Symbolic Unfoldings of Time Petri Nets

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joint work with Claude Jard

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Time Petri Nets

- Introduced by Merlin and Farber in 1976
- Specification of real-time concurrent systems
- Time constraints: intervals of possible firing delays
- Strong time semantics
- We consider safe time Petri nets
  - Several undecidable results in the general case...

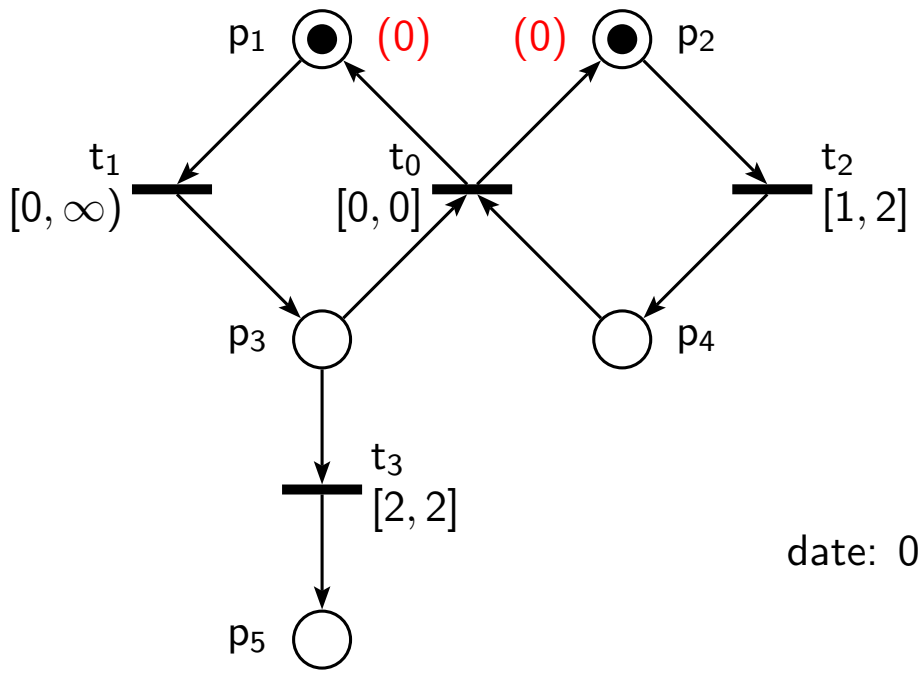
Safe Time Petri Nets: Definition

$\langle P, T, pre, post, efd, lfd \rangle$

- ${}^\bullet t \stackrel{\text{def}}{=} pre(t) \subseteq P$
- $t^\bullet \stackrel{\text{def}}{=} post(t) \subseteq P$
- earliest firing delay:
  $efd : T \longrightarrow Q$
- latest firing delay:
  $lfd : T \longrightarrow Q \cup \{\infty\}$

State $\langle M, dob, \theta \rangle$

- $M \subseteq P$ marking
- θ date
- date of birth:
  $\forall p \in M \quad dob(p) \leq \theta$



Safe Time Petri Nets: Semantics

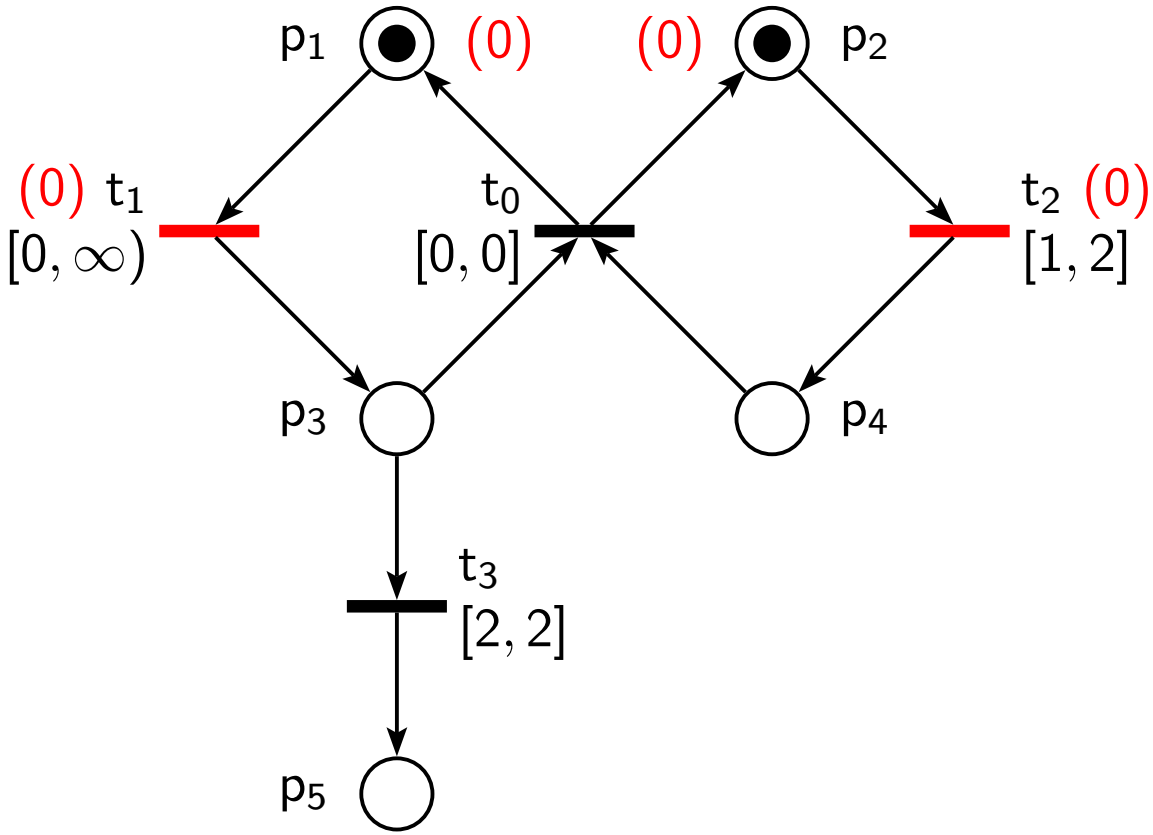
Transition t can fire at time $\theta' \geq \theta$ from state $\langle M, dob, \theta \rangle$ if:

- t is enabled: ${}^\bullet t \subseteq M$;
- the minimum delay is reached:
  $\theta' \geq doe(t) + efd(t)$;
- the enabled transitions do not overtake the maximum delays:
  $\forall t' \in T \quad {}^\bullet t' \subseteq M \implies \theta' \leq doe(t') + lfd(t')$.

where $doe(t) \stackrel{\text{def}}{=} \max_{p \in {}^\bullet t} dob(p)$.

