On Parametrical Sequences in Time Petri Nets

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Extended abstract

For more then 20 years time dependent Petri nets have been defined and have been used for modeling and analyzing of concurrent systems. At first sight both notions exclude each other. We use the time in order to define possible starts- and deadlines for the actions of a given system. Nevertheless the actions can happen concurrently in certain limits. Furthermore, for an amount of real processes, the time is an indispensable feature; only casual relations between the processes describe quite a few systems insufficient.

In general, the time dependent Petri Nets are more powerful than the classical Petri nets (they are equivalent to the Turing machine) and most of them (except the time dependent nets with stochastically defined time) can be simulated by Time Petri Nets, defined by Merlin[6] and further investigated by Berthomieu/Diaz[3], Popova[7], Popova[8], Boucheneb/Berthelot[2] , Aalst[1] etc.

Time Petri Nets are classical Petri Nets where to each transition t a time interval $[a_t, b_t]$ is associated. The times a_t and b_t are relative to the moment at which t was last enabled. Assuming that t was last enabled at time c_t, then t may fire only during the interval $[c_t + a_t, c_t + b_t]$ and must fire at the time $c_t + b_t$ at the latest, unless it is disabled before by the firing of another transition. Firing a transition takes no time. Thus, the philosophy of this kind of time dependent Petri Net is : when a transition become enabled it may not fire at once (in general) but during a certain time interval and at the end of the interval there is a force for firing. If the end of the interval is infinite then the second characteristic of this kind of net, the force for firing, is lost.

In general, there no is relation between the behaviour of a Time Petri Net and its skeleton (skeleton: the same net structure without time restrictions) (comp. Popova[8]). In Popova[9] we gave four classes of Time Petri Nets, which have the same boundedness and liveness behaviour as their skeletons.

Our goal now is: to find, for a given Petri Net, time boundaries for each transition thus (1) a given transition sequence , which is enabled in the timeless Petri Net remains enabled in the Time Petri net and (2) a possible branch in the timeless Petri Net becomes impossible in the Time Petri Net. These results can be used for computing time boundaries in order to control the liveness and the boundedness of a certain Time Petri Net.

Basic Notions and Definitions

We will use the following notations: $\mathcal{N}$ denotes the set of natural numbers, $\mathcal{N}^+$ is $\mathcal{N} \setminus \{\ 0\}$. $\mathcal{Q}$o is the set of nonnegative rational numbers. Let g be a given function from A to B. Then, the symbol # defines a special value from the set $g(A)$. T^* denotes the language of all words over the alphabet T, including the empty word e, $l(w)$ stands for length of the word w.

Definition 1.: The structure $N = (P, T, F, V, m_o)$ is called Petri net (PN), iff
(1) P, T, F are finite sets with
$P \cap T = \emptyset$, $\quad P \cup T \neq \emptyset$, $\quad F \subseteq (P \times T) \cup (T \times P)$ and
$dom(F) \cup cod(F) = P \cup T$ (net)
(2) $V : F \longrightarrow \mathcal{N}^+$ (weight of the arcs)
(3) $m_o : P \longrightarrow \mathcal{N}$ (initial marking)

A marking of a PN is a function $m : P \longrightarrow \mathcal{N}$, where $m(p)$ denotes the number of tokens in place p. The pre- and postsets of a transition t resp. of a place p are given by $Ft := \{p \mid p \in P \wedge pFt\}$ and $tF := \{p \mid p \in P \wedge tFp\}$ resp. Fp and pF. Each transition $t \in T$ induces the marking t^- und t^+, defined as follows:

$$t^- = \begin{cases} V(p,t) & \text{,iff} \quad (p,t) \in F \\ 0 & \text{,iff} \quad (p,t) \notin F \end{cases} \qquad t^+ = \begin{cases} V(t,p) & \text{,iff} \quad (t,p) \in F \\ 0 & \text{,iff} \quad (t,p) \notin F \end{cases}$$

By Δt we denote t^+- t^-. A transition $t \in T$ is enabled (may fire) by a marking m iff $t^- \leq m$ (i.g. $t^-(p) \leq m(p)$ for each place $p \in P$). When an enabled transition t by a marking m fires, there yields a new marking m' given by $m'(p) := m'(p) + \Delta t(p)$, denotes by $m \xrightarrow{t} m'$.

