Essential States in Time Petri Nets

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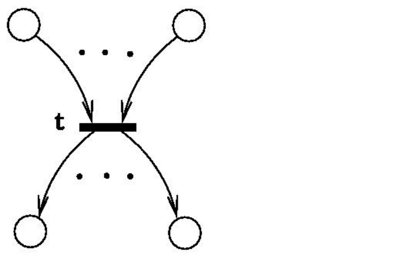
Outline

1. Time Petri Nets
   - Rounding of Runs
   - Essential States
   - Reachable Graph

Conception



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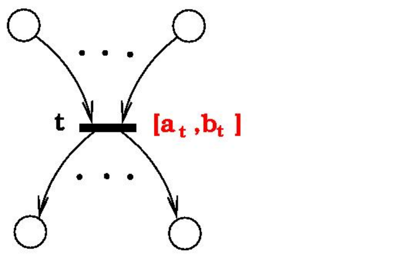

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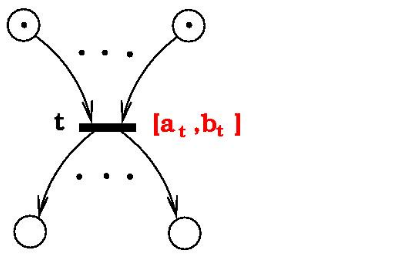

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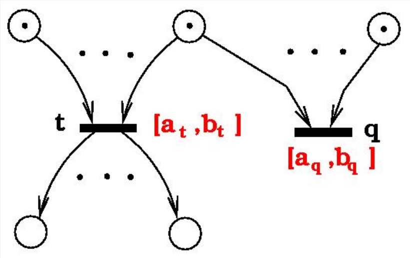

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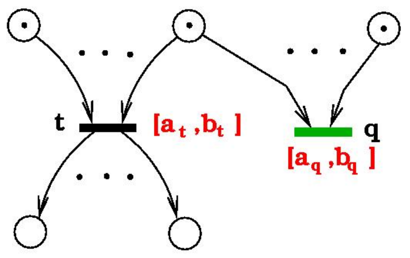

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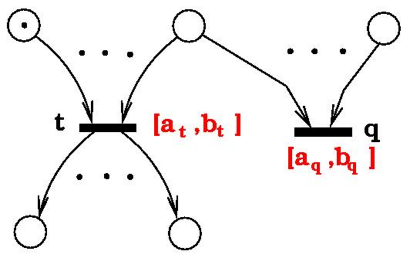

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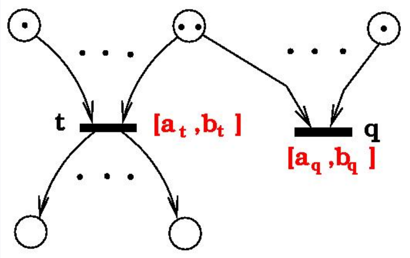

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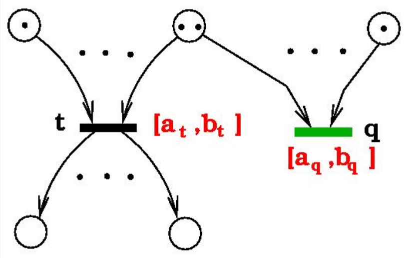

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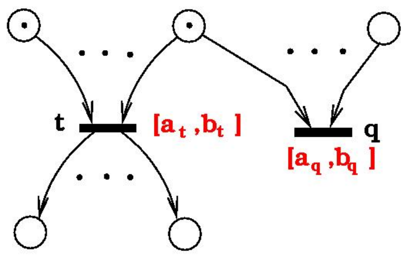
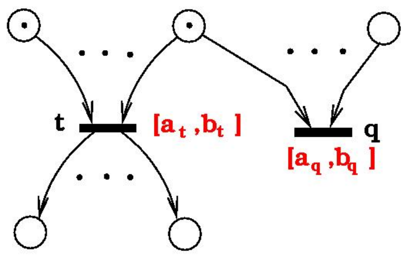


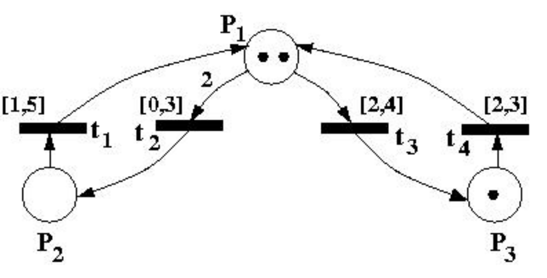
The local time of *t* is reset to zero!

Conception

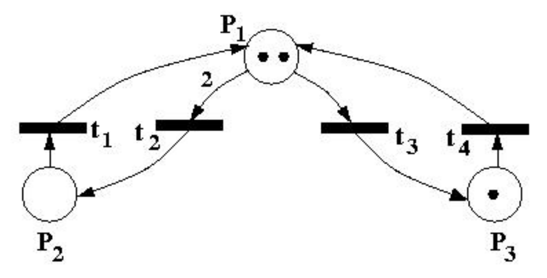


The local time of *t* is reset to zero! $\Longleftarrow$ static conflict

Statics: **Time Petri Net**



Statics: Petri Net (Skeleton)



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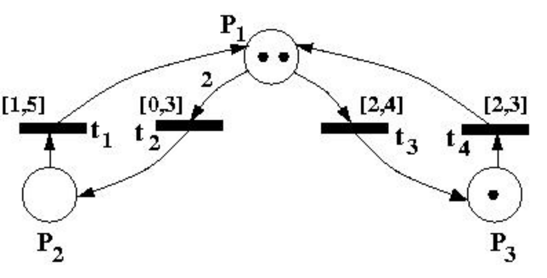