Example of Time Petri Net

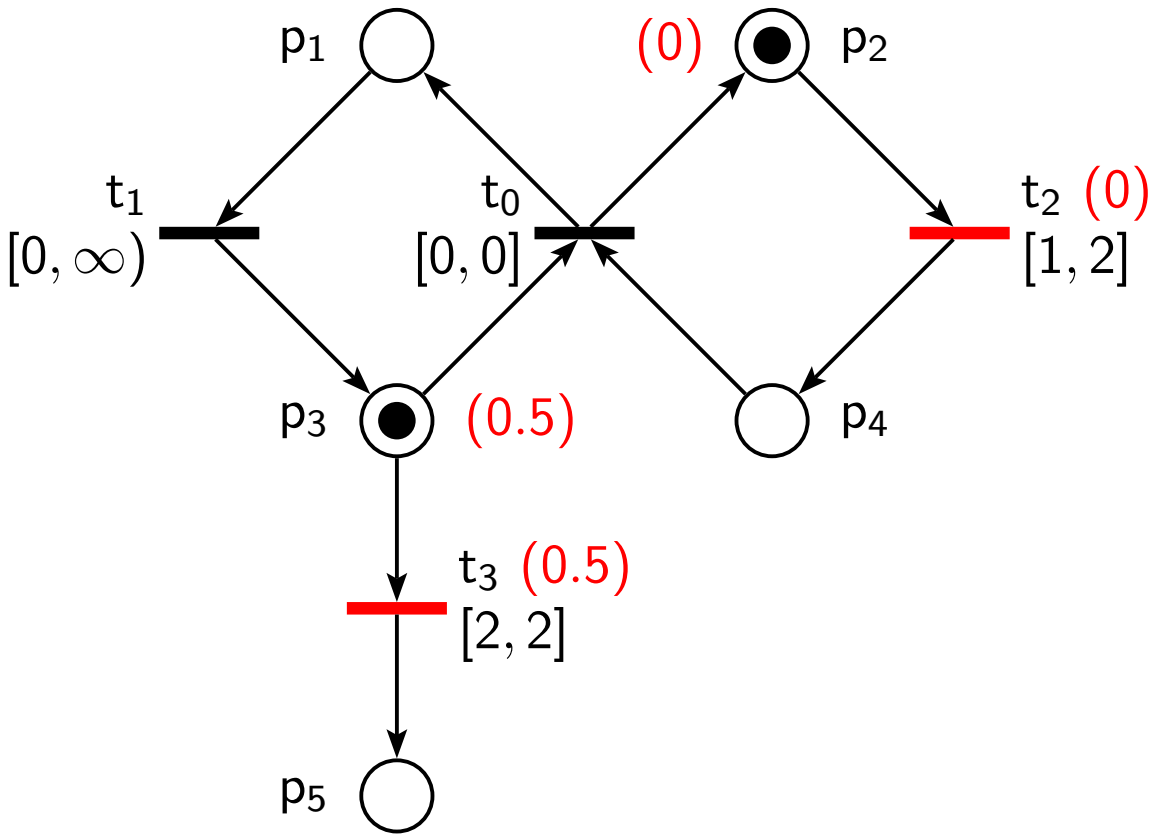
date: 0.0



Example of Time Petri Net

$(t_1, 0.5)$

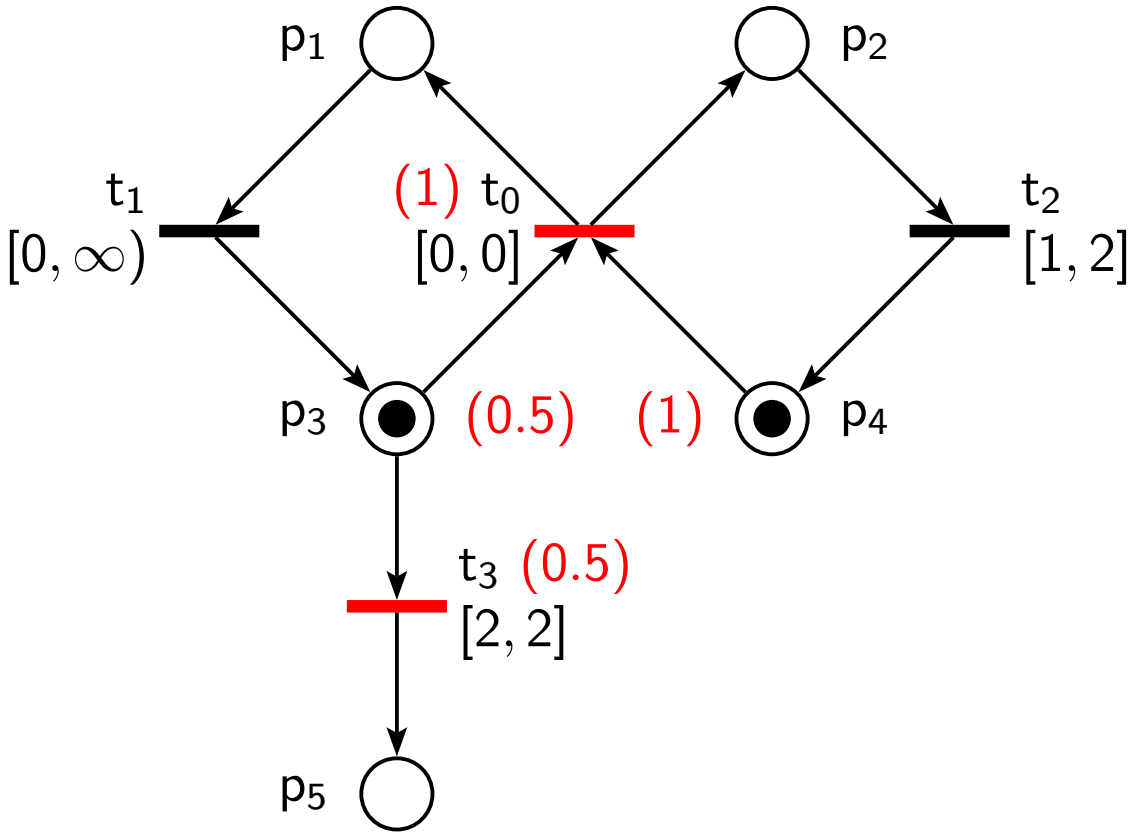
date: 0.5



Example of Time Petri Net

$(t_1, 0.5), (t_2, 1)$

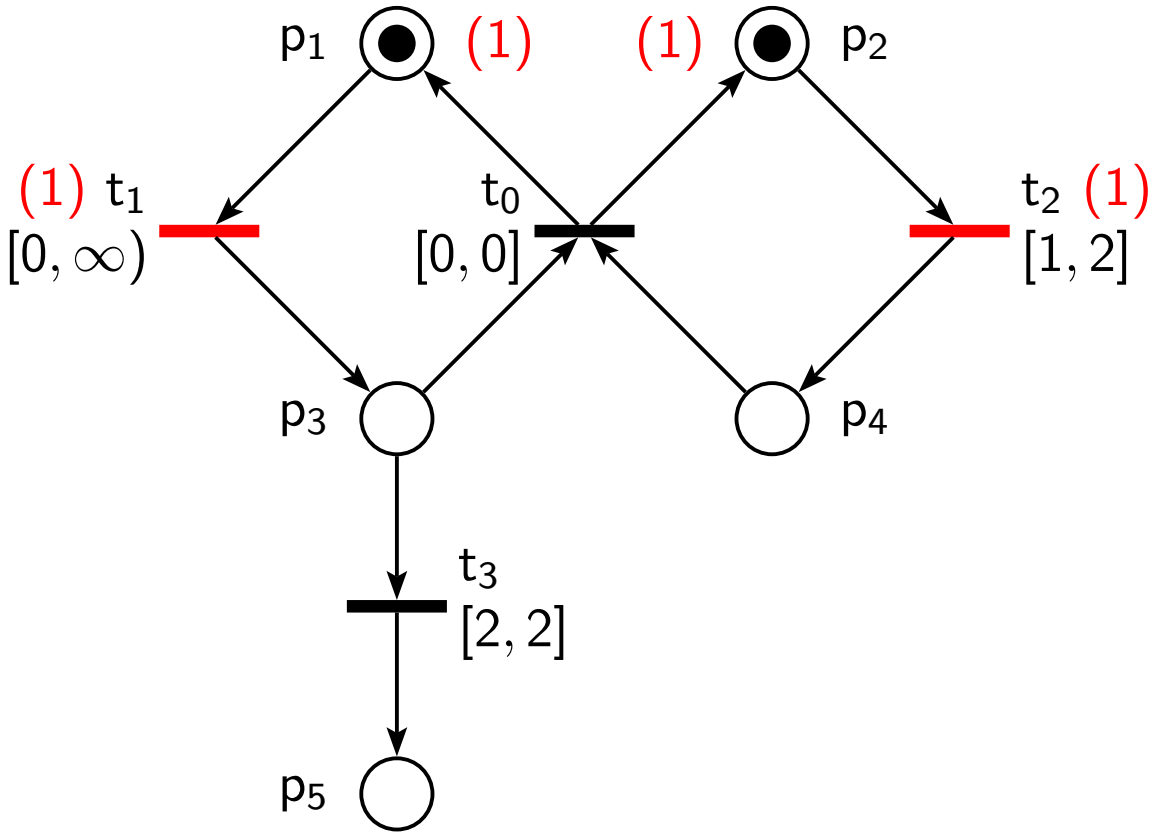
date: 1.0



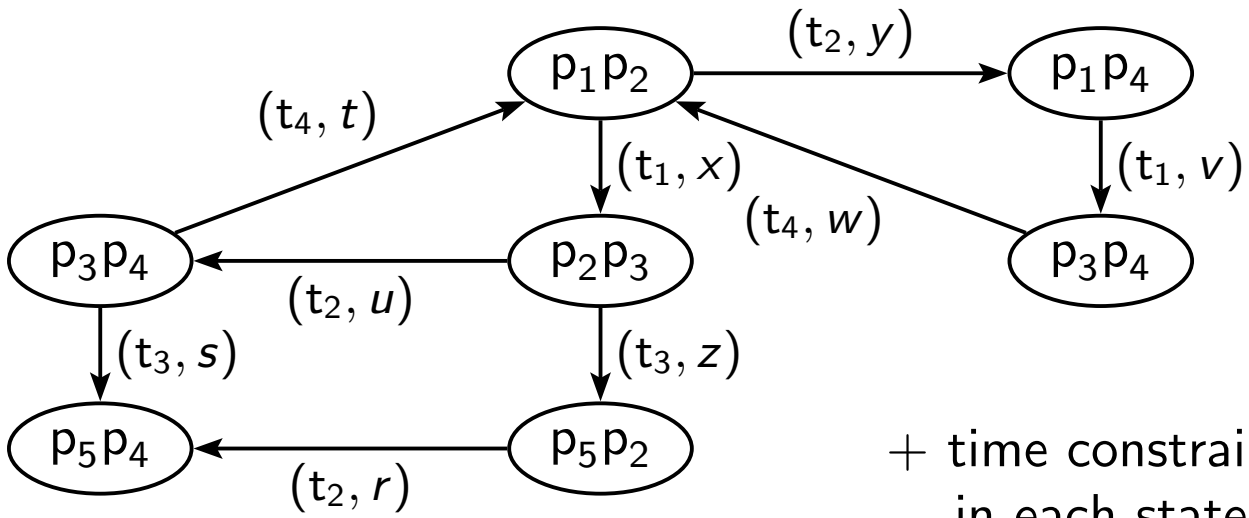
Example of Time Petri Net