Definition 2.: The structure $Z = (P, T, F, V, m_o, I)$ is called a Time Petri Net (TPN) iff:
(1) $S(Z) = (P, T, F, V, m_o)$ is a PN.
(2) $I : T \longrightarrow \mathcal{Q}_0 \times (\mathcal{Q}_0 \cup \{\infty\})$ and for each $t \in T$ is $I_1(t) \leq I_2(t)$, where $I(t) = (I_1(t), I_2(t))$.

I is called the time function of Z, $I_1(t)$ and $I_2(t)$ earliest firing time of t ($eft(t)$) and latest firing time of t ($lft(t)$), respectively. It is not difficult to see (comp. Popova[8]) that considerations of TPN with $I : T \longrightarrow \mathcal{N} \times (\mathcal{N} \cup \{\infty\})$ is not a restriction in general. In the following we always consider such time functions I. Furthermore, we use here the strong meaning of conflict: two transitions t_1 and t_2 are in conflict iff $Ft_1 \cap Ft_2 \neq \emptyset$. The PN $S(Z)$ is called the skeleton of Z.

The notion "state" is here a basic one and it is given by marking and the momentary local time for the enabled transitions and the empty sign ($\sharp$) for the disabled transitions. Thus, now we define "state" and "change from a state into another state" by firing a transition and by time duration.

Definition 3.: Let $Z = (P, T, F, V, m_o, I)$ be a TPN and $J : T \longrightarrow \mathcal{Q}_0 \cup \{\#\}$. $z = (m, J)$ is called a state in Z iff:
(1) m is a reachable marking in $S(Z)$.
(2) $\forall t (t \in T \wedge t^- \leq m \longrightarrow J(t) \leq lft(t))$.
(3) $\forall t (t \in T \wedge t^- \not\leq m \longrightarrow J(t) = \#)$.

An interpretation of the notion "state" is as follow: in the net each transition t has a watch. The watch does not work ($J(t) = \#$) at the marking m if t is disabled at m. If t is enabled at m, the watch of t shows the time ($J(t)$), elapsed since t was enabled last.

The state $z_o := (m_o, J_o)$ with $J_o(t) := \begin{cases} 0 & \text{,iff} \quad t \leq m \\ \# & \text{,iff} \quad t \not\leq m \end{cases}$ is set to be the initial state of the TPN Z. A transition t is ready to fire in the state $z = (m, J)$, denote by $z \xrightarrow{t}$, iff $t^- \leq m$ and $eft(t) \leq J(t)$. A transition t , being ready to fire in the state $z = (m, J)$ may fire yielding a new state $z' = (m', J')$ defined by $m' = m + \Delta t$ and $J'(t) =: \begin{cases} \# & \text{,iff} \quad t^- \not\leq m \\ J(t) & \text{,iff} \quad t^- \leq m \wedge t^- \leq m' \wedge Ft \cap Ft' = \emptyset \\ 0 & \text{,iff} \quad \text{otherwise} \end{cases}$.