- $m_0 = (2, 0, 1)$

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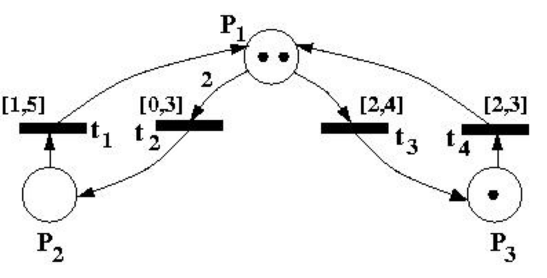

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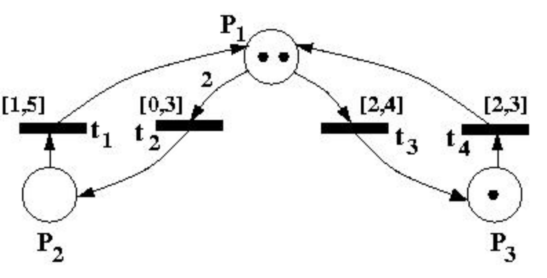

- $m_0 = (2, 0, 1)$ *p*-marking
- $h_0 = (\sharp, 0, 0, 0)$ *t*-marking

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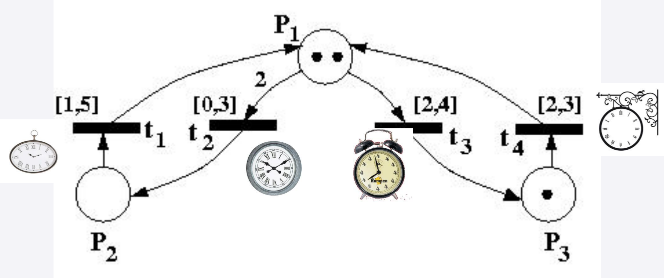


- $m_0 = (2, 0, 1)$ *p*-marking
- $h_0 = (\sharp, 0, 0, 0)$ *t*-marking

$h(t)$ is the time shown by the clock of t since the last enabling of t

State

The pair $z = (m, h)$ is called a **state** in a TPN $\mathcal{Z}$, iff:

- m is a p-marking in $\mathcal{Z}$.
- h is a t-marking in $\mathcal{Z}$.

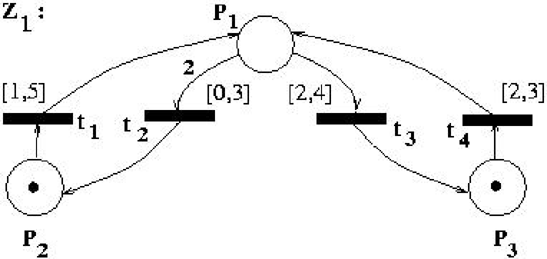
Dynamics: firing rules

Let $\mathcal{Z}$ be a TPN and let $z = (m, h)$, $z' = (m', h')$ be two states.
$\mathcal{Z}$ changes from state $z = (m, h)$ into the state $z' = (m', h')$ by:

firing a transition / \ **time elapsing**

Notation: $z \xrightarrow{t} z'$ $\qquad$ $z \xrightarrow{\tau} z'$

An example



$$(m_0, \begin{pmatrix} 0 \\ \sharp \\ \sharp \\ 0 \end{pmatrix})$$

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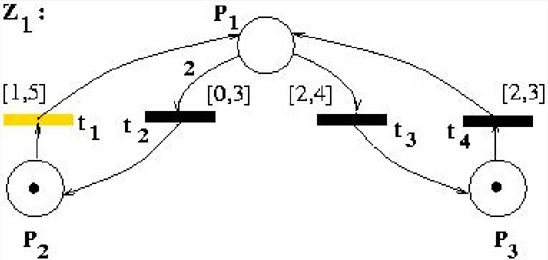
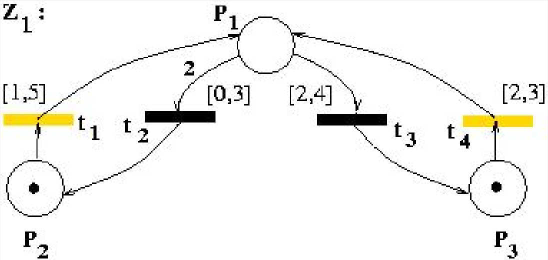


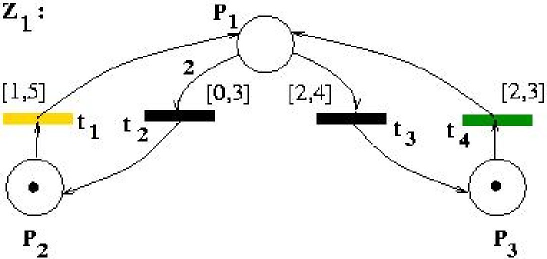
$$(m_0, \begin{pmatrix} 0 \\ \sharp \\ \sharp \\ 0 \end{pmatrix}) \xrightarrow{1.3} (m_1, \begin{pmatrix} 1.3 \\ \sharp \\ \sharp \\ 1.3 \end{pmatrix})$$

An example



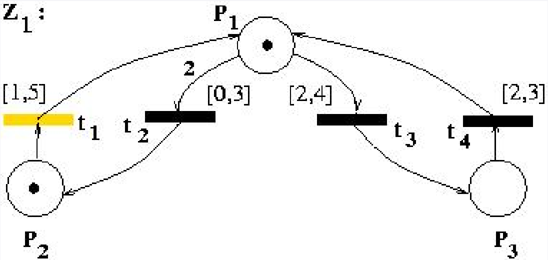
$$z_0 \xrightarrow{1.3} (m_1, \begin{pmatrix} 1.3 \\ \sharp \\ \sharp \\ 1.3 \end{pmatrix}) \xrightarrow{1.0} (m_2, \begin{pmatrix} 2.3 \\ \sharp \\ \sharp \\ 2.3 \end{pmatrix})$$