$(t_1, 0.5), (t_2, 1), (t_0, 1)$

date: 1.0

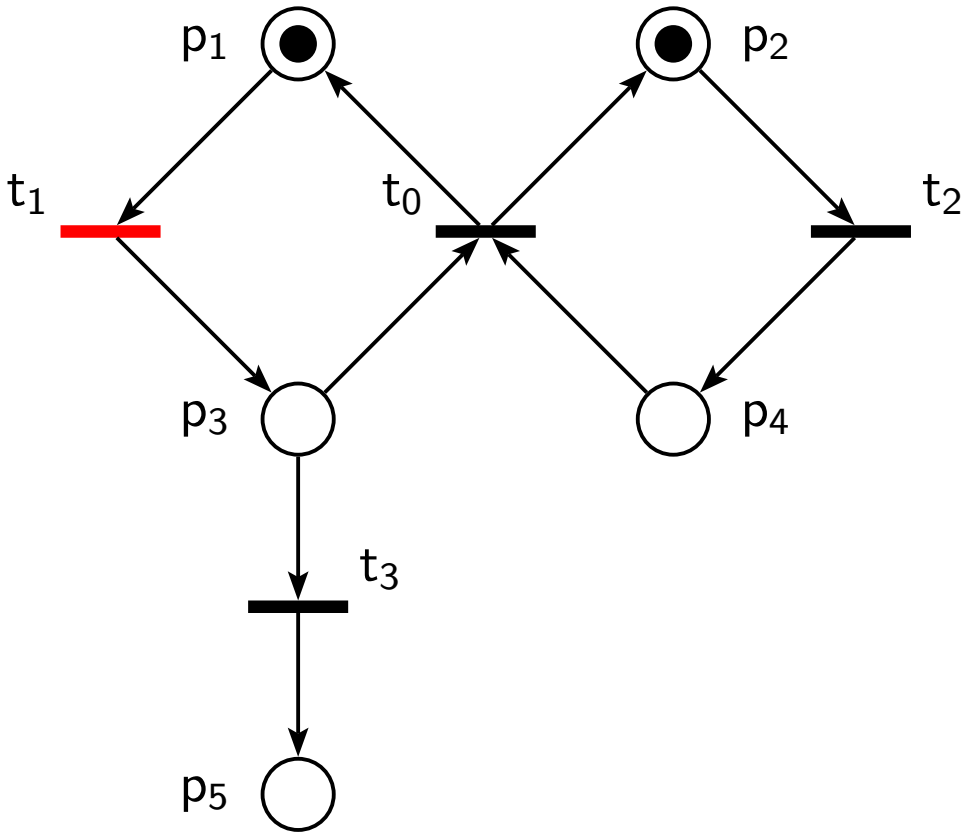


Executions in Interleaving Semantics

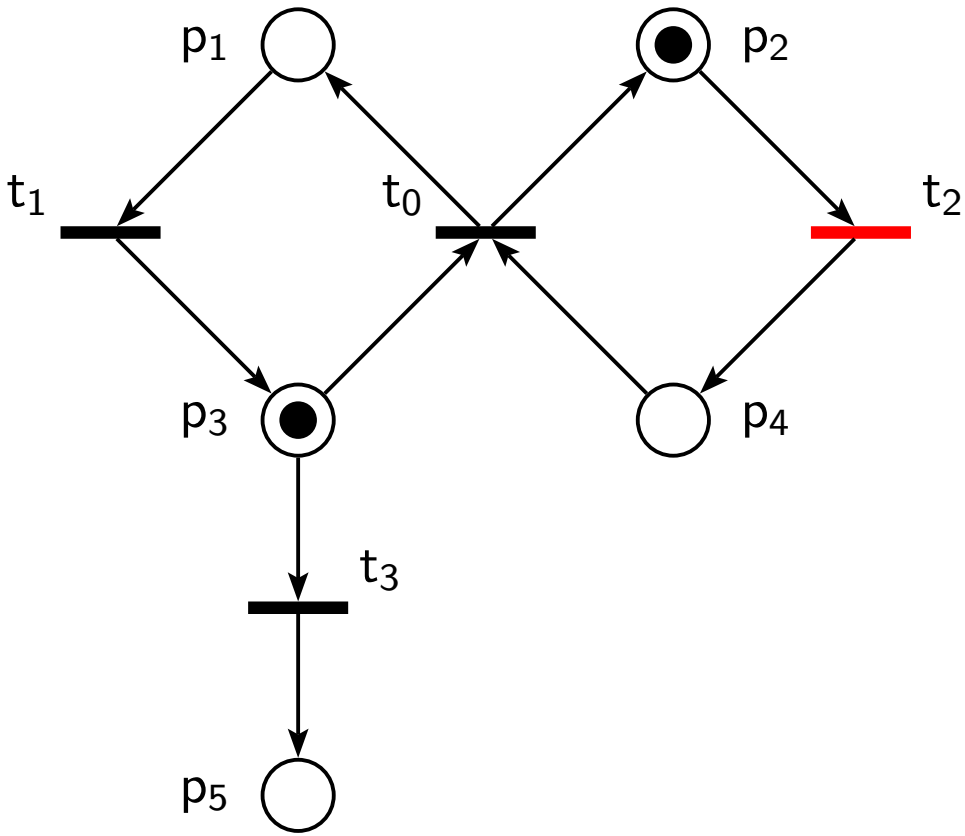


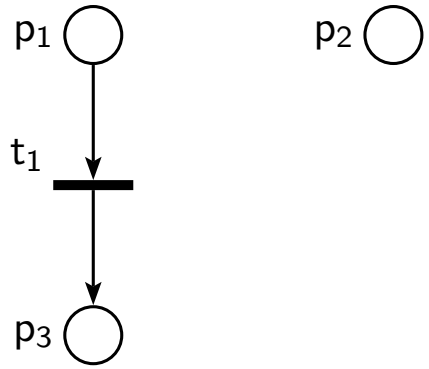
B. Berthomieu and M. Diaz, Modeling and verification of time dependent systems using time Petri nets, IEEE Transactions on Software Engineering, 17(3):259–273, 1991