The state $z = (m, J)$ changes into the state $z' = (m', J')$ by the time duration $\tau \in \mathcal{Q}^+$, denoded by $z \xrightarrow{\tau} z'$ iff $m' = m$ and the time duration τ is possible (formal: $\forall t (t \in T \wedge J(t) \neq$

$\# \longrightarrow J(t) + \tau \leq lft(t)$ and $J'(t) := \left\{ \begin{array}{ll} J(t)+\tau & \text{,iff} \quad t \leq m \\ \# & \text{,iff} \quad t \not\leq m \end{array} \right.$). $z = (m, J)$ is called an integer-state iff for each enabled transition t in m $J(t)$ is an integer. The graph $RG_Z(z_o)$ is called the reachability graph of the TPN Z iff its nodes are the reachable integer-states and its arcs are defined by the triples (z, τ, z') respectively (z, t, z'), where $z \xrightarrow{t} z'$, respectively $z \xrightarrow{\tau} z'$.Let $R_Z(z)$ be the set of all reachable states in Z started at z. A transition t is called live in z iff $\forall z'(z' \in R_Z(z) \longrightarrow \exists z''(z'' \in R_Z(z') \wedge t$ is ready to fire in $z''))$. The state z is live in the TPN Z iff all transitions $t \in T$ are live in z and Z is live iff z_o is live in Z.

Results

A transition sequence in a Time Petri Net is time depending: each transition fires at a certain moment during a certain interval. Therefore a transition sequence in a Time Petri Net is, in fact, an alternative sequence of times and transitions (e.g. $\tau_0 t_0 \tau_1 t_1 ...$, i.e. at the beginning the time τ_0 is elapsed. After that the transition t_0 fires. Than time τ_1 is elapses and afterwards the transition t_1 fires, etc.).

In order to study transition sequences in a Time Petri Net we will define the notion **parametrical canonical transition sequence** (in a Time Petri Net). The times in the sequence will be given by parameters which satisfy inequalities.

For this reason we define an elementary predicate calculus:

Let $\{X, P, F, K\}$ be a set of variable symbols, predicate symbols, function symbols and constants with

$X := \{x_i \mid i \in \mathcal{N}\}$, $P := \{A^2\}$, $F := \{F^2\}$, $K := \{u_t, v_t \mid t \in T\} \cup \{\$\}$.

We define $\mathcal{Q}_0 \cup \{\sharp\}$ as the domain for X and the Interpretation ω as the following:

$\omega(A^2) := \leq, \quad \omega(F^2) := +, \quad \omega(u_t) := eft(t), \quad \omega(v_t) := lft(t), \quad \omega(\$) := \sharp.$

Let SUM be the set of all terms, where each variable appears at most once and where no constants appear or the term consists of the constant \$ only. BED defines the set of all formulas $A^2 term_i term_j$, where $term_i \in SUM \setminus \{\$\}$ and $term_j$ is a constant ($term_j \neq \$$) or vice versa. Without loss of generality, $x_o, x_1, x_2, ...$ denote both the variable symbols and variables in the domain of X.

Definition 4.: (parametrical canonical description of a sequence)
Let $Z = (P, T, F, V, m_o, I)$ be a TPN. We define the map δ by induction as follows:
$\delta : T^* \longrightarrow R(m_0) \times SUM^T \times \wp(BED)$ and

Basis: $\delta(e) := [m_0, \Sigma_0, B_0]$ with

(1)$\Sigma_0(t) := \left\{ \begin{array}{ll} x_0 & \text{,iff} \quad t^- \leq m_0 \\ \$ & \text{,iff} \quad t^- \not\leq m_0 \end{array} \right.$ (i.e. $\omega(\Sigma_0(t)) := \left\{ \begin{array}{ll} x_0 & \text{,iff} \quad t^- \leq m_0 \\ \sharp & \text{,iff} \quad t^- \not\leq m_0 \end{array} \right.$)

(2) $B_0 := \{A^2 \Sigma_0 v_t \mid t^- \leq m_0\}$ (i.e. $\omega(B_0) := \{\omega(\Sigma_0(t)) \leq lft(t) \mid t^- \leq m_0\}$) .