An example



$$z_0 \xrightarrow{1.3} \xrightarrow{1.0} (m_2, \begin{pmatrix} 2.3 \\ \sharp \\ \sharp \\ 2.3 \end{pmatrix}) \xrightarrow{t_4}$$

An example



$$z_0 \xrightarrow{1.3} \xrightarrow{1.0} \left(m_2, \begin{pmatrix} 2.3 \\ \sharp \\ \sharp \\ 2.3 \end{pmatrix}\right) \xrightarrow{t_4} \left(m_3, \begin{pmatrix} 2.3 \\ \sharp \\ 0.0 \\ \sharp \end{pmatrix}\right)$$

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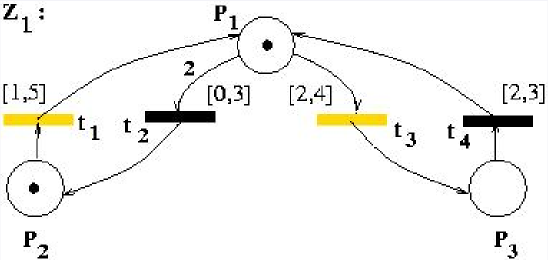


$$z_0 \xrightarrow{1.3}\xrightarrow{1.0}\xrightarrow{t_4} \left(m_3, \begin{pmatrix} 2.3 \\ \sharp \\ 0.0 \\ \sharp \end{pmatrix}\right) \xrightarrow{2.0} \left(m_4, \begin{pmatrix} 4.3 \\ \sharp \\ 2.0 \\ \sharp \end{pmatrix}\right)$$

Definitions:

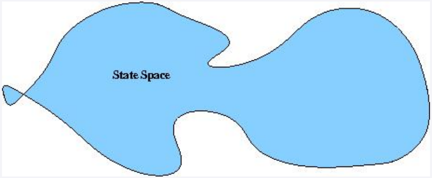
- **transition sequence:** $\sigma = t_1 \cdots t_n$
- **run:** $\sigma(\tau) = \tau_0\, t_1\, \tau_1\, \cdots\, \tau_{n-1}\, t_n\, \tau_n, \qquad \tau_i \in \mathbb{R}_0^+$
- **feasible run:** $z_0 \xrightarrow{\tau_0} z_0^* \xrightarrow{t_1} z_1 \xrightarrow{\tau_1} z_1^* \cdots \xrightarrow{t_n} z_n \xrightarrow{\tau_n} z_n^*$
- **feasible transition sequence :** σ is feasible if there ex. a feasible run $\sigma(\tau)$

Reachable state, Reachable marking, State space

Definitions:

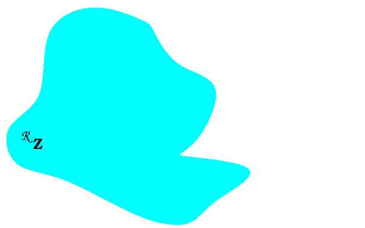
- z is a **reachable state** in $\mathcal{Z}$ if there ex. a feasible run $\sigma(\tau)$ and $z_0 \xrightarrow{\sigma(\tau)} z$
- m is a **reachable** ***p*****-marking** in $\mathcal{Z}$ if there ex. a reachable state z in $\mathcal{Z}$ with $z = (m, h)$
- The set of all reachable states in $\mathcal{Z}$ is the **state space** of $\mathcal{Z}$ (denoted: $StSp(\mathcal{Z})$).

Some Problems: The State Space



The set of all reachable states is dense.

Some Further Problems: Reachability of *p*-markings



$\mathcal{R}_{\mathcal{Z}}$ is the set if all reachable *p*-markings in Z.
$\mathcal{R}_{\mathcal{S}(\mathcal{Z})}$ is the set of all reachable markings in the skeleton of Z (the state space of the skeleton of Z).

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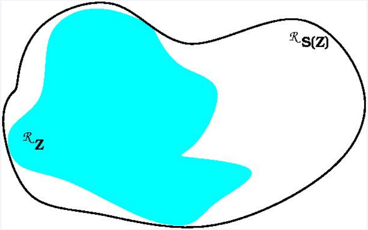

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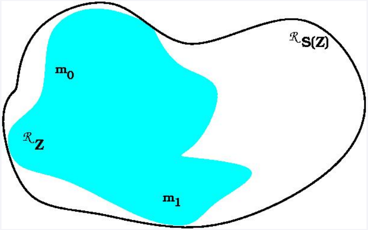

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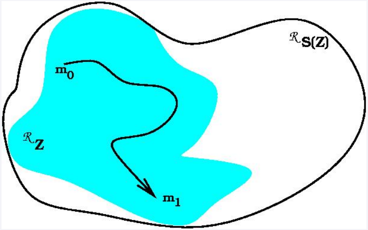

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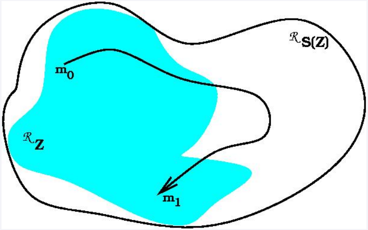

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Parametric Run, Parametric State

Let $\mathcal{Z} = \big(P, T, F, V, m_0, I\big)$ be a TPN and $\sigma = t_1 \cdots t_n$ be a transition sequence in $\mathcal{Z}$.

$(\sigma(x), B_\sigma)$ is a **parametric run** of σ and (z_σ, B_σ) is a **parametric state** in $\mathcal{Z}$ with $z_\sigma = (m_\sigma, h_\sigma)$, if

- $m_0 \xrightarrow{\sigma} m_\sigma$
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$h_\sigma(t)$ is a **term** and B_σ is a set of **formulas** in a predicate logic (Presburger Arithmetic - decidable !)

