Essential States in Time Petri Nets

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Abstract

This paper deals with studying the behaviour of Time Petri nets in a discrete way. It is shown that although a continuous time model is used here, only a few time points are important for the determination the behaviour of the net. The computation of the boundedness is shown as a recursive-enumerative process. Qu alitative and quantitative analysis can be done using only the essential states of the net.

1 Introduction

Time dependent Petri nets were first defined more then two decades ago and have been investigated and used for the modeling and analyzing of concurrent systems. Since then their variety has been permanently increasing according to many different possibilities of associating time with the individual net elements (places, transitions, arcs, tokens), to the different interpretations of time (as durations or intervals) and to whether times are static or stochastic.

In general, the time dependent Petri nets are more powerful than the classical Petri nets - they are equivalent to Turing machines, and most of them (except the time dependent nets with stochastically defined time) can be simulated by Time Petri nets, defined by Merlin [7].

We use the time in order to define possible starting points and deadlines for the actions of a given system. Within their time restrictions, the actions can occur concurrently. This kind of nets was introduced by Merlin [7] and studied by Berthomieu/Menasche [4], Berthomieu/Diaz [3], Popova [9], [8], Berthelot/Boucheneb [2], [5], Aalst [1] etc.

Time Petri nets (in the sense of Merlin) are classical Petri nets where to each transition t a time interval $[a_t, b_t]$ is associated. The times a_t and b_t are relative to the moment at which t was last enabled. When t becomes enabled it can not fire before a_t time units have elapsed and it has to fire not later than after b_t time units unless it is disabled by the firing of another transition. Firing a transition takes no time. Thus, the philosophy of this kind of time dependent Petri nets is: when a transition becomes enabled it may (in general) not fire at once but during a certain time interval, and at the end of that interval, firing is compulsory. If the end of the interval is infinite then the second characteristic of this kind of net, the obligation to fire disappears. In Popova [9] and Popova [8] we considered only time intervals, whose upper bounds are finite numbers ($b_t < \infty$ for each transition t).

For analyzing Time Petri nets there are two different methods: one has been defined by Berthomieu/Diaz/Menasche and the other has been defined by Popova. They both consist of the computation of a reachability graph of the Time Petri net but the two methods have different backgrounds.

By Berthomieu/Diaz/Menasche the notion "state" is used to describe (infinitely) many situations in Time Petri nets; it is defined by a marking and time intervals for the enabled transitions (comp. Berthomieu/ Menasche [4], Berthomieu/Diaz [3]) while in Popova [9] a state is a description of a snapshot of the net. A state is here given by a marking and a time vector of the current local time for the enabled transitions and a special symbol ($\sharp$) for the disabled transitions.

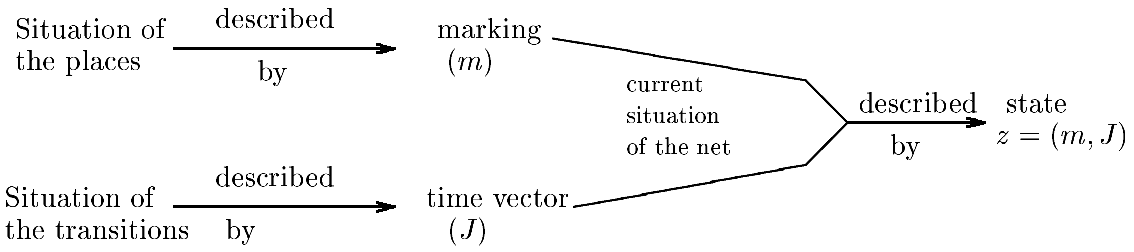


Figure 1: *Meaning and structure of a state in sense of Popova*

Therefore we develop a completely different method for building a reachability graph for a Time Petri net: a node of the Berthomieu/Diaz/ Menasche's reachability graph (a state class (comp. Berthomieu/ Menasche [4] , Berthomieu/Diaz [3]) includes an infinite number of possible situations of the Time Petri net and is given as a solution of a system of inequalities.The nodes of our reachability graph (comp. Popova [8]) are only the "essential" states. These are the integer-states where the current local time for all enabled transitions are integers. For example, $z = (\underbrace{(1,0,3)}_{marking}, \underbrace{(3,\sharp,0,\sharp)}_{timevec.})$ can be a state in a Time Petri net with 3 places and 4 transitions. The first and the third transitions are enabled. The first transition has been enabled for 3 time units, and the third transition has just become enabled. The second and the fourth transitions are disabled. Interpretation of the notion "state" is as follows: in the net, each transition t has a clock. If t is enabled at m, the clock of t shows the time elapsed since t was last enabled. The clock does not work (indicated by $\#$) at the marking m, if t is disabled at m. Obviously, this reachability graph includes only a discrete part of all the situations, which are possible for a given Time Petri Net. But the knowledge of the net behaviour in the integer-states is sufficient to determine the entire behaviour of the net (comp. Popova [9]). Computing the integer-states is easy.

By using this notion of state the behaviour of the Time Petri nets can be studied with the help of BDDs, simular to the classiocal Petri nets.

The reachability graph defined by Boucheneb/Berthelot is quite similar to the reachability graph defined by Berthomieu/Diaz/Menasche, especially with respect to the computation of the graph nodes: one has to solve a system of inequalities.

In general, there are no relations between the liveness behaviour of a given Time Petri net and the corr. timeless Petri net. Furthermore, for a bounded Time Petri net the corr. Petri net without time restrictions can be unbounded. An exception to this is formed by, at least, four different classes of Time Petri nets (comp. Popova [9] and Popova [8]).

In this paper we study the behaviour- both qualitative and quantitative - of Time Petri nets whose upper bounds b_t (t is a transition) are not necessarily finite using discrete methods.

For this purpose, we consistently extend the discrete algorithm for computing a reachability graph to Time Petri nets, given in Popova [8], where for all transitions the obligation to fire holds.

Again, the reachability graph contains all "essential" states. Which are they now? These are all those reachable states, in which the clock of each transition shows an integer time: for transitions with finite intervals, all reachable times, and for transitions with infinite intervals, all reachable times up to the corr. upper bounds.

This paper is organized as follows. In section 2 finite Time-Petri-Nets are defined and some basic properties, important for the proofs in section 3, are discussed. The existence of essential states in Time-Petri-Nets, not necessarily finite ones, is shown in section 3. Furthermore, it is proved that the knowledge of the net behaviour in the essential states is enough for knowing the whole net behaviour at **every** time. Finally, conclusions and outlook on further work will be given in section 4.