Step: We assume that $\delta(w)$ is defined and let $\delta(w) = [m, \Sigma, B]$ be. We define $\delta(w\hat{t}) := [m', \Sigma', B']$ as follows:
(1) $m' := m + \Delta(\hat{t})$.
(2) Let j be the smallest index such that the variable symbol x_j appears neither in Σ nor in B and

that j is greater than all indices of variable symbols appearing in Σ or B.

$$(3)\ \Sigma'(t) := \begin{cases} \$ & \text{iff} \quad t^- \not\leq m' \\ x_j & \text{iff} \quad (t^- \not\leq m \wedge t^- \leq m')\vee \\ & \quad\quad\ (t^- \leq m \wedge t^- \leq m' \wedge Ft \cap Ft' \neq \emptyset) \\ F^2\Sigma(t)x_j & \text{else} \end{cases}$$

$$(\text{i.e. } \omega(\Sigma'(t)) := \begin{cases} \sharp & \text{iff} \quad t^- \not\leq m' \\ x_j & \text{iff} \quad (t^- \not\leq m \wedge t^- \leq m')\vee \\ & \quad\quad\ (t^- \leq m \wedge t^- \leq m' \wedge Ft \cap F\hat{t} \neq \emptyset) \\ \omega(\Sigma(t)) + x_j & \text{else} \end{cases}\ )$$

$$(4)\ B' := B \cup \{A^2 u_t \Sigma(\hat{t})\} \cup \{A^2 \Sigma'(t) v_t \mid t^- \leq m'\}$$

$$(\text{ i.e. } \omega(B') := \omega(B) \cup \{eft(\hat{t}) \leq \omega(\Sigma(\hat{t}))\} \cup \{\omega(\Sigma'(t)) \leq lft(t) \mid t^- \leq m'\})$$

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Example:

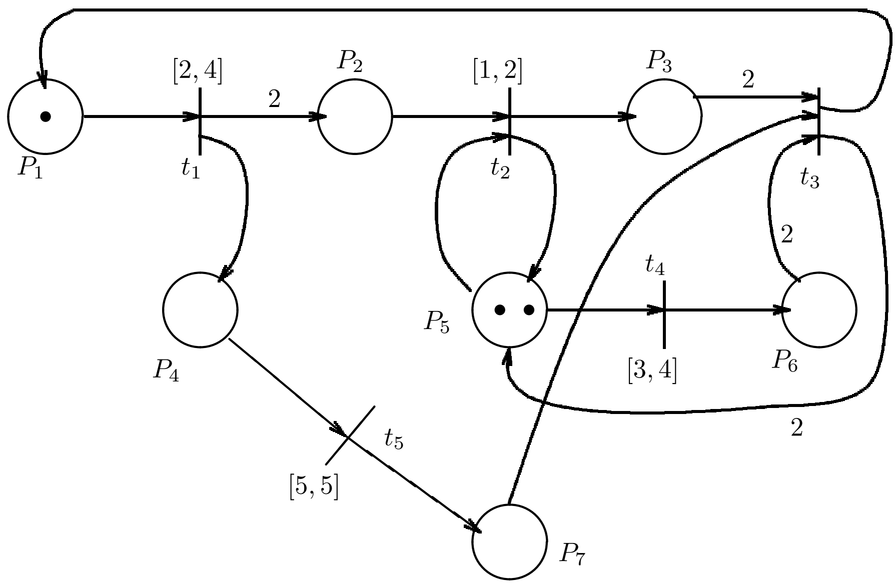

Figure 1:

$$\omega(\delta(e)) = \{(1,0,0,0,2,0,0), \begin{pmatrix} x_1 \\ \sharp \\ \sharp \\ x_1 \\ \sharp \end{pmatrix} \mid \begin{matrix} 0 \leq x_1 \\ x_1 \leq 4 \end{matrix}\}.$$

$$\omega(\delta(t_1)) = \{(0,2,0,1,2,0,0), \begin{pmatrix} \sharp \\ x_2 \\ \sharp \\ x_1+x_2 \\ x_2 \end{pmatrix} \mid \begin{array}{l} 2 \leq x_1 \\ x_1 \leq 4 \\ 0 \leq x_2 \\ x_2 \leq 2 \\ 0 \leq x_1+x_2 \\ x_1+x_2 \leq 4 \end{array} \}, ...$$