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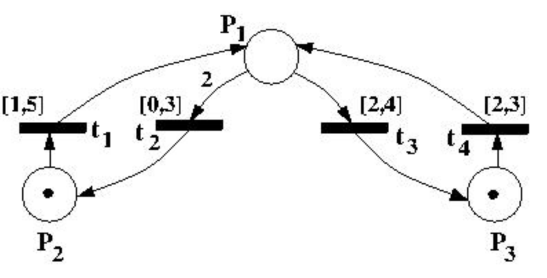
Obviously

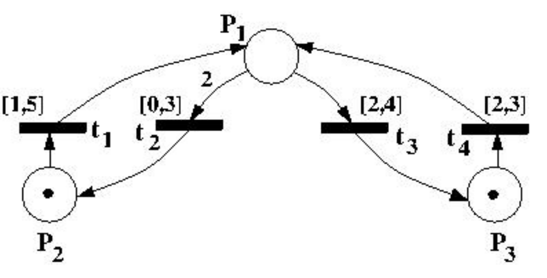
- $z_0 \xrightarrow{\sigma(x)} (z_\sigma, B_\sigma)$,
- $StSp(\mathcal{Z}) = \bigcup\limits_{\sigma,\beta} \{z_{\sigma(\beta(x))} \,|\, \beta : X \to \mathbb{R}_0^+, \beta(x) \text{ satisfies } B_\sigma\}$.

Example



$$\sigma = t_4 \ \ t_3$$

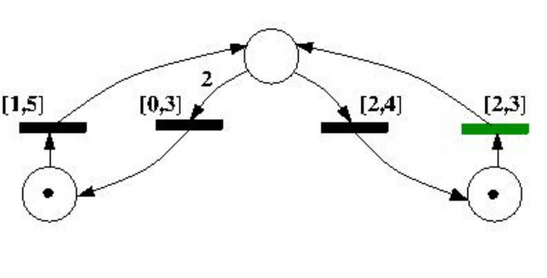
Example



$$\sigma = t_4 \;\; t_3 \qquad : \qquad x_0$$

$$(z_\epsilon, B_\epsilon) = (\underbrace{(\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} x_0 \\ \sharp \\ \sharp \\ x_0 \end{pmatrix})}_{z_\epsilon}, \underbrace{\{\ 0 \le x_0 \le 3\ \}}_{B_\epsilon}).$$

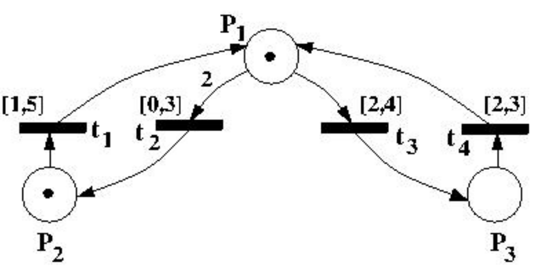
Example



$$\sigma = t_4 \ t_3 \qquad : \qquad x_0 \ t_4$$

$$\Big(\underbrace{\Big(\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} x_0 \\ \sharp \\ \sharp \\ x_0 \end{pmatrix} \Big)}, \underbrace{\{ \ 2 \le x_0 \le 3 \ \}} \Big).$$

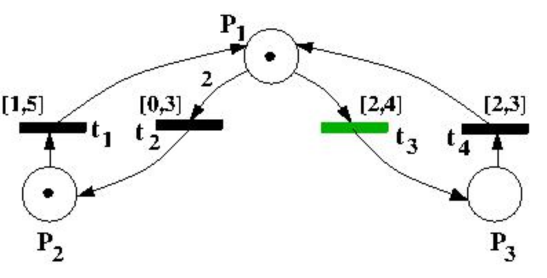
Example



$$\sigma = t_4 \;\; t_3 \qquad : \qquad x_0 \, t_4 \, x_1$$

$$(z_{t_4}, B_{t_4}) = (\underbrace{(\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} x_0 + x_1 \\ \sharp \\ x_1 \\ \sharp \end{pmatrix})}_{z_{t_4}}, \underbrace{\left\{ \begin{array}{ll} 2 \leq x_0 \leq 3, & x_0 + x_1 \leq 5, \\ 0 \leq x_1 \leq 4 & \end{array} \right\}}_{B_{t_4}}).$$

Example



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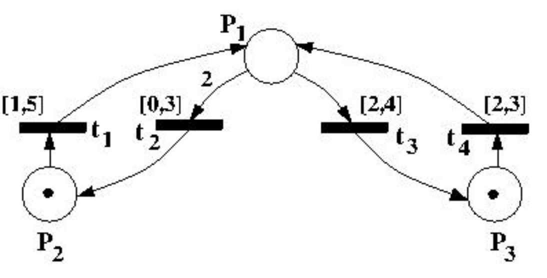
$$\Big(\underbrace{\left(\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} x_0 + x_1 \\ \sharp \\ x_1 \\ \sharp \end{pmatrix} \right)}, \underbrace{\left\{ \begin{matrix} 2 \leq x_0 \leq 3, & x_0 + x_1 \leq 5, \\ 2 \leq x_1 \leq 4 & \end{matrix} \right\}} \Big).$$