2 Notions and basic definitions

We will use the following notations: $\mathcal{N}$ denotes the set of natural numbers, $\mathcal{N}^+$ stands for $\mathcal{N} \setminus \{ 0\}$. $\mathcal{Q}_0$ is the set of nonnegative rational numbers. Let g be a given function from A to B. Then, the symbol # defines a special value from the set $g(A)$. T^* denotes the language of all words over the alphabet T, including the empty word e, $l(w)$ stands for length of the word w.

Definition 2.1 *The structure $N = (P, T, F, V, m_o)$ is called Petri net (PN), iff*
(1) P, T, F are finite sets with
$P \cap T = \emptyset$, $\quad P \cup T \neq \emptyset$, $\quad F \subseteq (P \times T) \cup (T \times P)$ *and*
$dom(F) \cup cod(F) = P \cup T$ *(net)*
(2) $V : F \longrightarrow \mathcal{N}^+$ (weight of the arcs)
(3) $m_o : P \longrightarrow \mathcal{N}$ (initial marking)

A marking of a PN is a function $m : P \longrightarrow \mathcal{N}$, where $m(p)$ denotes the number of tokens at place p. The pre- and postsets of a transition t resp. of a place p are given by $Ft := \{p \mid p \in P \wedge pFt\}$ and $tF := \{p \mid p \in P \wedge tFp\}$ resp. Fp and pF. Each transition $t \in T$ induces the marking t^- and t^+, defined as follows:

$$t^- = \begin{cases} V(p,t) & \text{,iff} \quad (p,t) \in F \\ 0 & \text{,iff} \quad (p,t) \notin F \end{cases} \qquad t^+ = \begin{cases} V(t,p) & \text{,iff} \quad (t,p) \in F \\ 0 & \text{,iff} \quad (t,p) \notin F \end{cases}$$

By Δt we denote $t^+ - t^-$. A transition $t \in T$ is enabled (may fire) at a marking m iff $t^- \leq m$ (i.g. $t^-(p) \leq m(p)$ for each place $p \in P$). When an enabled transition t at a marking m fires, there yields a new marking m' given by $m'(p) := m'(p) + \Delta t(p)$, denotes by $m \xrightarrow{t} m'$.

Definition 2.2 *The structure $Z = (P, T, F, V, m_o, I)$ is called a Time Petri Net (TPN) iff:*
(1) $S(Z) = (P, T, F, V, m_o)$ is a PN.
(2) $I : T \longrightarrow \mathcal{Q}_0 \times (\mathcal{Q}_0 \cup \{\infty\})$ and for each $t \in T$ is $I_1(t) \leq I_2(t)$, where $I(t) = (I_1(t), I_2(t))$.
The TPN Z is called FTPN, iff $I : T \longrightarrow \mathcal{Q}_0 \times \mathcal{Q}_0$.

I is called the time function of Z, $I_1(t)$ and $I_2(t)$ earliest firing time of t ($eft(t)$) and latest firing time of t ($lft(t)$), respectively. It is not difficult to see (comp. Popova[8]) that considerations of TPN with $I : T \longrightarrow \mathcal{N} \times (\mathcal{N} \cup \{\infty\})$ is not a restriction in general. In the following we always consider such time functions I. Furthermore, we use here the strong meaning of conflict: two transitions t_1 and t_2 are in conflict iff $Ft_1 \cap Ft_2 \neq \emptyset$. The PN $S(Z)$ is called the skeleton of Z.

The notion "state" is here a basic one and it is given by marking and the momentary local time for the enabled transitions and the sign $\sharp$ for the disabled transitions. Thus, now we define "state" and "change from a state into another state" by firing a transition and by time duration.

Definition 2.3 *Let $Z = (P, T, F, V, m_o, I)$ be a TPN and $J : T \longrightarrow \mathcal{Q}_0 \cup \{\#\}$. $z = (m, J)$ is called a state in Z iff:*
(1) m is a reachable marking in $S(Z)$.
(2) $\forall t (t \in T \wedge t^- \leq m \longrightarrow J(t) \leq lft(t))$.
(3) $\forall t (t \in T \wedge t^- \not\leq m \longrightarrow J(t) = \#)$.

An interpretation of the notion "state" is as follow: in the net each transition t has a clock. The clock does not work ($J(t) = \#$) at the marking m if t is disabled at m. If t is enabled at m, the clock of t shows the time ($J(t)$), elapsed since t was enabled last.

The state $z_o := (m_o, J_o)$ with $J_o(t) := \begin{cases} 0 & \text{,iff} \quad t \leq m_0 \\ \# & \text{,iff} \quad t \not\leq m_0 \end{cases}$ is set to be the initial state of the TPN Z. A transition t is ready to fire in the state $z = (m, J)$, denote by $z \xrightarrow{t}$, iff $t^- \leq m$ and $eft(t) \leq J(t)$. A transition $\hat{t}$, being ready to fire in the state $z = (m, J)$ may fire yielding a new state $z' = (m', J')$ defined by $m' = m + \Delta\hat{t}$ and

$$J'(t) =: \begin{cases} \# & \text{,iff} \quad t^- \not\leq m \\ J(t) & \text{,iff} \quad t^- \leq m \wedge t^- \leq m' \wedge Ft \cap F\hat{t} = \emptyset \\ 0 & \text{,iff} \quad \text{otherwise} \end{cases} .$$

The state $z = (m, J)$ changes into the state $z' = (m', J')$ by the time progress $\tau \in \mathcal{Q}^+$, denoted by $z \xrightarrow{\tau} z'$ iff $m' = m$ and the time duration τ is possible (formal: $\forall t(t \in T \wedge J(t) \neq \# \longrightarrow J(t) + \tau \leq lft(t)$ and $J'(t) := \begin{cases} J(t) + \tau & ,\text{iff} \quad t \leq m \\ \# & ,\text{iff} \quad t \not\leq m \end{cases}$). $z = (m, J)$ is called an integer-state iff for each enabled transition t in m $J(t)$ is an integer. The graph $RG_Z(z_o)$ is called the reachability graph of the TPN Z iff its nodes are the reachable integer-states and its arcs are defined by the triples (z, t, z') respectively (z, τ, z'), $\tau = 1$, where $z \xrightarrow{t} z'$, respectively $z \xrightarrow{\tau} z'$. Let $R_Z(z)$ be the set of all reachable states in Z started at z. A transition t is called live in z iff $\forall z'(z' \in R_Z(z) \longrightarrow \exists z''(z'' \in R_Z(z') \wedge t$ is ready to fire in $z'')$. The state z is live in the TPN Z iff all transitions $t \in T$ are live in z and Z is live iff z_o is live in Z.