$$\omega(\delta(t_1t_4t_2t_5)) = \{(0,1,1,0,1,1,1), \begin{pmatrix} \sharp \\ x_4+x_5 \\ \sharp \\ x_4+x_5 \\ \sharp \end{pmatrix} \mid \begin{array}{l} 2 \leq x_1,\ x_1 \leq 4,\ 0 \leq x_1+x_2,\ x_1+x_2 \leq \\ 2,\ 0 \leq x_2,\ x_2 \leq 2,\ 3 \leq x_1+x_2,\ x_2+x_3 \leq \\ 5,\ x_3 \leq 2,\ 1 \leq x_3,\ x_2+x_3+x_4 = 5,\ 0 \leq \\ x_4,\ x_4 \leq 2,\ 0 \leq x_4+x_5 \leq 2 \end{array} \}.$$

etc...

Thus, $\omega(\delta(\sigma))$ is the set of all states, which can be reached when the transition sequence σ has fired in the TPN Z starting at the initial state z_0. That is why we will sometimes use the notion C_0 resp. $C_{l(\sigma)}$, where hold: $C_0 = \omega(\delta(e))$ and $C_{l(\sigma)} = \omega(\delta(\sigma))$ and we write $C_0 \xrightarrow{\sigma} C_{l(\sigma)}$.

Properties

(1) Let Z be a TPN, $\sigma \in T^*, C_0 \xrightarrow{\sigma} C_{l(\sigma)}$ ($\omega(\delta(\sigma)) = [m, \Sigma, B]$). Then, for each state $z \in C$ there is a value assignment β of the variables under the interpretation ω such that: $z = (m, \beta(\Sigma))$ and $\bigwedge_{b \in B} \beta \ sat\ b$.

(2) (inversion of (1))
Let Z be a TPN, $\sigma \in T^*, C_0 \xrightarrow{\sigma} C_{l(\sigma)}$ ($\omega(\delta(\sigma)) = [m, \Sigma, B]$). For each value assignment β of the variables under the interpretation ω with $\bigwedge_{b \in B} \beta \ sat\ b$ holds: $z := (m, \beta(\Sigma)) \in C$.

(3) Let Z be a TPN, $\sigma \in T^*, C_0 \xrightarrow{\sigma} C_{l(\sigma)}$ ($\omega(\delta(\sigma)) = [m, \Sigma, B]$) and $z := (m, J)$ be an arbitrary state in $C_{l(\sigma)}$. Then there are states $\underline{z} = (m, \underline{J})$ and $\overline{z} = (m, \overline{J})$ in $C_{l(\sigma)}$ with
$\forall t (t \in T \wedge \leq m \longrightarrow \underline{J}(t) \in \mathcal{N} \wedge 0 \leq J(t) - \underline{J}(t) < 1 \wedge \overline{J}(t) \in \mathcal{N} \wedge 0 \leq \overline{J}(t) - J(t) < 1)$.

(4) Let Z be a TPN and $z = (m, J)$ (with $m \in R_Z(z_0)$). When $\underline{z}$ or $\overline{z}$ are not reachable in the TPN Z, then z is not reachable in Z, too.
The reverse is not true (exactly: When $\underline{z}$ and $\overline{z}$ are two states in the set C, it does not follow that each z with $\underline{z} < z < \overline{z}$ is a state in C).

For the proof of the four properties see Popova[7].

Computation of the time boundaries

It is well known, that the liveness, boundedness, T- and S- Invariants etc. in a given TPN are depended essentially on the time boundaries of its transitions. Our goal now is to give an algorithm for computing eft's and lft's for a given PN such that the arising TPN has a certain behaviour. We will give here a method, which computes time boundaries like that a given enabled transition sequence in the PN remains enabled in the TPN and a branch in the PN disappears in the TPN.