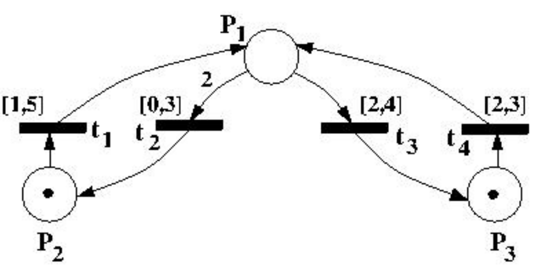
Example



$$\sigma = t_4 \ \ t_3 \qquad : \qquad x_0 \, t_4 \, x_1 \, t_3 \ \ x_2$$

$$(z_{t_4 t_3}, B_{t_4 t_3}) = \underbrace{\left(\left(\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} x_0 + x_1 + x_2 \\ \sharp \\ \sharp \\ x_2 \end{pmatrix} \right) \right.}_{z_{t_4 t_3}}, \underbrace{\left\{ \begin{array}{l} 2 \leq x_0 \leq 3, \ \ x_0 + x_1 \leq 5, \\ 2 \leq x_1 \leq 4, \\ 0 \leq x_2 \leq 3, \\ x_0 + x_1 + x_2 \leq 5 \end{array} \right\} \Bigg).}_{B_{t_4 t_3}}$$

Example



$$\sigma = t_4\, t_3 \quad \leadsto \quad (z_\sigma, B_\sigma) = (z_{t_4 t_3}, B_{t_4 t_3}) =$$

$$\Bigg(\underbrace{\left(\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} x_0 + x_1 + x_2 \\ \sharp \\ \sharp \\ x_2 \end{pmatrix} \right)}_{z_\sigma}, \underbrace{\left\{ \begin{array}{l} 2 \leq x_0 \leq 3, \\ 2 \leq x_1 \leq 4, \quad x_0 + x_1 + x_2 \leq 5 \\ 0 \leq x_2 \leq 3 \end{array} \right\}}_{B_\sigma} \Bigg).$$

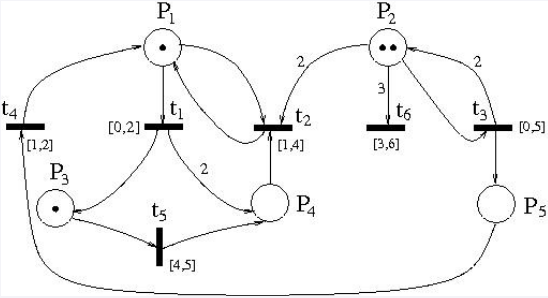
Bounds for the number of equalities in B_σ:

Let $\mathcal{Z} = (P, T, F, V, m_0, I)$ be a Petri net. Furthermore let (z_σ, B_σ) be a parametric state and $\sigma = t_1 \cdots t_n$ a transition sequence in $\mathcal{Z}$.

- The number of variables appearing in B_σ is at most $n + 1$.
- The number of non-redundant inequalities in B_σ is at most

$$\min\{2 \cdot \big(n \cdot |T| + 1\big), (n + 1) \cdot (\frac{n}{2} + 2)\}$$

Runs



$$\sigma = t_1 \ t_3 \ t_4 \ t_2 \ t_3$$

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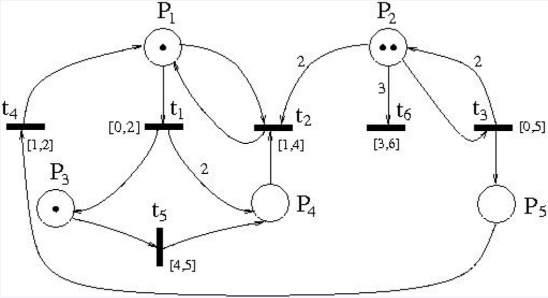

$$\sigma = t_1\ t_3\ t_4\ t_2\ t_3$$

$$z_0 \xrightarrow{0.7}$$

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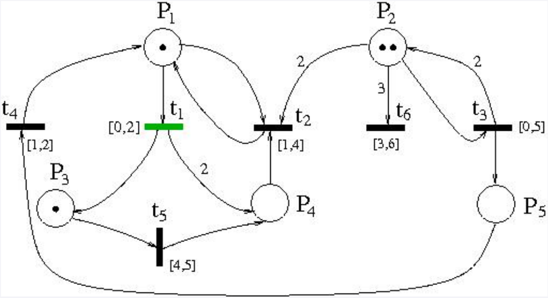

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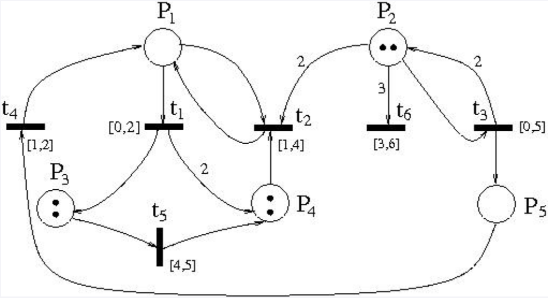
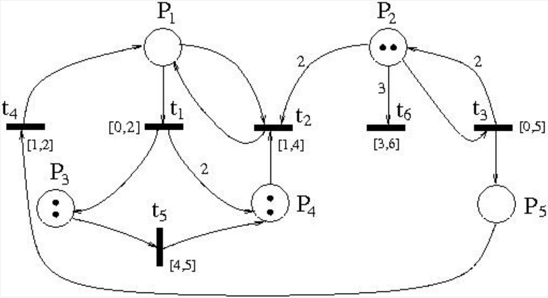