Processes of Untimed Petri Nets

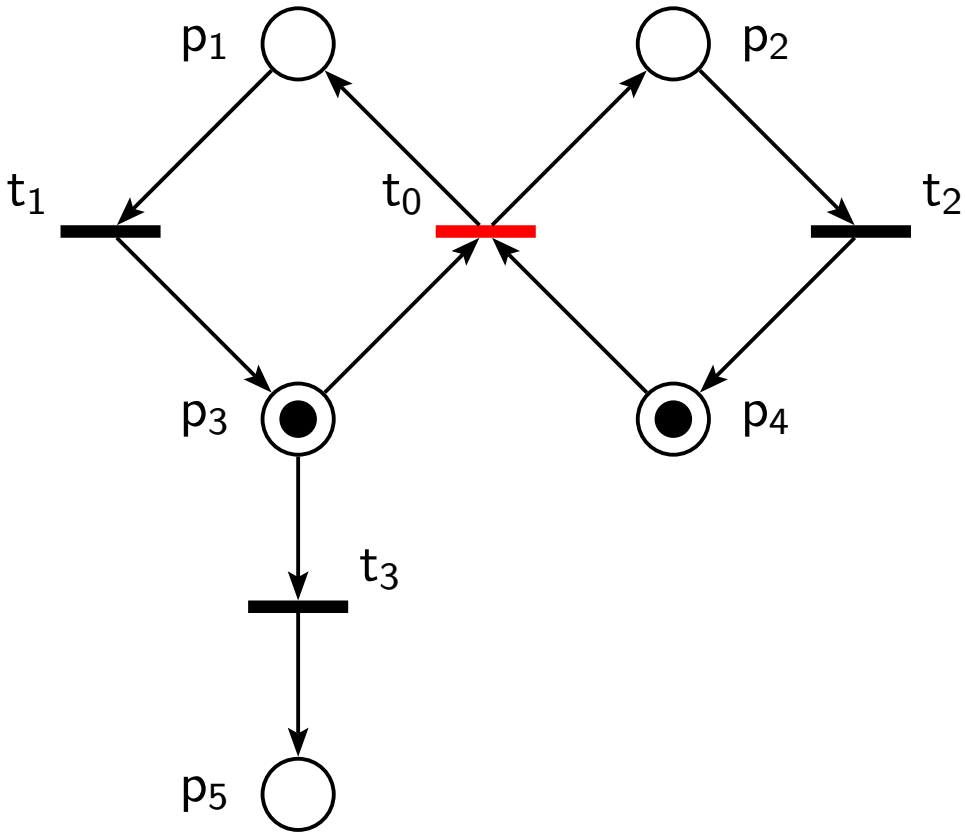


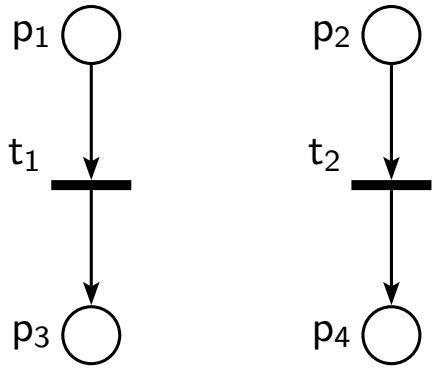
Processes of Untimed Petri Nets



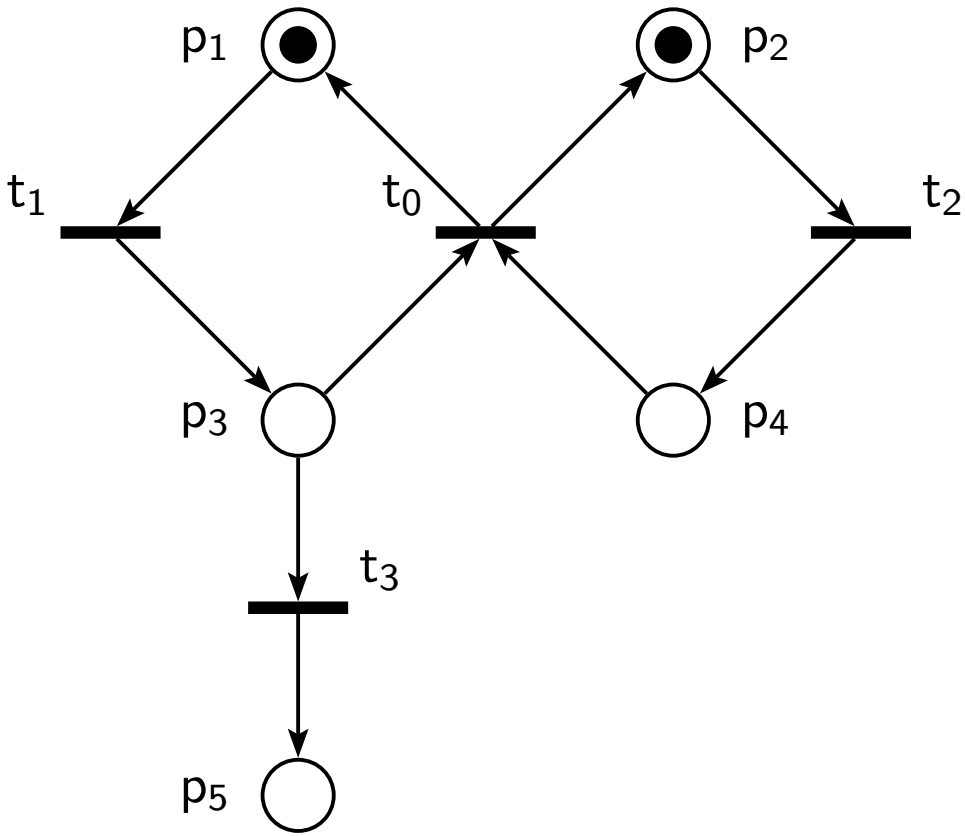


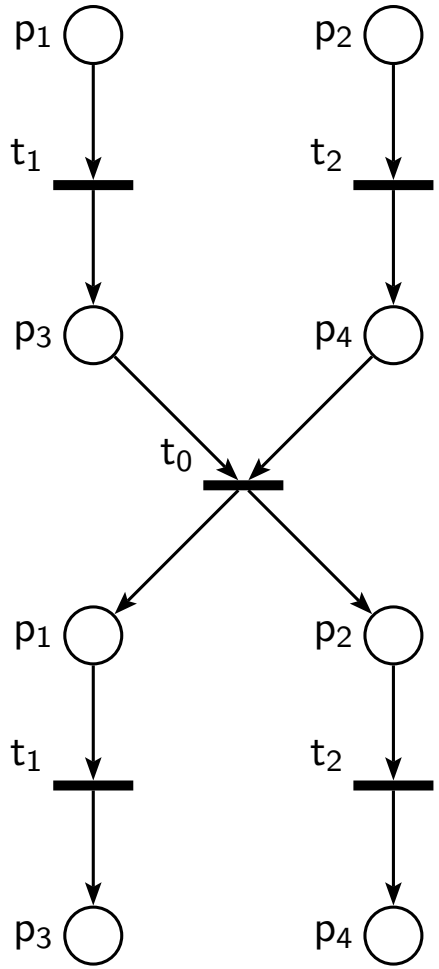
Processes of Untimed Petri Nets





Processes of Untimed Petri Nets





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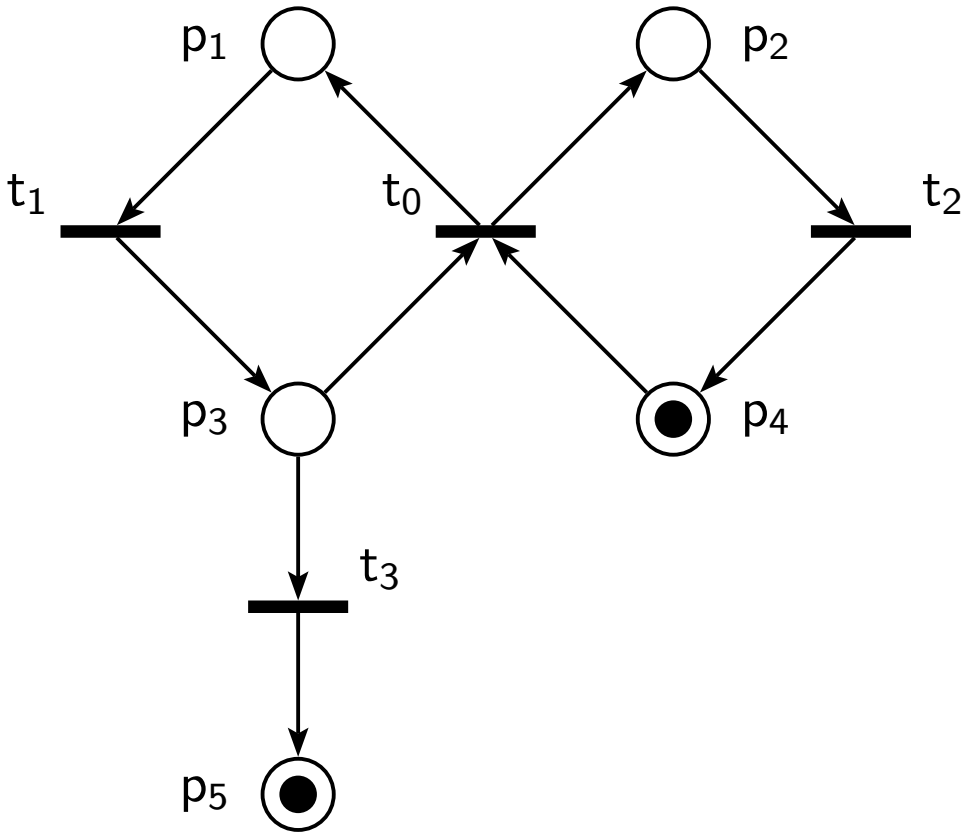
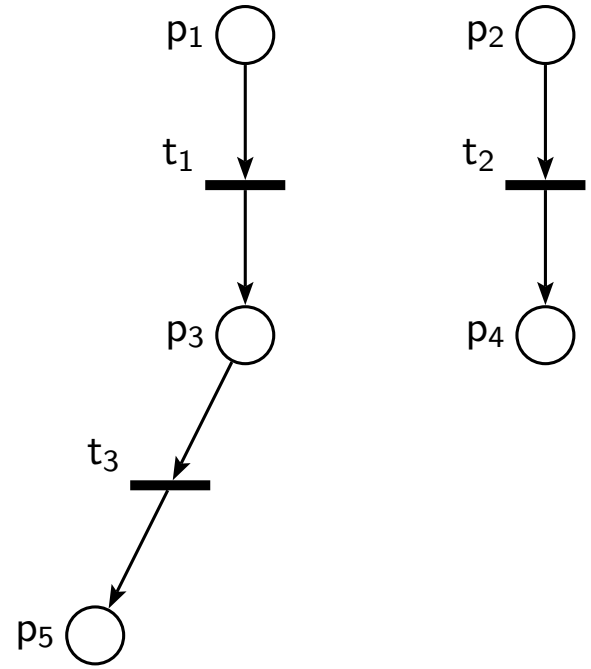
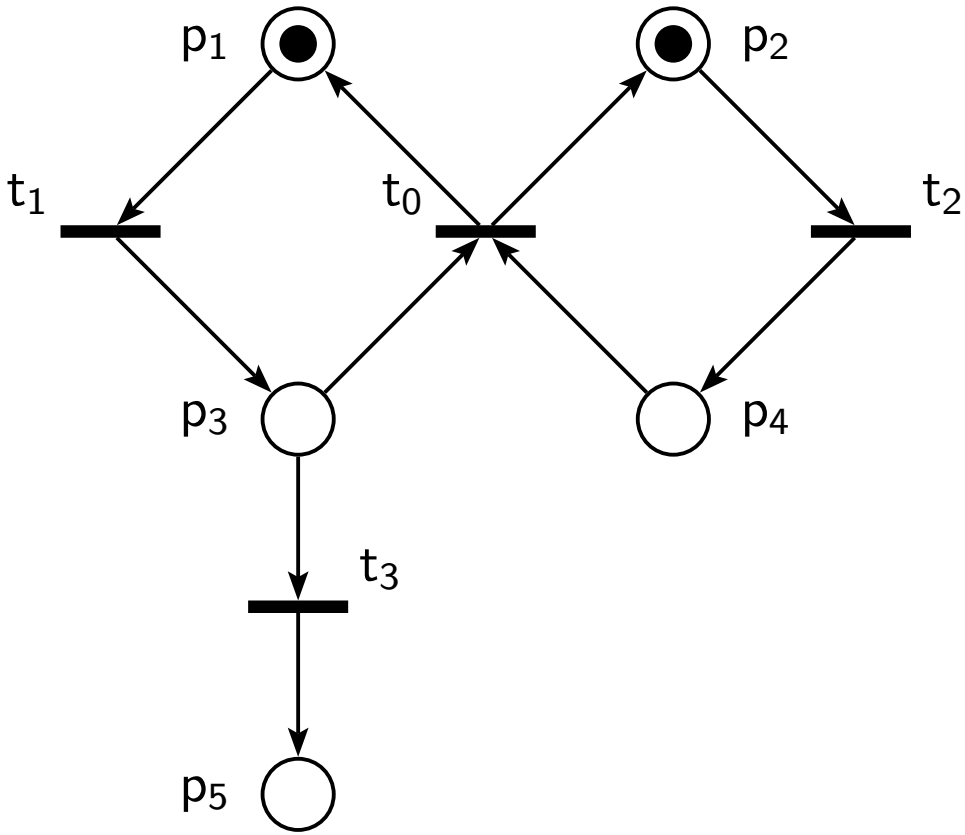
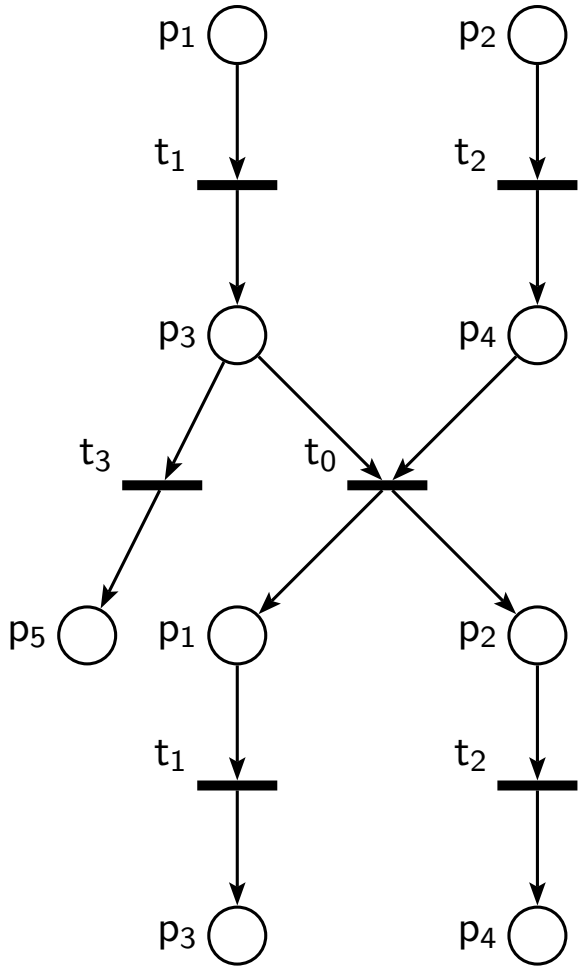




