (1)) need generally the computation of a suitable data base representing all possible behaviours (like reachability graph or finite prefix of a branching process), or at least all possible behaviours relevant for the properties to be proven (e. g. stubborn set reduction techniques may be used by a LTL model checker to decide whether a given net is a behaviourally free choice one). Nevertheless, property 4 should be practically decidable in many cases by applying a suitable analysis tool kit.

Using these results presented above, we are now able to summarize in general terms the interrelations (as known up to now) between qualitative and quantitative analyses in case of using interval Petri nets:

- An unbounded Petri net remains unbounded under any timing,
  if the earliest firing time of all transitions is zero, **or**
  if the latest firing time of all transitions is infinite.
- A bounded Petri nets remains bounded under any timing.
- A live Petri net remains live under any timing,
  if it is persistent, **or**
  if the earliest firing time of all transitions is zero, **or**
  if the latest firing time of all transitions is infinite, **or**
  if it is an homogeneous and timely homogeneous
  extended free choice net **or**
  behaviourally free choice net
  without purely immediate transitions.

4 Final Remarks

For technical reasons, some of the above conditions are apparently too restrictive. E. g. conditions (3) and (4) of property 4 might be replaced by dynamically equivalent ones.

Results similar to property 3 have been presented for Generalized Stochastic Petri Nets (GSPN) and Queueing Petri Nets (QPN) in [Bause 96]. Moreover, it has been proven for duration Petri nets that even extended simple nets are time-independently live [Starke 90]. It would be worth reasoning whether this more general result might be transferred to other time-dependent net types.

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