$$\sigma = t_1 \ t_3 \ t_4 \ t_2 \ t_3$$

$$z_0 \xrightarrow{0.7} \xrightarrow{t_1}$$

Runs



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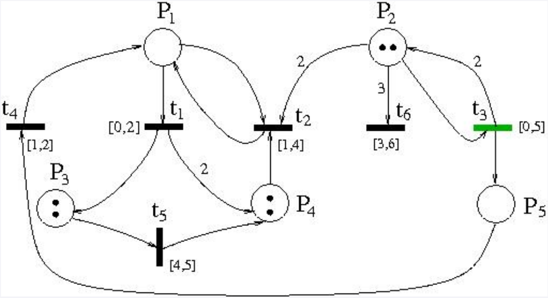
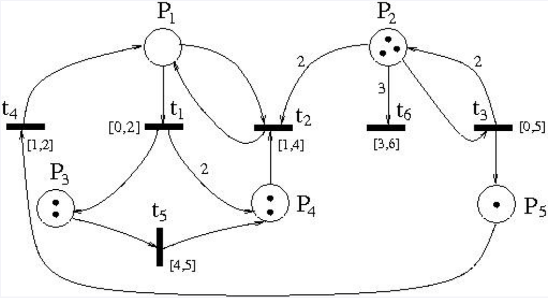
Runs



$$\sigma = t_1\ t_3\ t_4\ t_2\ t_3$$

$$z_0 \xrightarrow{0.7} \xrightarrow{t_1} \xrightarrow{0.0}$$

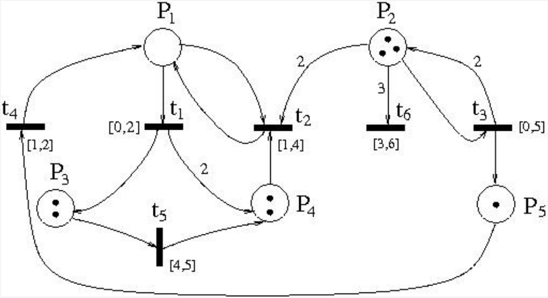
Runs



$$\sigma = t_1 \; t_3 \; t_4 \; t_2 \; t_3$$

$$z_0 \xrightarrow{0.7} \xrightarrow{t_1} \xrightarrow{0.0} \xrightarrow{t_3}$$

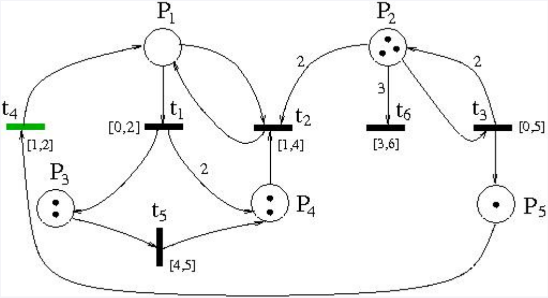
Runs



$$\sigma = t_1 \ t_3 \ t_4 \ t_2 \ t_3$$

$$z_0 \xrightarrow{0.7} \xrightarrow{t_1} \xrightarrow{0.0} \xrightarrow{t_3} \xrightarrow{0.4}$$

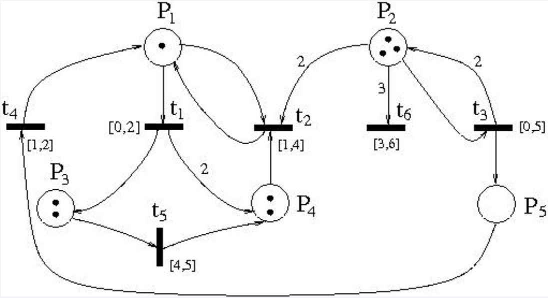
Runs



$$\sigma = t_1 \; t_3 \; t_4 \; t_2 \; t_3$$

$$z_0 \xrightarrow{0.7} \xrightarrow{t_1} \xrightarrow{0.0} \xrightarrow{t_3} \xrightarrow{0.4}$$

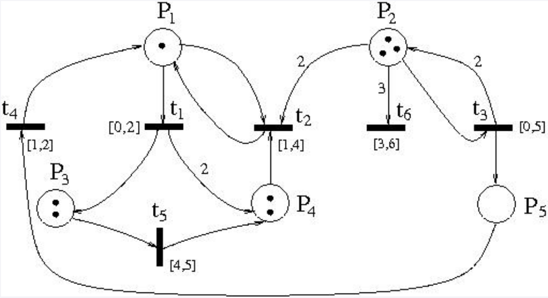
Runs



$$\sigma = t_1 \ t_3 \ t_4 \ t_2 \ t_3$$

$$z_0 \xrightarrow{0.7} \xrightarrow{t_1} \xrightarrow{0.0} \xrightarrow{t_3} \xrightarrow{0.4} \xrightarrow{t_4}$$

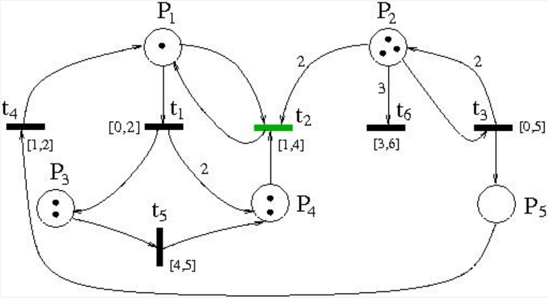
Runs



$$\sigma = t_1\ t_3\ t_4\ t_2\ t_3$$

$$z_0 \xrightarrow{0.7} \xrightarrow{t_1} \xrightarrow{0.0} \xrightarrow{t_3} \xrightarrow{0.4} \xrightarrow{t_4} \xrightarrow{1.2}$$

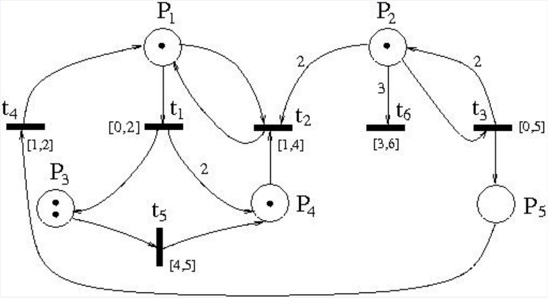
Runs



$$\sigma = t_1 \; t_3 \; t_4 \; t_2 \; t_3$$

$$z_0 \xrightarrow{0.7} \xrightarrow{t_1} \xrightarrow{0.0} \xrightarrow{t_3} \xrightarrow{0.4} \xrightarrow{t_4} \xrightarrow{1.2}$$