Unfoldings of Untimed Petri Nets

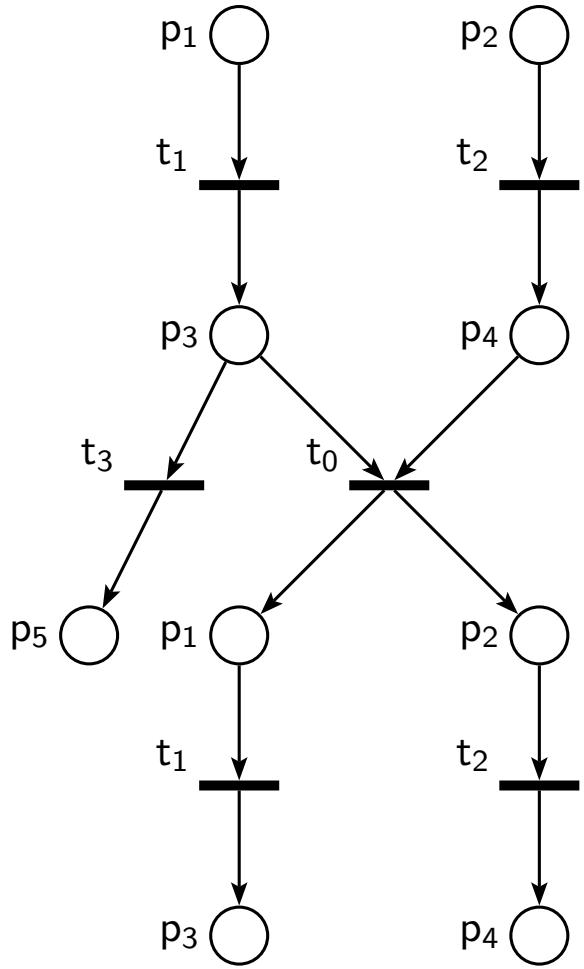




Unfoldings of Untimed Petri Nets

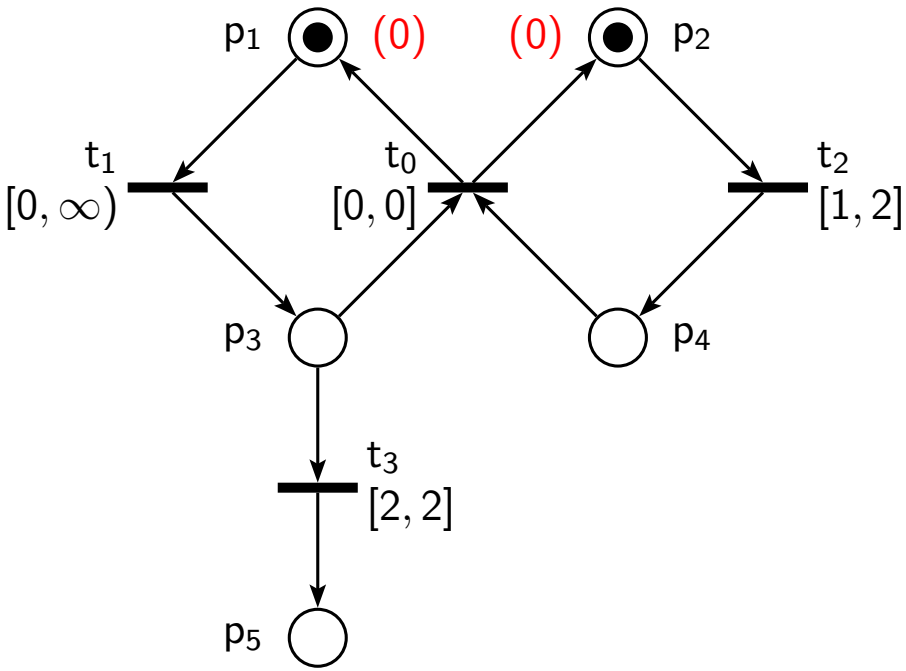
All the processes have been superimposed.

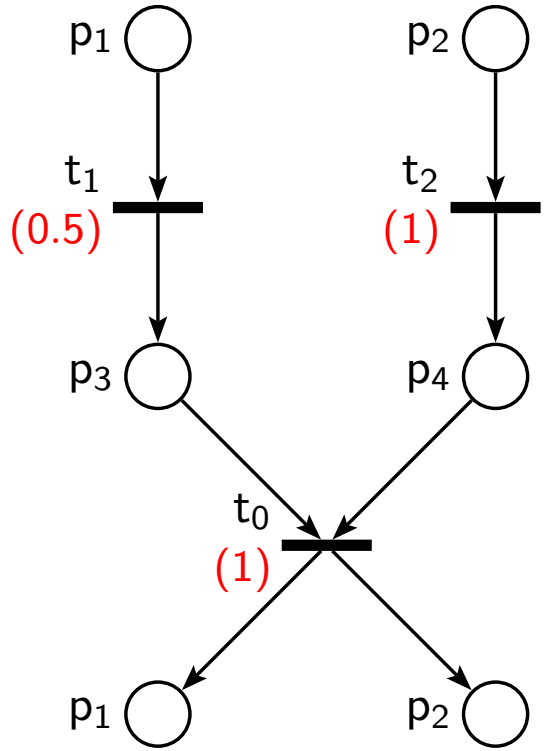
- Given a set E of events, is it a process?
- Given a set E of events, does there exist a process F such that $E \subseteq F$?
- Given a set C of conditions, does there exist a process E such that C are final conditions of E? (useful for construction)



Processes of Time Petri Nets

$(t_1, 0.5), (t_2, 1), (t_0, 1)$





Other dates are possible with the same structure $\rightarrow$ parameters

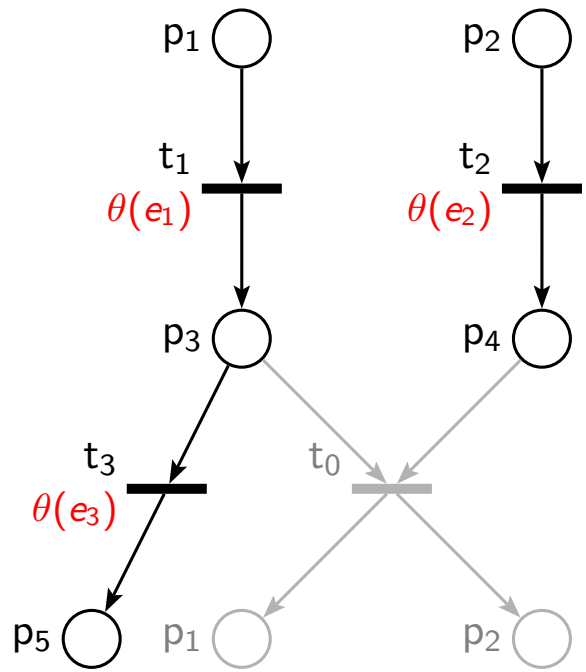
Process of Time Petri Nets – Finding the Possible Dates

Symbolic representation of the dates as parameters.

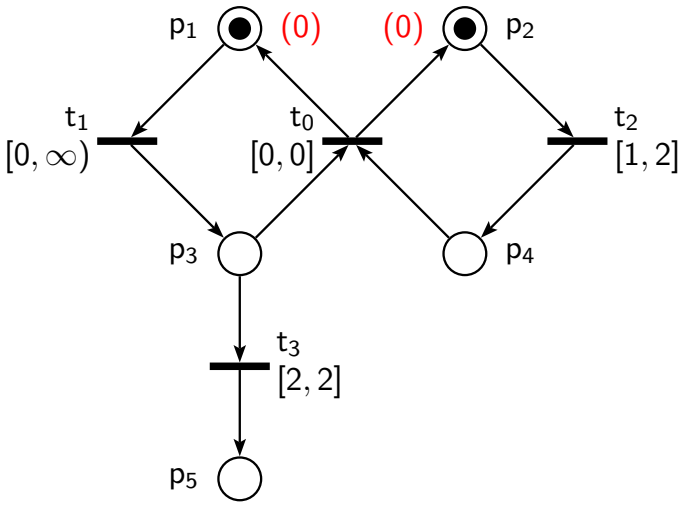
Question: given a process of the underlying Petri net, what are the possible dates for the events?

> Tuomas Aura and Johan Lilius. A causal semantics for time Petri nets. Theoretical Computer Science, 243(2):409-447, (2000)

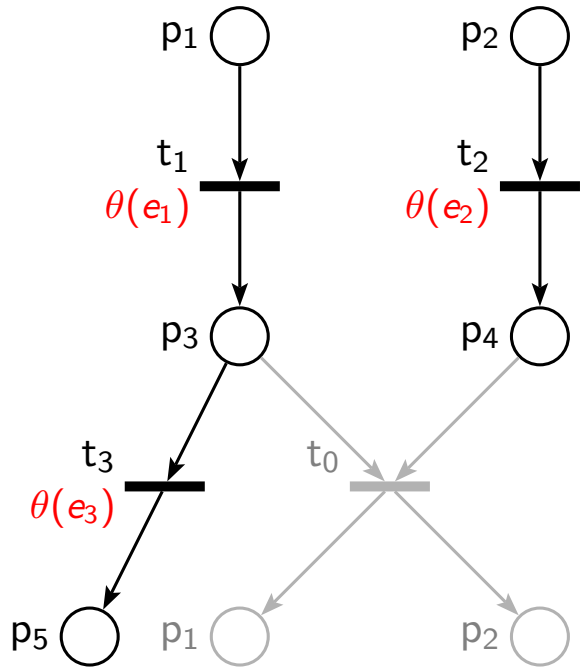
- System of inequalities on the dates of the events.
- Some events that are not in the process must be taken into account.