Example 2.1

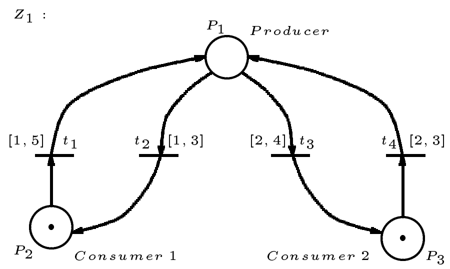


Figure 2: *An easy example for finite TPN*

The initial state is $z_0 = ((0, 1, 1)(0, \sharp, \sharp, 0))$. At the initial state the transitions t_1 and t_4 are enabled but neither t_1 nor t_4 may fire because of their time restrictions. Thus, in Z z_o can change into another state only by elapsing time. E.g., in Z the state change $z_o \xrightarrow{1.7} z_1$ is possible, where z_1 is given by $m_1 = m_0$ and $J_1 = (1.7, \sharp, \sharp, 1.7)$. Furthermore, z_1 can change, e.g., into the state z_2 with $z_1 \xrightarrow{1.0} z_2$, where the state z_2 is given with $m_2 = m_1$ and $J_2 = (2.7, \sharp, \sharp, 2.7)$. At z_2 the transition t_4 can fire - the state z_3 is yielded, then. Thereby it holds: $m_3 = m_2 + \Delta t_4 = (1, 1, 0)$ and $J_3 = (2.7, 0, 0, \sharp)$. Now, when the state z_3 changes into the state z_4 by elapsing time $\tau_3 = 2$ and z_4 changes into the state z_5 by firing t_1, then it is: $m_4 = m_3$, $J_4 = (4.7, 2.0, 2.0, \sharp)$, $m_5 = (2, 0, 0)$ and $J_5 = (\sharp, 2.0, 2.0, \sharp)$. The initial state and the state z_5 are integer-states whereas the states z_1, z_2, z_3 and z_4 are not. One says, the states-transition(-times) sequence or path $z_o \xrightarrow{1.7} z_1 \xrightarrow{1.0} z_2 \xrightarrow{t_4} z_3 \xrightarrow{2.0} z_4 \xrightarrow{t_1} z_5$ is a feasible one in Z. The transition sequence $\sigma = (t_4 t_1)$ (resp. the transitions-times sequence $\theta = (\tau_1 \tau_2 t_4 \tau_3 t_1)$, where $\tau_1 = 1.7, \tau_2 = 1.0$, and $\tau_3 = 2.0$) are executable in Z. $\blacksquare$

As already mentioned we developed an algorithm for investigating the qualitative and quantitative behaviour of a given Time-Petri-net with obligation to fire for each transition, here called finite Time-Petri-net (FTPN). The algorithm is discrete, i.e. we pick out a finite set of "essential" states, which are "carriers" of the properties of the net and define them as the nodes of the reachability graph of the net. For this reason, we first show the basic property, that if in a given FTPN Z an arbitrary firing sequence can fire, then it is always possible for the same transitions sequence to fire in the same net in integer-states. These integer-states are the next neighborhood of the origin states. Exactly, we proof the following basic property for a FTPN Z: Let $z_0 \xrightarrow{\tau_o} z'_0 \xrightarrow{t_o} z_1 \xrightarrow{\tau_1} z'_1 \xrightarrow{t_1} \ldots \xrightarrow{\tau_n} z'_n \xrightarrow{t_n} z_{n+1}$ be an feasible path in Z, where $\tau_i \in \mathcal{Q}_0$ for $i = 0 \ldots n$ and therefore not necessarily natural numbers. Then there exist reachable integer-states z^*_i $(i = 0 \ldots n+1)$, z^{**}_i $(i = 0 \ldots n)$ and $\tau^*_i \in \mathcal{N}$ $(i = 0 \ldots n)$, so that the path $z_0 = z^*_0 \xrightarrow{\tau^*_0} z^{**}_0 \xrightarrow{t_0} z^*_1 \xrightarrow{\tau^*_1} z^{**}_1 \xrightarrow{t_1} \ldots \xrightarrow{\tau^*_n} z^{**}_n \xrightarrow{t_n} z^*_{n+1}$ is feasible one in Z. Hereby it holds: (1) $m_i = m'_i = m^*_i = m^{**}_i$ for $i = 0 \ldots n$ resp. $n+1$ and (2) for each $t \in T$, which is enabled by $m_i \longrightarrow \mid J_i(t) - J^*_i(t) \mid < 1$ and $\mid J'_i(t) - J^{**}_i(t) \mid < 1$ (neighborhood). The proof of this theorem is constructive one (comp. Popova [8]).

Example 2.2

We consider the FTPN Z_1, given in the example 2.1 again. The following path, containing not integer-states, is feasible in Z_1 :

$$(\underbrace{(0,1,1)}_{\mathfrak{m}_o}, \underbrace{(0,\sharp,\sharp,0)}_{\mathfrak{J}_o}) \xrightarrow{2.3} (\underbrace{(0,1,1)}_{\mathfrak{m}'_o}, \underbrace{(2.3,\sharp,\sharp,2.3)}_{\mathfrak{J}'_o}) \xrightarrow{t_4} (\underbrace{(1,1,0)}_{\mathfrak{m}_1}, \underbrace{(2.3,0,0,\sharp)}_{\mathfrak{J}_1}) \xrightarrow{2.1}$$

$$(\underbrace{(1,1,0)}_{\mathfrak{m}'_1}, \underbrace{(4.4,2.1,2.1,\sharp)}_{\mathfrak{J}'_1}) \xrightarrow{t_3} (\underbrace{(0,1,1)}_{\mathfrak{m}_2}, \underbrace{(4.4,\sharp,\sharp,0)}_{\mathfrak{J}_2}) \xrightarrow{0.1} (\underbrace{(0,1,1)}_{\mathfrak{m}'_2}, \underbrace{(4.5,\sharp,\sharp,0.1)}_{\mathfrak{J}'_2})$$

$$\xrightarrow{t_1} (\underbrace{(1,0,1)}_{\mathfrak{m}_3}, \underbrace{(\sharp,0,0,0.1)}_{\mathfrak{J}_3})$$

is a feasible path of pseudo-states in Z_1. Using the preceding basic property we get a path, consisting of the same transitions (in the same order) and exclusive of integer-statets:

$$(\underbrace{(0,1,1)}_{\mathfrak{m}^*_o}, \underbrace{(0,\sharp,\sharp,0)}_{\mathfrak{J}^*_o}) \xrightarrow{2} (\underbrace{(0,1,1)}_{\mathfrak{m}^{**}_o}, \underbrace{(2,\sharp,\sharp,2)}_{\mathfrak{J}^{**}_o}) \xrightarrow{t_4} (\underbrace{(1,1,0)}_{\mathfrak{m}^*_1}, \underbrace{(2,0,0,\sharp)}_{\mathfrak{J}^*_1}) \xrightarrow{2}$$

$$(\underbrace{(1,1,0)}_{\mathfrak{m}^{**}_1}, \underbrace{(4,2,2,\sharp)}_{\mathfrak{J}^{**}_1}) \xrightarrow{t_3} (\underbrace{(0,1,1)}_{\mathfrak{m}^*_2}, \underbrace{(4,\sharp,\sharp,0)}_{\mathfrak{J}^*_2}) \xrightarrow{0} (\underbrace{(0,1,1)}_{\mathfrak{m}^{**}_2}, \underbrace{(4,\sharp,\sharp,0)}_{\mathfrak{J}^{**}_2})$$

$$\xrightarrow{t_1} (\underbrace{(1,0,1)}_{\mathfrak{m}^*_3}, \underbrace{(\sharp,0,0,0)}_{\mathfrak{J}^*_3}) \qquad \blacksquare$$