For example, in the timeless PN $S(Z)$ in figure 1 (the skeleton of Z) the sequence $\sigma = t_1t_4t_2t_5$ is enabled. After firing t_5 there is a branch $\begin{smallmatrix} \nearrow t_2 \\ \searrow t_4 \end{smallmatrix}$. We want to define such a TPN with the skeleton $S(Z)$ so that the transition sequence σ is enabled in the TPN and that the branch $\begin{smallmatrix} \nearrow t_2 \\ \searrow t_4 \end{smallmatrix}$ is solved in favour of t_4.

For this purpose we draw up the parametrical canonical description of the sequence $\sigma = t_1t_4t_2t_5$ $\omega(\delta(\sigma)) = C_{l(\sigma)}$. The $eft(t)$ and the $lft(t)$ of each transition t are now unknown. a_t and b_t stand for them. So far it is easy to see, that:

(1) The sequence σ is enabled in Z, iff the inequality system, described by $\omega(B)$ has a solution.

(2) The branch $\begin{smallmatrix}\nearrow t_2\\ \searrow t_4\end{smallmatrix}$ is solved in favour of t_4, iff the value of the optimal solution of the parametrical linear program (PLP) P1 is less than the value of the optimal solution of the PLP P2:

$$\max \quad x_4 + x_5 \qquad\qquad\qquad\qquad \min x_4 + x_5$$

$$(P1)\begin{cases} x_1 & \le b_1\\ x_1 & \le b_4\\ -x_1 & \le -a_1\\ x_2 & \le b_2\\ x_2 & \le b_5\\ x_1+x_2 & \le b_4\\ -x_1-x_2 & \le -a_4\\ x_3 & \le b_2\\ x_3 & \le b_4\\ x_2+x_3 & \le b_5\\ -x_3 & \le -a_2\\ x_4 & \le b_2\\ x_4 & \le b_4\\ x_2+x_3+x_4 & \le b_5\\ -x_2-x_3-x_4 & \le -a_5\\ x_4+x_5 & \le b_2\\ x_4+x_5 & \le b_4\\ -x_4-x_5 & \le -a_2\\ x_i \ge 0, i=1,...,5 & \end{cases} \qquad (P2)\begin{cases} x_1 & \le b_1\\ x_1 & \le b_4\\ -x_1 & \le -a_1\\ x_2 & \le b_2\\ x_2 & \le b_5\\ x_1+x_2 & \le b_4\\ -x_1-x_2 & \le -a_4\\ x_3 & \le b_2\\ x_3 & \le b_4\\ x_2+x_3 & \le b_5\\ -x_3 & \le -a_2\\ x_4 & \le b_2\\ x_4 & \le b_4\\ x_2+x_3+x_4 & \le b_5\\ -x_2-x_3-x_4 & \le -a_5\\ x_4+x_5 & \le b_2\\ x_4+x_5 & \le b_4\\ -x_4-x_5 & \le -a_4\\ x_i \ge 0, i=1,...,5 & \end{cases}.$$

In general, in a PN a sequence $\sigma = t_1...t_r$ is given and followed by a branch $\begin{smallmatrix}\nearrow t_{r+1}\\ \searrow t_{r+2}\end{smallmatrix}$. We search time bounderies for the transitions thus σ is enabled in the yielded TPN and the branch is solved in favour of t_{r+1}. For this reason we have to solve the inequality:

$$\lambda \; < \; \mu,$$

where λ is the value of the optimal solution of the PLP P1 and μ is the value of the optimal solution of the PLP P2

(P1) $\max\ \omega(\Sigma(t_{r+1}))$ subject to $\omega(B) \cup \{eft(t_{r+1}) \le \omega(\Sigma(t_{r+1}))\}$

and

(P2) $\min\ \omega(\Sigma(t_{r+2}))$ subject to $\omega(B) \cup \{eft(t_{r+2}) \le \omega(\Sigma(t_{r+2}))\}$.

For solving the parametrical linear programs see Kausmann/Lommatzsch/Nožička.

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