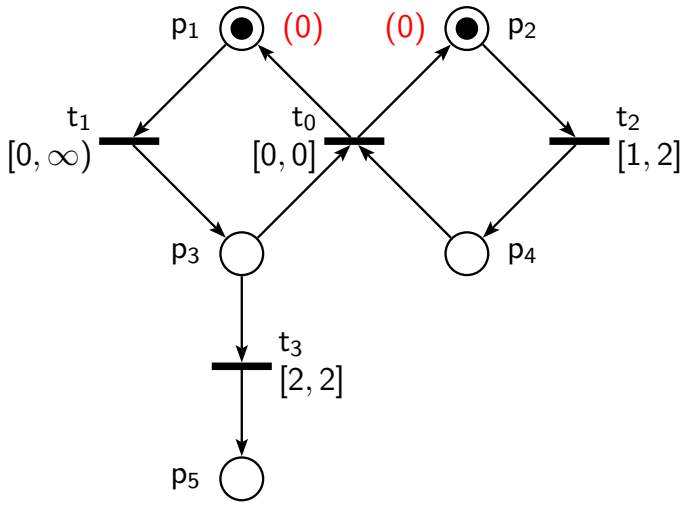
Process of Time Petri Nets – Finding the Possible Dates



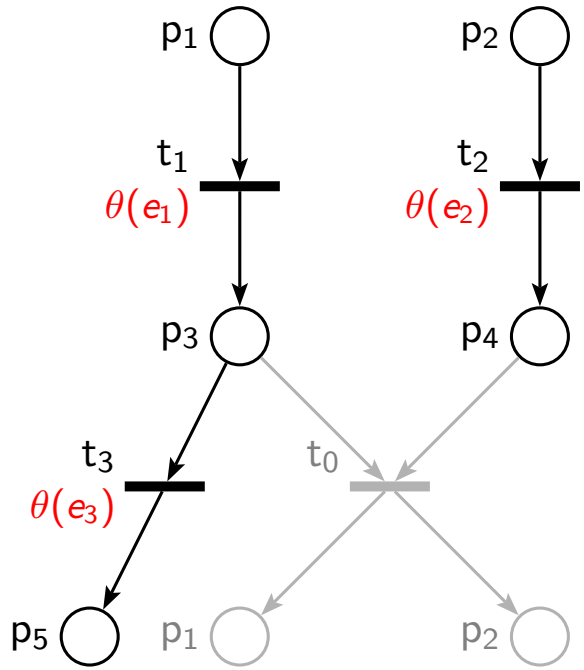
- Firing delays respected
  - $0 \leq \theta(e_1)$
  - $1 \leq \theta(e_2) \leq 2$
  - $\theta(e_3) = \theta(e_1) + 2$



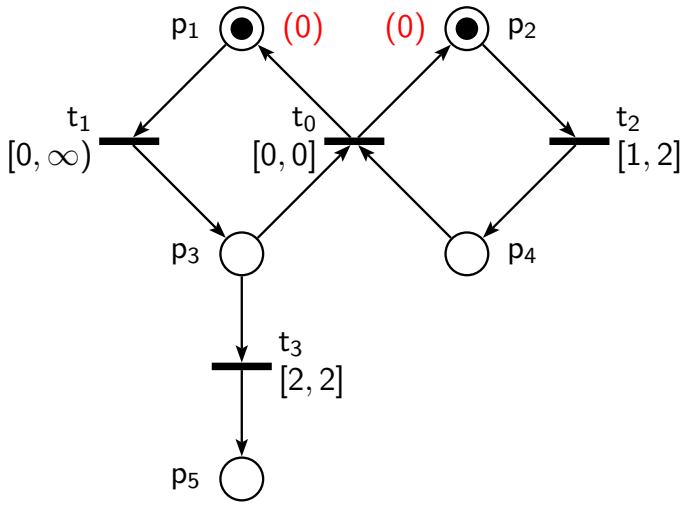
Process of Time Petri Nets – Finding the Possible Dates

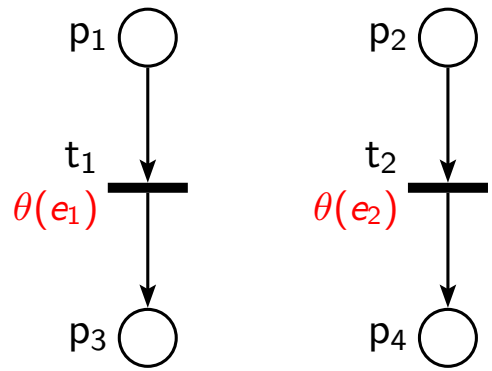


- Firing delays respected
  - $0 \leq \theta(e_1)$
  - $1 \leq \theta(e_2) \leq 2$
  - $\theta(e_3) = \theta(e_1) + 2$
- Disabled events have not overtaken their latest firing delay
  - $\theta(e_3) \leq \max\{\theta(e_1), \theta(e_2)\} + 0$



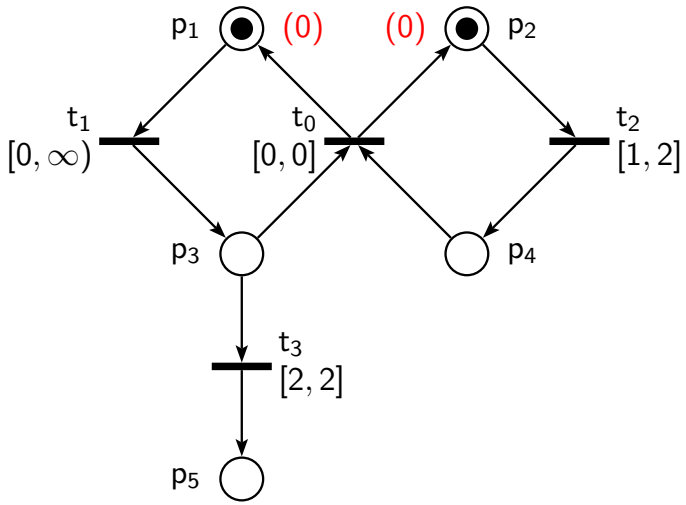
Process of Time Petri Nets – Finding the Possible Dates



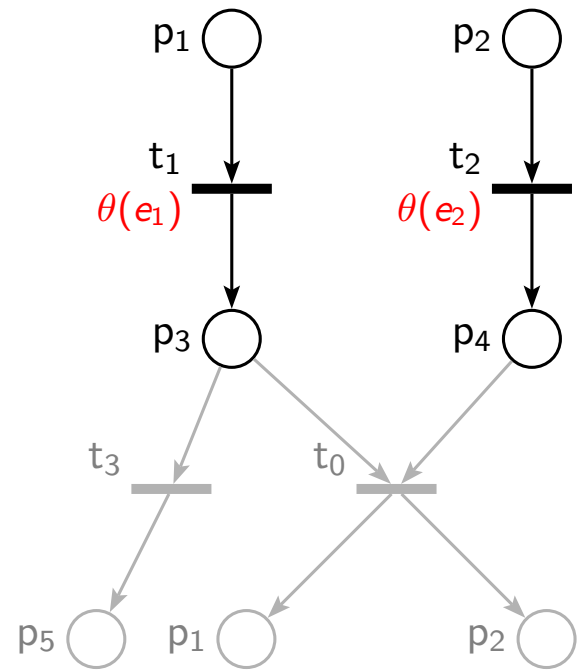


- Firing delays respected
  - $0 \leq \theta(e_1)$
  - $1 \leq \theta(e_2) \leq 2$
- Disabled events have not overtaken their latest firing delay
  - None here

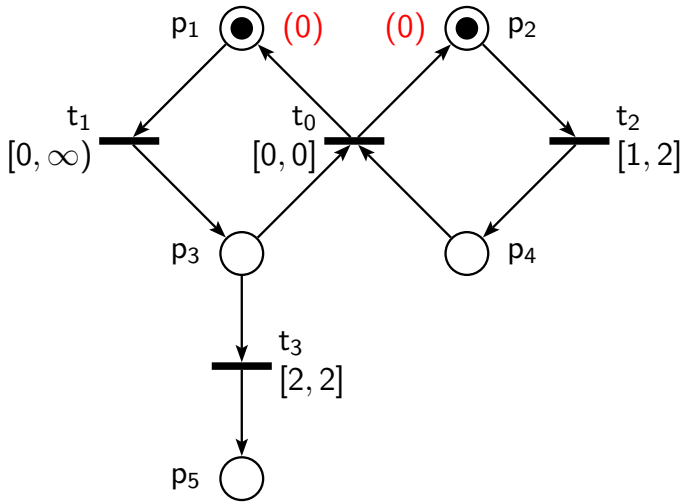
Process of Time Petri Nets – Finding the Possible Dates



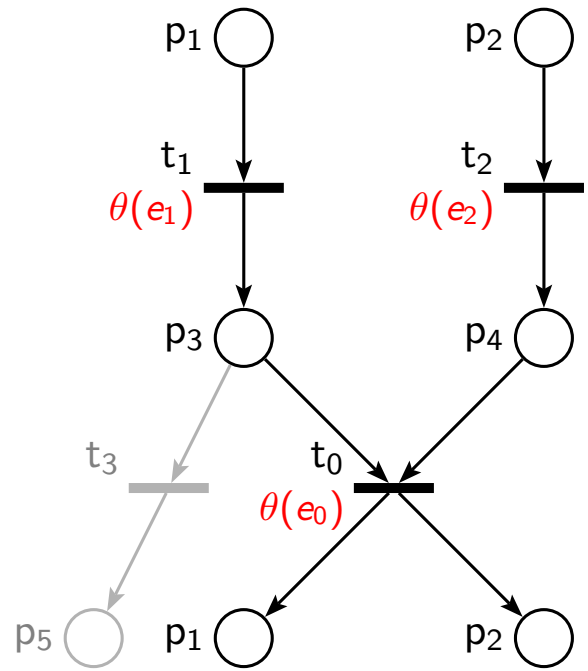
- Firing delays respected
  - $0 \leq \theta(e_1)$
  - $1 \leq \theta(e_2) \leq 2$
- Disabled events have not overtaken their latest firing delay
  - None here
- Events enabled in the final configurations have not fired yet
  - $\max\{\theta(e_1), \theta(e_2)\} \leq \theta(e_1) + 2$ for t_3
  - $\max\{\theta(e_1), \theta(e_2)\} \leq \max\{\theta(e_1), \theta(e_2)\} + 0$ for t_0



Exercise – Finding the possible dates



- Firing delays respected
  - ?
- Disabled events have not overtaken their latest firing delay
  - ?
- Events enabled in the final configurations have not fired yet
  - ?