We usually use a "reduced" reachability graph, where such paths as

$$\xrightarrow{t} z_i \xrightarrow{1} z_{i+1} \xrightarrow{1} \ldots \xrightarrow{1} z_{i+k} \xrightarrow{t'}$$

without branching are substituted by

$$\xrightarrow{t} z_i \xrightarrow{k} z_{i+k} \xrightarrow{t'} .$$

The reduced reachability graph of Z_1 contains 53 nodes and 122 arcs (computed by INA, comp. Starke [14]).

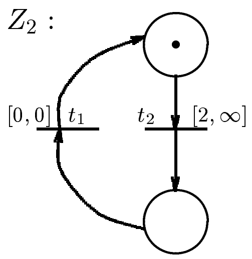
Example 2.3

Using the considerations above, we obtain that the FTPN Z_1 is live, bounded ($K_{p_1} = K_{p_2} = K_{p_3} = 2$) etc. ■

Using the basic property we have shown, that a FTPN is bounded iff its reachability graph is finite. The liveness of the net can be decided for bounded FTPN, viz. a FTPN is live iff for each transition t it holds: (1) in the RG there is an arc labeled by this transition t and (2) from each node in the RG it is possible to get another note, which has an output arc, labeled by this transition t. (comp. Popova [8]). Of course, this method is not fast. That is why, we use already further methods in order to investigate a FTPN resp. develop new (comp. Popova [10], [11], [12] [13]).

However, this approach for defining the reachability graph of a FTPN cannot be applied for any TPN, viz. when in the net, at least one transition t has as lft the infinity ($lft(t) = \infty$). Obviously, in such TPN it does not hold that the net is bounded if the number of reachable integer-states is finite:

Example 2.4



Evidently, this net is bounded, but the number of the reachable integer-states (all integer states $z_i = ((1,0)(i,\sharp))$ with $i \in \mathcal{N}$ and $((0,1)(\sharp,0))$) is infinite.

Figure 3: *A bounded infinite TPN with a infinite number of integer-states*

Still we wish to keep the idea of picking an appropriate set of states that carries the net properties. But then the questions arise: Do such "essential" states exists in general in not necessarily finite TPN? If so, which are they?

3 Essential states and qualitative behaviour in TPN

Our aim is to find as small as possible set of reachable states for a given TPN such that using these states to receive as much as possible knowledge about the behaviour of the net.

First we define the "new" essential states and then we show that these states are the carriers of the net behaviour actually. For this reason we define the notion pseudo-states. This notion is a consistent extension of the notion state in the FTPN. For defining pseudo-states we consider the time in the TPN as follow: the clock of each enabled transition t is going on until reaching the $lft(t)$ if $lft(t) < \infty$ and the clock is going on until the $eft(t)$, else.

Definition 3.1 *Let* $Z = (P, T, F, V, m_0, I)$ *be a TPN and* $\mathfrak{J}\colon T \longrightarrow \mathcal{Q}_0 \cup \{\#\}$. $\mathfrak{z}_\mathfrak{i} = (\mathfrak{m}_\mathfrak{i}, \mathfrak{J}_\mathfrak{i})$ *is called a pseudo-state in* Z, *iff*
Basis. *i=0.* $\mathfrak{z}_\mathfrak{o} := z_0$
Step. *We assume that* $\mathfrak{z}_\mathfrak{i}$ *is already defined. Then* $\mathfrak{z}_{\mathfrak{i}+1}$ *is yielded from* $\mathfrak{z}_\mathfrak{i}$ *by*

1. *elapsing time* $\tau \in \mathcal{Q}_0$ $\quad (\mathfrak{z}_\mathfrak{i} \xrightarrow{\tau} \mathfrak{z}_{\mathfrak{i}+1})$, *iff*

   (a) $\mathfrak{m}_{\mathfrak{i}+1} := \mathfrak{m}_\mathfrak{i}$

   (b) *the change is possible, i.e.* $\forall t (t \in T \wedge \mathfrak{J}_\mathfrak{i}(t) \neq \# \longrightarrow \mathfrak{J}_\mathfrak{i}(t) + \tau \leq lft(t))$

   (c) $$\mathfrak{J}_{\mathfrak{i}+1}(t) := \begin{cases} \# & , t \not\leq \mathfrak{m}_{\mathfrak{i}+1} \\ \mathfrak{J}_\mathfrak{i}(t) + \tau & , (t \leq \mathfrak{m}_{\mathfrak{i}+1} \wedge lft(t) < \infty) \vee \\ & \ (t^- \leq \mathfrak{m}_{\mathfrak{i}+1} \wedge lft(t) = \infty \wedge \mathfrak{J}_\mathfrak{i}(t) + \tau \leq eft(t)) \\ eft(t) & , else \end{cases} .$$

2. *firing the transition* t' *in the pseudo-state* $\mathfrak{z}_\mathfrak{i}$ $\quad (\mathfrak{z}_\mathfrak{i} \xrightarrow{t'} \mathfrak{z}_{\mathfrak{i}+1})$ *and it holds:*

   (a) $\mathfrak{m}_{\mathfrak{i}+1} = \mathfrak{m}_\mathfrak{i} + \Delta t'$

   (b) $$\mathfrak{J}_{\mathfrak{i}+1}(t) =: \begin{cases} \# & , iff \quad t^- \not\leq \mathfrak{m}_{\mathfrak{i}+1} \\ \mathfrak{J}_\mathfrak{i}(t) & , iff \quad t^- \leq \mathfrak{m}_\mathfrak{i} \wedge t^- \leq \mathfrak{m}_{\mathfrak{i}+1} \wedge Ft \cap Ft' = \emptyset \\ 0 & , iff \quad otherwise \end{cases} .$$

Example 3.1

We consider Z_2 again. The path $(\mathfrak{z}_\mathfrak{o}\tau_1\mathfrak{z}_1\tau_2\mathfrak{z}_2\tau_3\mathfrak{z}_3\mathfrak{z}_4)$, where $\tau_1 = 1.3, \tau_2 = 3.2, \tau_3 = 7.5$ and
$$(\underbrace{(1,0)}_{\mathfrak{m}_\mathfrak{o}}, \underbrace{(0,\sharp)}_{\mathfrak{J}_\mathfrak{o}}) \xrightarrow{1.3} (\underbrace{(1,0)}_{\mathfrak{m}_1}, \underbrace{(1.3,\sharp)}_{\mathfrak{J}_1}) \xrightarrow{3.2} (\underbrace{(1,0)}_{\mathfrak{m}_2}, \underbrace{(2,\sharp)}_{\mathfrak{J}_2}) \xrightarrow{7.5} (\underbrace{(1,0)}_{\mathfrak{m}_3}, \underbrace{(2,\sharp)}_{\mathfrak{J}_3}) \xrightarrow{t_2} (\underbrace{(1,1)}_{\mathfrak{m}_4}, \underbrace{(\sharp,0)}_{\mathfrak{J}_4})$$
is a feasible path of pseudo-states in Z_2.