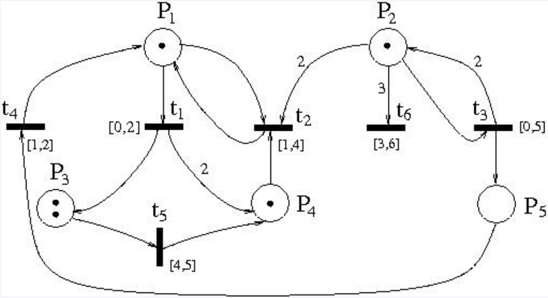
Runs



$$\sigma = t_1\ t_3\ t_4\ t_2\ t_3$$

$$z_0 \xrightarrow{0.7} \xrightarrow{t_1} \xrightarrow{0.0} \xrightarrow{t_3} \xrightarrow{0.4} \xrightarrow{t_4} \xrightarrow{1.2} \xrightarrow{t_2}$$

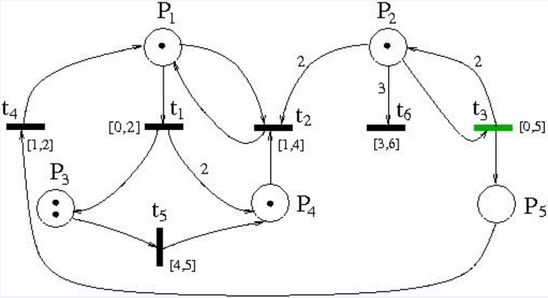
Runs



$$\sigma = t_1\ t_3\ t_4\ t_2\ t_3$$

$$z_0 \xrightarrow{0.7} \xrightarrow{t_1} \xrightarrow{0.0} \xrightarrow{t_3} \xrightarrow{0.4} \xrightarrow{t_4} \xrightarrow{1.2} \xrightarrow{t_2} \xrightarrow{0.5}$$

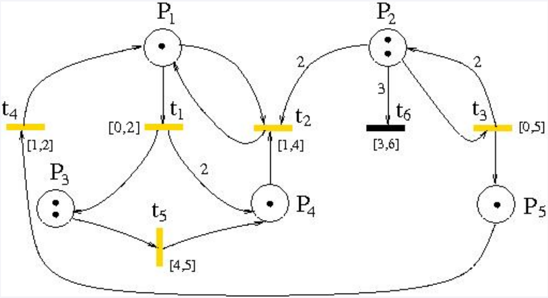
Runs



$$\sigma = t_1\ t_3\ t_4\ t_2\ t_3$$

$$z_0 \xrightarrow{0.7} \xrightarrow{t_1} \xrightarrow{0.0} \xrightarrow{t_3} \xrightarrow{0.4} \xrightarrow{t_4} \xrightarrow{1.2} \xrightarrow{t_2} \xrightarrow{0.5}$$

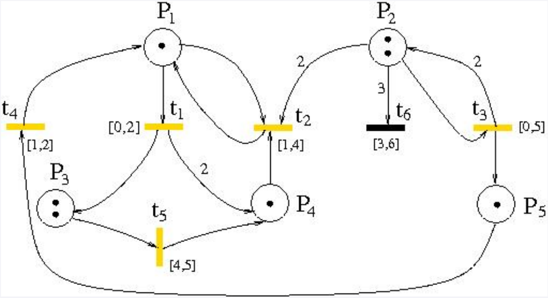
Runs



$$\sigma = t_1\ t_3\ t_4\ t_2\ t_3$$

$$z_0 \xrightarrow{0.7} \xrightarrow{t_1} \xrightarrow{0.0} \xrightarrow{t_3} \xrightarrow{0.4} \xrightarrow{t_4} \xrightarrow{1.2} \xrightarrow{t_2} \xrightarrow{0.5} \xrightarrow{t_3}$$

Runs

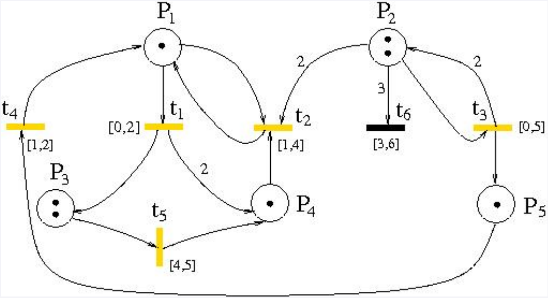


$$\sigma = t_1 \; t_3 \; t_4 \; t_2 \; t_3$$

$$z_0 \xrightarrow{0.7} \xrightarrow{t_1} \xrightarrow{0.0} \xrightarrow{t_3} \xrightarrow{0.4} \xrightarrow{t_4} \xrightarrow{1.2} \xrightarrow{t_2} \xrightarrow{0.5} \xrightarrow{t_3} \xrightarrow{1.4} z$$

$$\tau = 0.7 \; 0.0 \;\; 0.4 \;\; 1.2 \;\; 0.5 \;\; 1.4$$

Runs

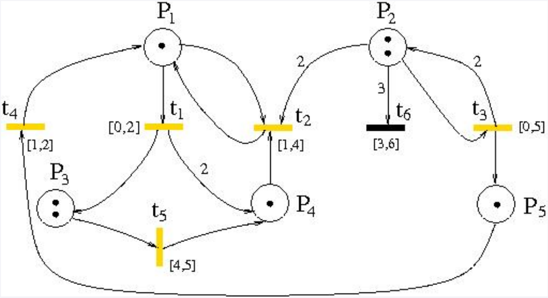


$$\sigma = t_1\ t_3\ t_4\ t_2\ t_3$$

$$\sigma(\tau) := z_0 \xrightarrow{0.7} \xrightarrow{t_1} \xrightarrow{0.0} \xrightarrow{t_3} \xrightarrow{0.4} \xrightarrow{t_4} \xrightarrow{1.2} \xrightarrow{t_2} \xrightarrow{0.5} \xrightarrow{t_3} \xrightarrow{1.4} z$$