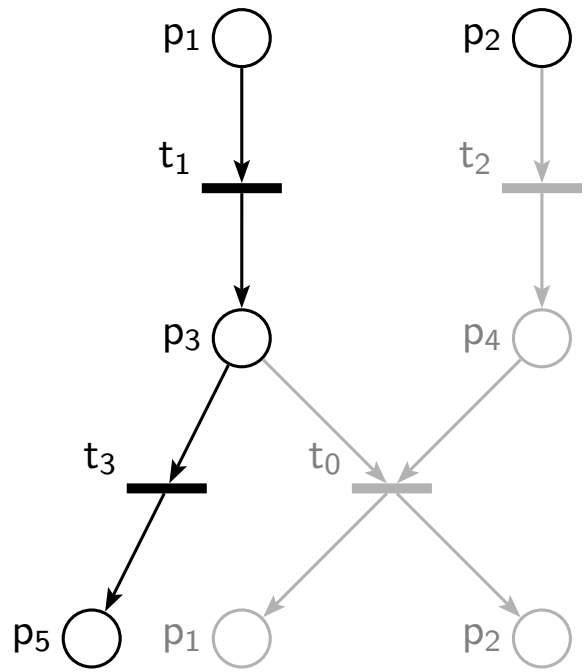


Superimposition of the Processes of a Time Petri Net

Superimposition of all the processes of a time Petri net $\rightarrow$ prefix of the unfolding of the underlying untimed Petri net.

- Given a set E of events, is it a process? $\rightarrow$ answered
- Given a set E of events, does there exist a process F such that $E \subseteq F$?
- Given a set C of conditions, does there exist a process E such that C are final conditions of E? (useful for construction)

Look at what can happen in other parts of the net.
The information that allows a transition to fire is not entirely coded in the corresponding event.
$\rightarrow$ add enough information in the events, but not the global state.



Simple Unfoldings for Time Petri Nets (and Other Models)

Simple cases

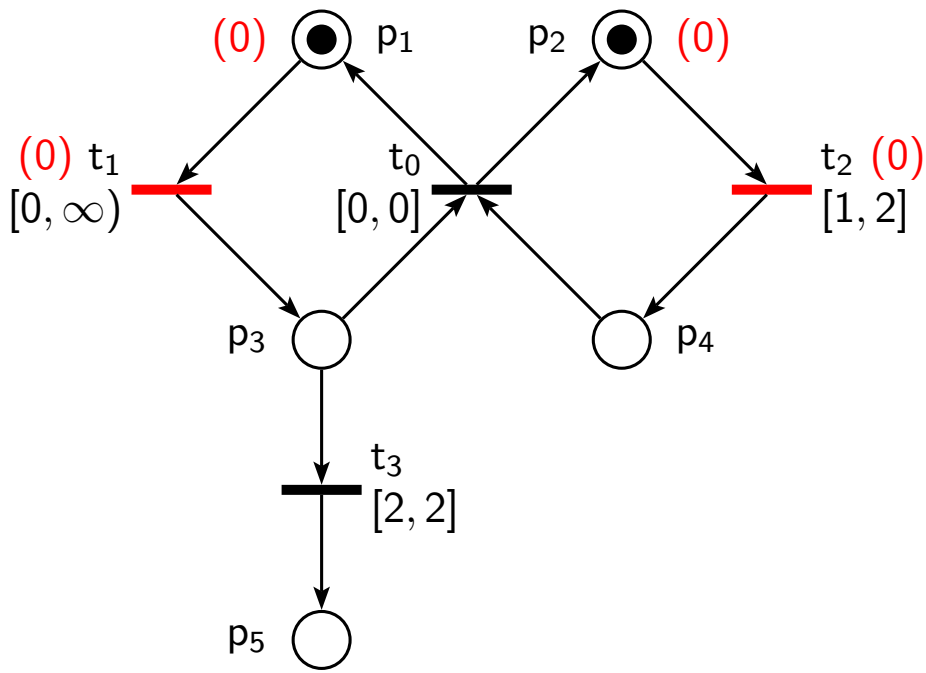
- models without urgency
- models where the choices are local: e.g. extended free choice Time Petri nets

The superimposition of the processes is sufficient for these cases.

Concurrent Operational Semantics for Time Petri Nets

How to simulate a time Petri net without using clocks,
but with as much concurrency as possible?

- Look for local conditions to fire a transition
- Notion of partial state $\langle L, dob, lrd \rangle$
- Partition of P into sets of mutually exclusive places (here $\{p_1, p_3, p_5\}$ and $\{p_2, p_4\}$)
  $\rightarrow$ test the absence of a token
- Executions must map into prefixes of processes



Local Conditions to Fire Transitions

To fire t at θ' from $\langle L, dob, lrd \rangle$, we want (intuitively):

$$\text{for any global state } S \text{ that extends } \langle L, dob, lrd \rangle,$$
$$t \text{ can fire at } \theta' \text{ from } S.$$

We have:

$$t \text{ can fire at } \theta' \text{ from } S$$
$$\text{iff}$$

- t is enabled: ${}^\bullet t \subseteq M$;
- the minimum delay is reached: $\theta' \geq doe(t) + efd(t)$;
- all the enabled transitions in S do not overtake the maximum delays: $\forall t' \in T \quad {}^\bullet t' \subseteq M \implies \theta' \leq doe(t') + lfd(t')$.

$$\text{where } doe(t) \stackrel{\text{def}}{=} \max_{p \in {}^\bullet t} dob(p).$$

Local Firing Condition

Transition t can fire at date θ from the partial state $\langle L, dob, lrd \rangle$ if

- t is enabled: ${}^\bullet t \subseteq L$;
- the minimum delay is reached: $\theta \geq doe(t) + efd(t)$;
- the partial state $\langle L, dob, lrd \rangle$ can remain until θ (whatever happens in other parts of the net)

local stability condition

$$LSC(\langle L, dob, lrd \rangle, \theta)$$

Local Stability Condition

Intuition:

$$LSC(\langle L, dob, lrd \rangle, \theta)$$
iff
for all context S of $\langle L, dob, lrd \rangle$, $\langle L, dob, lrd \rangle$ is stable in S until θ.

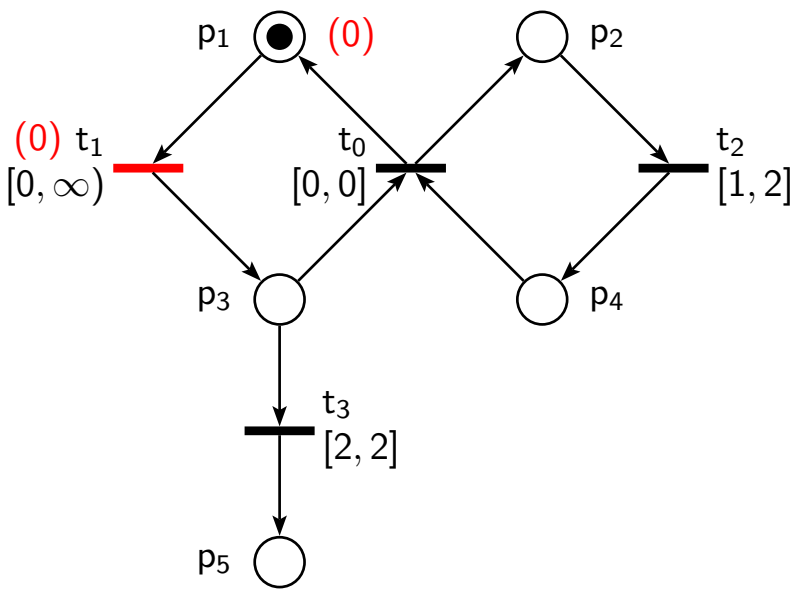
Local Stability Condition

Several choices to define $LSC(\langle L, dob, lrd\rangle, \theta)$:

- trivial choice: $\langle L, dob, lrd\rangle$ is a global state:
  stable until one of the enabled transitions reaches its latest firing delay;
- $\langle L, dob, lrd\rangle$ contains enough information to check that all the transitions that consume tokens in L can wait until θ.

$$\forall t' \in T \quad {}^\bullet t' \cap L \neq \emptyset \implies \begin{cases} & \exists p \in {}^\bullet t' \quad \bar{p} \cap L \neq \emptyset \\ \vee & \theta \leq \max\limits_{p \in {}^\bullet t' \cap L} dob(p) + lfd(t') \end{cases}$$

where $\bar{p}$ denotes the set of places mutually exclusive with p. E.g. $\bar{p_1} = \{p_3, p_5\}$.



$\{(p_1, 0)\}$ is stable forever

Local Stability Condition

Several choices to define $LSC(\langle L, dob, lrd\rangle, \theta)$:

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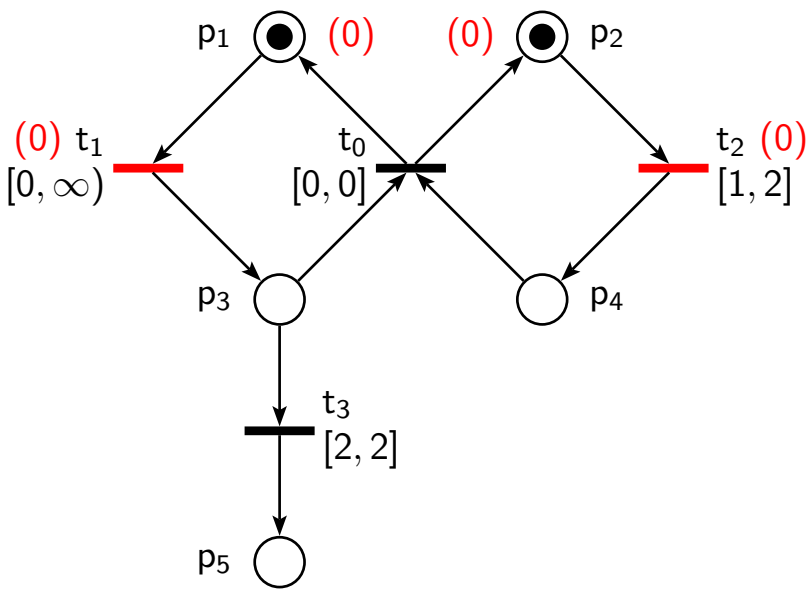

$\{(p_1, 0), (p_2, 0)\}$ is stable until $\theta = 2$

Local Stability Condition

Several choices to define $LSC(\langle L, dob, lrd\rangle, \theta)$:

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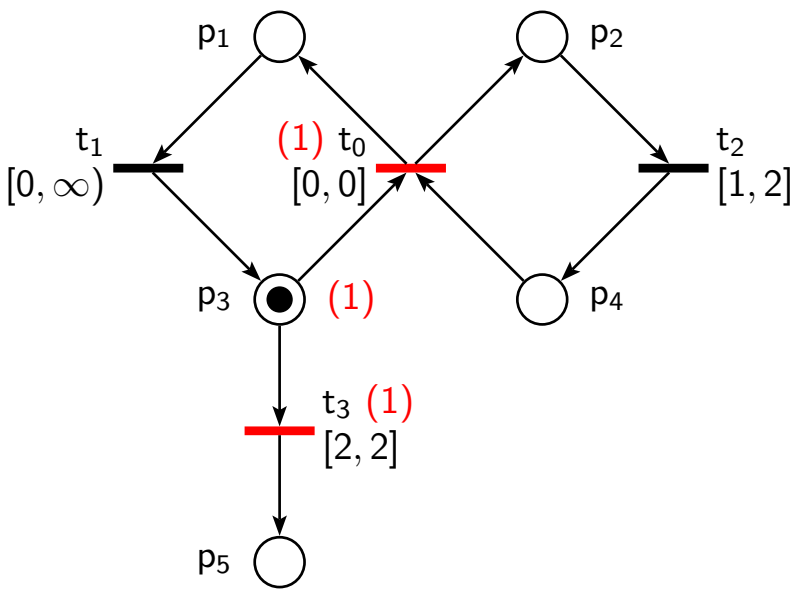

$\{(p_3, 1)\}$ is stable until $\theta = 1$

Local Stability Condition

Several choices to define $LSC(\langle L, dob, lrd\rangle, \theta)$:

- trivial choice: $\langle L, dob, lrd\rangle$ is a global state:
  stable until one of the enabled transitions reaches its latest firing delay;
- $\langle L, dob, lrd\rangle$ contains enough information to check that all the transitions that consume tokens in L can wait until θ.

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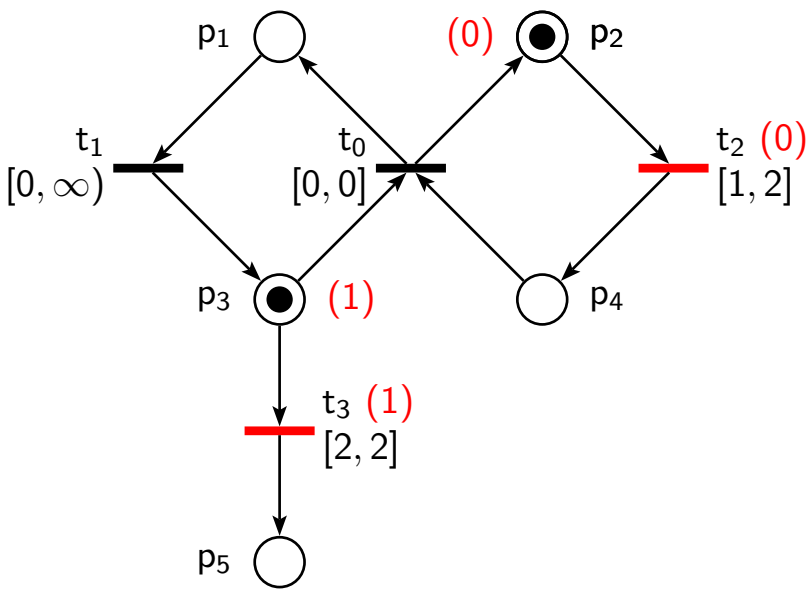


$\{(p_3, 1), (p_2, 0)\}$ is stable until $\theta = 2$

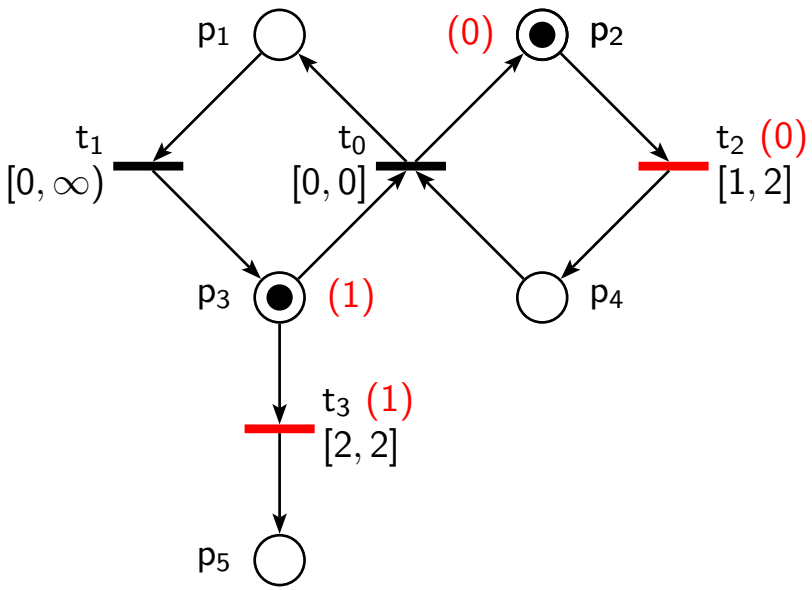
Local Stability Condition

Several choices to define $LSC(\langle L, dob, lrd\rangle, \theta)$:

- trivial choice: $\langle L, dob, lrd\rangle$ is a global state:
  stable until one of the enabled transitions reaches its latest firing delay;
- $\langle L, dob, lrd\rangle$ contains enough information to check that all the transitions that consume tokens in L can wait until θ.

$$\forall t' \in T \quad {}^\bullet t' \cap L \neq \emptyset \implies \begin{cases} & \exists p \in {}^\bullet t' \quad \bar{p} \cap L \neq \emptyset \\ \vee & \theta \leq \max\limits_{p \in {}^\bullet t' \cap L} dob(p) + lfd(t') \end{cases}$$

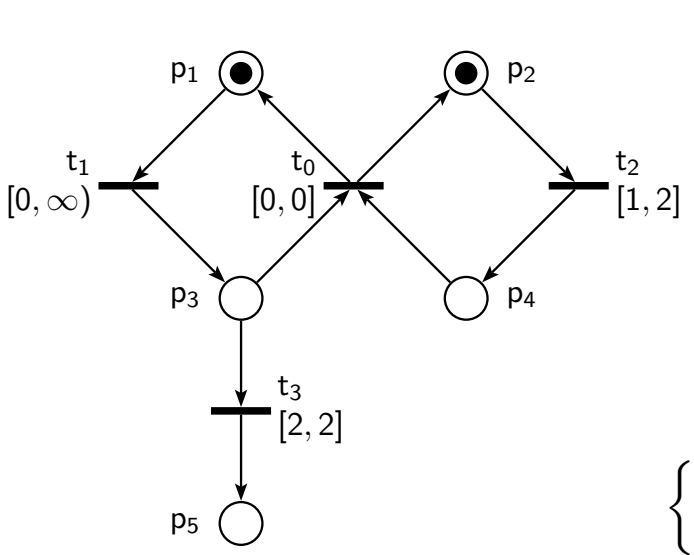
where $\bar{p}$ denotes the set of places mutually exclusive with p. E.g. $\bar{p_1} = \{p_3, p_5\}$.

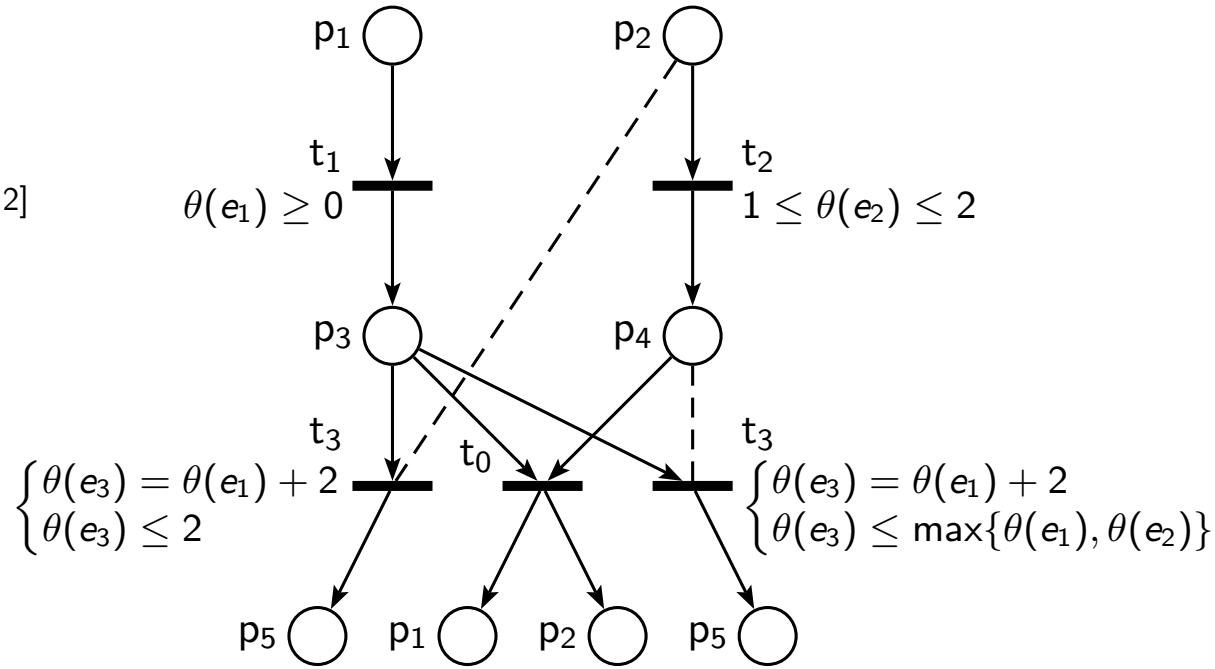


Exercise

Play with local stability conditions!

Symbolic Unfolding





- In symbolic unfoldings: keep track of all the places in L (not only those that are consumed) $\rightarrow$ use read arcs (or consume and rewrite).
- Use only minimal sets L to increase concurrency.
- Redundancy
- Solve constraints on the dates of the events

Conclusion

- Concurrent operational semantics for TPN
- (Parameterized) local stability condition
- Solve constraints on the dates of the events
- Study of the form of the constraints
  $\rightarrow$ finite complete prefix of the unfolding
- If there is no urgency or the choices are local (extended free choice Petri net), the unfolding is simply the superimposition of the processes