Furthermore it is evidently, that the set $\{ \ ((0,1),(\sharp,0)) \ , ((1,0),(0,\sharp)) \ , \ ((1,0),(1,\sharp)) \ , ((1,0),(2,\sharp)) \ \}$ contains all reachable-integer-pseudo-states in Z_2. ■

It is easy to see, that for FTPN the notions "states" and "pseudo-states" agree. Furthermore, the following interdependence between states(-path) and pseudo-states(-path) is valid:

Lemma 3.1 *Let* $Z = (P, T, F, V, I, m_0)$ *be a TPN and let*
$z_0 = \mathfrak{z}_\mathfrak{o} \xrightarrow{\tau_o} \mathfrak{z}'_\mathfrak{o} \xrightarrow{t_o} \mathfrak{z}_1 \xrightarrow{\tau_1} \mathfrak{z}'_1 \xrightarrow{t_1} \ldots \xrightarrow{\tau_n} \mathfrak{z}'_\mathfrak{n} \xrightarrow{t_n} \mathfrak{z}_{\mathfrak{n}+1}$ *be a feasible path in* Z. *Then the following sequence*
$z_0 \xrightarrow{\tau_o} z'_0 \xrightarrow{t_o} z_1 \xrightarrow{\tau_1} z'_1 \xrightarrow{t_1} \ldots \xrightarrow{\tau_n} z'_n \xrightarrow{t_n} z_{n+1}$ *is feasible in* Z, *too.*

Furthermore, the states $z_i = (m_i, J_i)$ *and* $\mathfrak{z}_i = (\mathfrak{m}_i, \mathfrak{J}_i)$, *resp.* $z'_i = (m'_i, J'_i)$ *and* $\mathfrak{z}'_i = (\mathfrak{m}'_i, \mathfrak{J}'_i)$ *are almost the same (later on called* **adequately** *to each other resp. both paths are adequately); it holds:*

1. $m_i = m'_i = \mathfrak{m}_i = \mathfrak{m}'_i$ *for each* $i = 0 \dots n$ *and* $\mathfrak{m}_{n+1} = \mathfrak{m}'_{n+1}$
2. *for each* $t (t \in T)$ *it is*
   (a) $\mathfrak{J}_i(t) = \sharp$ *iff* $J_i(t) = \sharp$ *and* $\mathfrak{J}'_i(t) = \sharp$ *iff* $J'_i(t) = \sharp$.
   (b) $\mathfrak{J}_i(t) = J_i(t)$ *iff* $(t^- \le m_i \wedge lft(t) < \infty) \vee (t^- \le m_i \wedge lft(t) = \infty \wedge J_i(t) \le eft(t))$ *and*
   $\mathfrak{J}'_i(t) = J'_i(t)$ *iff* $(t^- \le m_i \wedge lft(t) < \infty) \vee (t^- \le m_i \wedge lft(t) = \infty \wedge J'_i(t) \le eft(t))$.

Proof: It can be done straightforward by induction on n:
Basis. $i = 0$. $\mathfrak{z}_0 = z_0$ is true according to definition 3.1 and obviously (1), (2a) and (2b) are satisfied.
Step. We assume that both paths $\mathfrak{z}_0 \xrightarrow{\tau_0} \mathfrak{z}'_0 \xrightarrow{t_0} \mathfrak{z}_1 \xrightarrow{\tau_1} \mathfrak{z}'_1 \xrightarrow{t_2} \mathfrak{z}_2 \dots \xrightarrow{\tau_n} \mathfrak{z}'_n \xrightarrow{t_n} \mathfrak{z}_{n+1}$ and $z_0 \xrightarrow{\tau_0} z'_0 \xrightarrow{t_0} z_1 \xrightarrow{\tau_1} z'_1 \xrightarrow{t_1} z_2 \dots \xrightarrow{\tau_n} z'_n \xrightarrow{t_n} z_{n+1}$ are adequate.
We **want** to show that $\mathfrak{z}_0 \xrightarrow{\tau_0} \mathfrak{z}'_0 \xrightarrow{t_0} \mathfrak{z}_1 \xrightarrow{\tau_1} \mathfrak{z}'_1 \xrightarrow{t_2} \mathfrak{z}_2 \dots \xrightarrow{\tau_n} \mathfrak{z}'_n \xrightarrow{t_n} \mathfrak{z}_{n+1} \xrightarrow{\tau_{n+1}} \mathfrak{z}'_{n+1} \xrightarrow{t_{n+1}} \mathfrak{z}_{n+2}$ and $z_0 \xrightarrow{\tau_0} z'_0 \xrightarrow{t_0} z_1 \xrightarrow{\tau_1} z'_1 \xrightarrow{t_1} z_2 \dots \xrightarrow{\tau_n} z'_n \xrightarrow{t_n} z_{n+1} \xrightarrow{\tau_{n+1}} z'_{n+1} \xrightarrow{t_{n+1}} z_{n+2}$ are adequate, too.
We consider the state z_{n+1} (and in particular the time vector $J_{n+1}(t)$) and the time τ_{n+1}. For each transition t, enabled in m_{n+1} there are two possible cases: **case 1:** $lft(t) < \infty$. Because of the inductive assumption it is true that $J_{n+1}(t) + \tau_{n+1} = \mathfrak{J}_{n+1}(t) + \tau_{n+1}$ and because $\mathfrak{z}_{n+1} \xrightarrow{\tau_{n+1}}$ is executable in Z, it follows that $\mathfrak{J}_{n+1}(t) + \tau_{n+1} \le lft(t)$. Hence $J_{n+1}(t) + \tau_{n+1} \le lft(t)$. **case 2:** $lft(t) = \infty$. Than it is obviously true: $J_{n+1}(t) + \tau_{n+1} \le lft(t)$. Now, because of the inductive assumption the state z_{n+1} satisfy (1), (2a) and (2b) the change $z_{n+1} \xrightarrow{\tau_{n+1}}$ is possible in Z. Let $z_{n+1} \xrightarrow{\tau_{n+1}} z'_{n+1}$ be. It holds $m'_{n+1} = m_{n+1}$ and $m_{n+1} = \mathfrak{m}_{n+1} = \mathfrak{m}'_{n+1}$ and thus we yield that $m'_{n+1} = \mathfrak{m}'_{n+1}$, i.e. (1) is satisfy. (2a) follows from (1). Furthermore it holds for each transition t with $t^- \le m'_{n+1}$ and $lft(t) < \infty$ or $lft(t) = \infty \wedge J'_{n+1} \le eft(t)$: $\mathfrak{J}'_{n+1}(t) = \mathfrak{J}_{n+1}(t) + \tau_{n+1} = J'_{n+1}(t)$ and this is what is claimed in (2).
Now we consider the state $z'_{n+1} = (m'_{n+1}, J'_{n+1})$ and the transition t_{n+1}. Since it is true that $m'_{n+1} = \mathfrak{m}'_{n+1}$ and $t^-_{n+1} \le \mathfrak{m}'_{n+1}$ it follows:

$$t^-_{n+1} \le m'_{n+1}. \tag{*}$$

Furthermore and because t_{n+1} can fire in $\mathfrak{z}'_{n+1}$, it is:

$$\mathfrak{J}'_{n+1}(t_{n+1}) \ge eft(t_{n+1}). \tag{**}$$

Because of the last two inequations (*) and (**) it is clear that t_{n+1} can fire in the state z'_{n+1}. Let $z'_{n+1} \xrightarrow{t_{n+1}} z_{n+2}$ be. Than it is: $m_{n+2} = m'_{n+1} + \Delta t_{n+1} = \mathfrak{m}'_{n+1} + \Delta t_{n+1} = \mathfrak{m}_{n+2}$ and therefore (2a) is satisfied. In order to show (2b) let t be an arbitrary transition in Z, enabled in m_{n+2}, i.e. $t^- \le m_{n+2}$. In **case** of $t^- \le m_{n+2} \wedge t^- \le m'_{n+1} \wedge Ft \cap Ft_{n+1} = \emptyset$ it holds: $J_{n+2}(t) = J'_{n+2}(t)$. Furthermore, it is $J'_{n+2}(t) = \mathfrak{J}'_{n+1}(t)$ because of the inductive assumption and $\mathfrak{J}'_{n+1}(t) = \mathfrak{J}_{n+2}(t)$ because of the definition 3.1. Hence $J_{n+2}(t) = \mathfrak{J}_{n+2}(t)$ is true. If $t^- \le m_{n+2} \wedge t^- \le m'_{n+1} \wedge Ft \cap Ft_{n+1} \ne \emptyset$ is the **case**, than it holds $\mathfrak{J}_{n+2}(t) = 0 = J_{n+2}(t)$. Thus the proposition (2b) is satisfied, too. ■

The notion "bounded" is used as usual: a TPN is bounded iff for each place $p \in P$ there is a natural number K_p and for each reachable marking reachable marking m in the net it is true: $m(p) \leq K_p$.

Next property is analogous to the main property for FTPN, viz. when a transitions sequence $\sigma = (t_0 t_1 \dots t_n)$ is executible in Z in arbitrary states (i.e. not necessarely integer-states) then the same transitions sequence σ can fire at integer-states in Z, too:

Theorem 3.1 *Let $Z = (P, T, F, V, I, m_0)$ be a TPN and*

$$z_0 = \mathfrak{z}_0 \xrightarrow{\tau_o} \mathfrak{z}_0' \xrightarrow{t_o} \mathfrak{z}_1 \xrightarrow{\tau_1} \mathfrak{z}_1' \xrightarrow{t_1} \dots \xrightarrow{\tau_n} \mathfrak{z}_n' \xrightarrow{t_n} \mathfrak{z}_{n+1} \qquad (1)$$

be a feasible path in Z with $\tau_0 \in \mathcal{Q}_0$. Then it is true: For each i there exist states $\mathfrak{z}_i^*$ and $\mathfrak{z}_i^{**}$, and times $\tau_i^* \in \mathcal{N}$ with:

1. $\mathfrak{m}_i^* = \mathfrak{m}_i^{**} = \mathfrak{m}_i = \mathfrak{m}_i'$
2. $\mathfrak{z}_i^*$ *and* $\mathfrak{z}_i^{**}$ *are integer-states*
3. $z_0 = \mathfrak{z}_0^* \xrightarrow{\tau_o^*} \mathfrak{z}_0^{**} \xrightarrow{t_0} \mathfrak{z}_1^* \xrightarrow{\tau_1^*} \mathfrak{z}_1^{**} \xrightarrow{t_1} \dots \xrightarrow{\tau_n^*} \mathfrak{z}_n^{**} \xrightarrow{t_n} \mathfrak{z}_{n+1}^* \qquad (2)$
   is a feasible path in Z
4. *for each t ($t \in T$) it holds:* $(\mathfrak{J}_i(t) = \sharp \longleftrightarrow \mathfrak{J}_i^*(t) = \sharp) \wedge (\mathfrak{J}_i'(t) = \sharp \longleftrightarrow \mathfrak{J}_i^{**}(t) = \sharp)$
5. $\forall t \, (t^- \leq \mathfrak{m}_i \quad \longrightarrow \quad (\mid \mathfrak{J}_i(t) - \mathfrak{J}_i^*(t) \mid < 1 \wedge \mid \mathfrak{J}_i'(t) - \mathfrak{J}_i^{**}(t) \mid < 1))$.

Proof: First we consider the path

$$\mathfrak{z}_0 = z_0 \xrightarrow{\tau_0} z_0' \xrightarrow{t_0} z_1 \xrightarrow{\tau_1} z_1' \xrightarrow{t_1} \dots \xrightarrow{\tau_n} z_n' \xrightarrow{t_n} z_{n+1} \qquad (3),$$

adequate to the path (1). According to the Lemma 3.1 the path (3) is feasible one in Z, too, and it holds: $\mathfrak{m}_i = m_i = m_i' = \mathfrak{m}_i'$. Furthermore, the path is finite and hence we can apply the algorithm defined by the proof of the basic property for FTPN (comp Popova [8]. Now, we can find integer-states z_i^*, z_i^{**}, times $\tau_i^* \in \mathcal{N}$, so that the transitions sequence $(t_0 t_1 \dots t_n)$ can fire in Z in integer-states, i.e. the path

$$z_0 = z_0^* \xrightarrow{\tau_0^*} z_0^{**} \xrightarrow{t_0} z_1^* \xrightarrow{\tau_1^*} z_1^{**} \xrightarrow{t_1} \dots \xrightarrow{\tau_n^*} z_n^{**} \xrightarrow{t_n} z_{n+1}^* \qquad (4)$$

is feasible one in Z, $\mid J_i(t) - J_i^*(t) \mid < 1$ and $\mid J_i'(t) - J_i^{**}(t) \mid < 1$ if $t^- \leq m_i$ and $m_i = m_i' = m_i^* = m_i^{**}$. Now, we consider the pseudo-states, adequate to the states z_i^* and z_i^{**}; let they be $\mathfrak{z}_i^*$ and $\mathfrak{z}_i^{**}$. It is easy to proof, that $\mathfrak{z}_i^* \xrightarrow{\tau_i^*} \mathfrak{z}_i^{**} \xrightarrow{t_i} \mathfrak{z}_{i+1}^*$ is executable in Z because of (3), (4) and Lemma 3.1. Thereby it follows that the sequence $z_0 = \mathfrak{z}_0^* \xrightarrow{\tau_o^*} \mathfrak{z}_0^{**} \xrightarrow{t_o} \mathfrak{z}_1^* \xrightarrow{\tau_1^*} \mathfrak{z}_1^{**} \xrightarrow{t_1} \dots \xrightarrow{\tau_n^*} \mathfrak{z}_n^{**}$ is feasible one in Z. This is exactly the path (2) whose existence is claimed here. ■

Let z_1 and z_2, resp. $\mathfrak{z}_1$ and $\mathfrak{z}_2$ be two states resp. pseudo-states in a TPN Z. $z_1 \xrightarrow{*} z_2$, resp. $\mathfrak{z}_1 \xrightarrow{*} \mathfrak{z}_2$ means that there exists a **feasible finite** states-transitions-times sequence, resp. **feasible finite** pseudo-states-transitions-times sequence in Z, which connects z_1 and z_2, resp. $\mathfrak{z}_1$ and $\mathfrak{z}_2$.

Theorem 3.2 *Let Z be a TPN, not necessarily a finite one. Then, Z is bounded iff the set $\mathcal{E} = \{\mathfrak{z}_0 \xrightarrow{*} \mathfrak{z}, \mathfrak{z}$ - integer $\}$ is finite.*

Proof: ($\longrightarrow$) This direction is obviously.

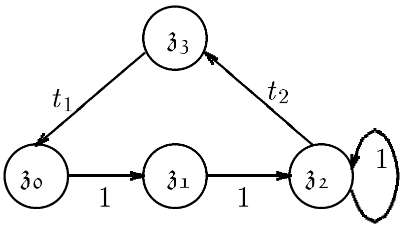
($\longleftarrow$) Let $\mathcal{E}$ be a finite set. Assuming that Z is unbounded, than there is a state $\hat{z} = (\hat{m}, \hat{J})$, reachable in Z such that for each element $\mathfrak{z} = (\mathfrak{m}, \mathfrak{J}) \in \mathcal{E}$ (i.e. $\mathfrak{z}$ is a integer-pseudo-state, reachable in Z) $\hat{m} \neq \mathfrak{m}$ is. Because $\hat{z}$ is reachable in Z there exists a sequence, feasible in Z : $z_0 \xrightarrow{\tau_0} z'_0 \xrightarrow{t_0} z_1 \ldots \xrightarrow{t_n} z_{n+1} \xrightarrow{\tau_{n+1}} z'_{n+1} = \hat{z}$. According to lemma 3.1 there exists the sequence $z_0 = \mathfrak{z}_o \xrightarrow{\tau_o} \mathfrak{z}'_o \xrightarrow{t_o} \mathfrak{z}_1 \xrightarrow{\tau_1} \mathfrak{z}'_1 \xrightarrow{t_1} \ldots \xrightarrow{\tau_n} \mathfrak{z}'_n \xrightarrow{t_n} \mathfrak{z}_{n+1} \xrightarrow{\tau_{n+1}} \mathfrak{z}'_{n+1}$, feasible in Z, too. Applying for the last path the theorem 3.1 we get the new path $z_0 = \mathfrak{z}^*_o \xrightarrow{\tau^*_o} \mathfrak{z}^{**}_o \xrightarrow{t_o} \mathfrak{z}^*_1 \xrightarrow{\tau^*_1} \mathfrak{z}^{**}_1 \xrightarrow{t_1} \ldots \xrightarrow{t^*_n} \mathfrak{z}^{**}_n \xrightarrow{\tau^*_n} \mathfrak{z}^{**}_n \xrightarrow{t_n} \mathfrak{z}^*_{n+1} \xrightarrow{\tau_{n+1}} \mathfrak{z}^{**}_{n+1}$, feasible in Z, too and $\mathfrak{z}^*_i$ and $\mathfrak{z}^{**}_i$ are integer-pseudo-states. Hence, $\mathfrak{z}^{**}_{n+1}$ is an element of the set $\mathcal{E}$ and this is a contradiction to the choose of $\hat{m}$. ■

The theorem 3.2 shows that the integer-pseudo-states are the carriers of the property "boundedness" for an arbitrary, not necessary finite TPN, in fact.

Definition 3.2 *The graph $EG_z(\mathfrak{z}_o)$ is called a reachability graph of the TPN Z iff its nodes are the reachable integer-pseudo-states from Z and its arcs are defined by the triple $(\mathfrak{z}, t, \mathfrak{z}')$ resp. $(\mathfrak{z}, \tau, \mathfrak{z}')$, where $\mathfrak{z} \xrightarrow{t} \mathfrak{z}'$ resp. $\mathfrak{z} \xrightarrow{\tau} \mathfrak{z}'$*

Example 3.2

We consider the TPN Z_2 defined in example 2.4, again. As already mentioned, the set of all integer-states of this net is finite, whilst the set of integer pseudo-states is finite (comp. example 3.1). Thus, the (unreduced) RG of Z_2 is the following graph:



$\mathfrak{z}_o$= (1.0), (0, ♯))
$\mathfrak{z}_1$= ((1, 0), (1, ♯))
$\mathfrak{z}_2$= (1, 0), (2, ♯))
$\mathfrak{z}_3$= ((0, 1), (♯, 0))

Figure 4: *The unreduced reachability graph $RG_{Z_1(\mathfrak{z}_o)}$*

■

In the following we study the liveness of a TPN, not necessarily a finite one, using the integer-pseudo-states. For this reason, first we show that the both languages, the language of the TPN and the language of their reachability graph, which are defined in a usual way, are the same. Then we derive an algorithm for checking the liveness by means of the RG.

Definition 3.3 *The language $L_{Z(z_0)}$ is called the language of the TPN $Z = (P, T, F, V, I, m_0)$ iff $L_{Z(z_0)} := \{\, w \mid w \in T^* \wedge z_0 \xrightarrow[*]{w} \,\}$.*

Definition 3.4 *The language $L_{EG_{Z(z_0)}}$ of the reachability graph EG of the TPN Z is defined as follows:*
$L_{EG_{Z(z_0)}} := \{\, w \mid w \in T^* \wedge \mathfrak{z}_0 \xrightarrow[*-i]{w} \,\}$

Lemma 3.2 $L_{Z(z_0)} = L_{EG_{Z(z_0)}}$.

Proof: ($\subseteq$) Let $w = (t_1 t_2 \ldots t_n) \in T^*$ be a word in $L_{Z(z_0)}$, e.i. w is an executible transitions sequence in Z. Then, the path

$$z_0 \xrightarrow{\tau_o} z_0' \xrightarrow{t_o} z_1 \xrightarrow{\tau_1} z_1' \xrightarrow{t_1} \ldots \xrightarrow{\tau_n} z_n' \xrightarrow{t_n} z_{n+1}$$

is feasible in Z for some $\tau_i \in \mathcal{N}$. Then according to lemma 3.1 there exists a further feasible sequence in Z: $z_0 = \mathfrak{z}_0^* \xrightarrow{\tau_o^*} \mathfrak{z}_0^{**} \xrightarrow{t_0} \mathfrak{z}_1^* \xrightarrow{\tau_1^*} \mathfrak{z}_1^{**} \xrightarrow{t_1} \ldots \xrightarrow{\tau_n^*} \mathfrak{z}_n^{**} \xrightarrow{t_n} \mathfrak{z}_{n+1}^*$ where $\mathfrak{z}_i^*$ and $\mathfrak{z}_i^{**}$ are integer-pseudo-states and hence - nodes in the reachability graph of Z. Therefore w is a word in the language $L_{EG(Z(z_0))}$, too.

($\supseteq$) Let $w = (t_0 t_1 \ldots t_n)$ be a word in $L_{EG(Z(z_0))}$. Hence there is a path
$z_0 = \mathfrak{z}_0 \xrightarrow{\tau_o} \mathfrak{z}_0' \xrightarrow{t_o} \mathfrak{z}_1 \xrightarrow{\tau_1} \mathfrak{z}_1' \xrightarrow{t_1} \ldots \xrightarrow{\tau_n} \mathfrak{z}_n' \xrightarrow{t_n} \mathfrak{z}_{n+1}$ $(\tau_i \in \mathcal{N})$
in the reachability graph of Z. Now, according to lemma 3.1 there exists the feasible sequence
$z_0 \xrightarrow{\tau_o} z_0' \xrightarrow{t_o} z_1 \xrightarrow{\tau_1} z_1' \xrightarrow{t_1} \ldots \xrightarrow{\tau_n} z_n' \xrightarrow{t_n} z_{n+1}$ in Z
and with that holds: $w \in L_{Z(z_0)}$. ■

The equality of the both languages means, that the (infinite many) integer-states, we disregarded are not essential for investigating the liveness behaviour of the net.

We call a language L over the alphabet T **live** iff for each word $w \in L$ and each letter $t \in T$ the word w can be progressed to a further word $wut \in L$, formally:

$$\forall w \forall t (w \in L \wedge t \in T \longrightarrow \exists u (u \in T^* \wedge wut \in L)).$$

Theorem 3.3 *The TPN Z is live iff the language L_Z is live.*

Proof: ($\longrightarrow$) Let Z be live and let $w \in L_Z$ be. Then $z_0 \xrightarrow[*]{w} z$ is feasible in Z. Let t be an arbitrary but fixed transition in T. Since Z is live, there exists a further word $u \in T^*$ and state z' such that $z_0 \xrightarrow[*]{u} z' \xrightarrow{t}$, i.e. the sequence $z_0 \xrightarrow[*]{w} z \xrightarrow[*]{u} z' \xrightarrow{t}$ is executable in Z and therefore $wut \in L_Z$ is.

($\longleftarrow$) Let L_Z be live, $t \in T$ and z an arbitrary reachable (w.e.o.g. integer-) states in Z. Then there exists a transitions sequence $w = (t_0 t_1 \ldots t_n), t_i \in T$ which is executable in Z and lead from state z_0 to the state z: $z_0 \xrightarrow[*]{u} z' \xrightarrow{t}$ is executible, i.e. Z is live. ■

Now we can give an algorithm for checking the liveness property for a TPN, if their reachability graph is finite:

Theorem 3.4 *Let $Z = (P, T, F, V, I, m_0)$ be a TPN. Z is live iff*
$\forall t (t \in T \longrightarrow$

1. $\exists(\mathfrak{z}, t, \mathfrak{z}')$ ($(\mathfrak{z}, t, \mathfrak{z}'$ *is an arc in* $EG_{Z(z_0)}$ $\wedge$

2. $\forall\mathfrak{z}(\mathfrak{z}$ *is a node in* $EG_{Z(z_0)} \longrightarrow \exists\sigma(\sigma = (\mathfrak{z}\sigma^*\mathfrak{z}^*\mathfrak{z}^{**})$ *path in* $EG_{Z(z_0)})$)).

The **Proof** follows immediately by definition 3.3, 3.4 and theorem 3.3. ∎

Using the reachability graph of a bounded TPN the deadlock freedom can be proved easily. Furthermore it is not difficult to realize that the reachability of a certain state $z = (m, J)$ in a bounded TPN, not necessarily finite, is decidable, even if the decision algorithm might be protracted: first, the adequate pseudo-states $\mathfrak{z} = (\mathfrak{m}, \mathfrak{J})$ has to be determined. If the pseudo-state is an integer one z is reachable iff the pseudo-state $\mathfrak{z}$ is a node in the reachable graph. If the pseudo-state $\mathfrak{z}$ is not an integer one, the TPN has to be streched by a factor c, i.e. all eft and lft have to be multiplied by c. c is computed as the largest common multiplier of all $\mathfrak{J}(t) \neq \sharp$. Now, $z = (m, J)$ is reachable in Z iff $\mathfrak{z}_\mathfrak{c} = (\mathfrak{m}, c \cdot \mathfrak{J})$ is a node in the reachability graph of the streched net.

4 Final remarks

A new discrete algorithm for studying the properties of a given TPN, not necessarily a finite one, has been introduced here. It is true that the time in the nets is described continuously, but it is shown that only a few points in time are important for the determination the behaviour of the nets.

Furthermore, it can be proved that the so defined reachability graph can be used for the computing the shortest and the largest path between two arbitrary states in the TPN. The proof has to be done similar to the case of FTPN, given in Heiner/Popova [6].

Further work should be done, similar to the result given in Popova [13] to get knowledge about the qualitative properties of a given TPN without computing the reachability graph of the net.

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