$$\tau = 0.7\ 0.0\ 0.4\ 1.2\ 0.5\ 1.4$$

Runs

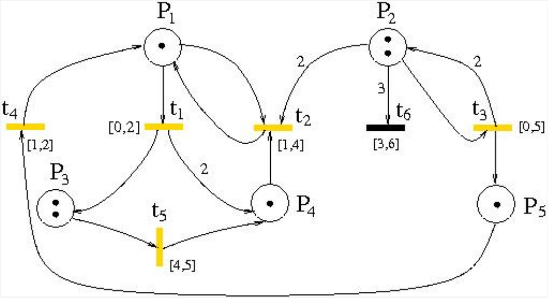


$$\sigma = t_1\ t_3\ t_4\ t_2\ t_3$$

$$\sigma(\tau) := z_0 \xrightarrow{\mathbf{0.7}} \xrightarrow{t_1} \xrightarrow{\mathbf{0.0}} \xrightarrow{t_3} \xrightarrow{\mathbf{0.4}} \xrightarrow{t_4} \xrightarrow{\mathbf{1.2}} \xrightarrow{t_2} \xrightarrow{\mathbf{0.5}} \xrightarrow{t_3} \xrightarrow{\mathbf{1.4}} z$$

$$\tau = 0.7\ 0.0\ 0.4\ 1.2\ 0.5\ 1.4$$

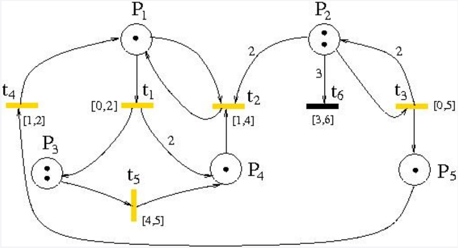
Example



$$\sigma = t_1 \ t_3 \ t_4 \ t_2 \ t_3$$

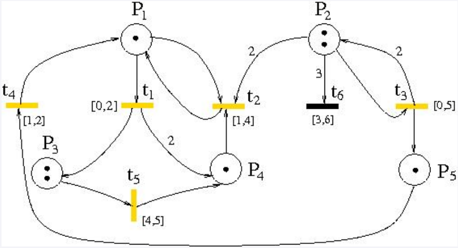
$$m_\sigma = (1, 2, 2, 1, 1)$$

Example - Continiation



$$h_\sigma = \begin{pmatrix} x_4 + x_5 \\ x_5 \\ x_5 \\ x_5 \\ x_0 + x_1 + x_2 + x_3 + x_4 + x_5 \\ \sharp \end{pmatrix}$$ and

Example - Continuation



$$B_\sigma = \left\{ \begin{array}{ll} 0 \leq x_0, & x_0 \leq 2, \\ 0 \leq x_1, & x_2 \leq 2, \\ 0 \leq x_2, & x_3 \leq 2, \\ 1 \leq x_3, & x_0 + x_1 + x_2 + x_3 + x_4 + x_5 \leq 5, \\ 0 \leq x_4, & x_4 + x_5 \leq 2, \\ 0 \leq x_5 & \end{array} \right\}.$$

Example - Continuation

The run $\sigma(\tau)$ with

$$z_0 \xrightarrow{\mathbf{0.7}} \xrightarrow{t_1} \xrightarrow{\mathbf{0.0}} \xrightarrow{t_3} \xrightarrow{\mathbf{0.4}} \xrightarrow{t_4} \xrightarrow{\mathbf{1.2}} \xrightarrow{t_2} \xrightarrow{\mathbf{0.5}} \xrightarrow{t_3} \xrightarrow{\mathbf{1.4}} (m_\sigma, \begin{pmatrix} 1.9 \\ 1.4 \\ 1.4 \\ 1.4 \\ 4.2 \\ \sharp \end{pmatrix})$$

is feasible.

Example - Continuation

$$\underbrace{\left(m_\sigma, \begin{pmatrix} \mathbf{1.9} \\ \mathbf{1.4} \\ \mathbf{1.4} \\ \mathbf{1.4} \\ \mathbf{4.2} \\ \sharp \end{pmatrix}\right)}_{z_0 \xrightarrow{\sigma(\tau)} z}$$

Example - Continuation

$$\underbrace{\left(m_\sigma, \begin{pmatrix} \mathbf{1.0} \\ \mathbf{1.0} \\ \mathbf{1.0} \\ \mathbf{1.0} \\ \mathbf{4.0} \\ \sharp \end{pmatrix}\right)}_{z_0 \xrightarrow{\sigma(?)} \lfloor z \rfloor} \qquad \underbrace{\left(m_\sigma, \begin{pmatrix} \mathbf{1.9} \\ \mathbf{1.4} \\ \mathbf{1.4} \\ \mathbf{1.4} \\ \mathbf{4.2} \\ \sharp \end{pmatrix}\right)}_{z_0 \xrightarrow{\sigma(\tau)} z}$$

Example - Continuation

$$\underbrace{\left(m_\sigma, \begin{pmatrix} \mathbf{1.0} \\ \mathbf{1.0} \\ \mathbf{1.0} \\ \mathbf{1.0} \\ \mathbf{4.0} \\ \sharp \end{pmatrix}\right)}_{z_0 \xrightarrow{\sigma(?)} \lfloor z \rfloor} \qquad \underbrace{\left(m_\sigma, \begin{pmatrix} \mathbf{1.9} \\ \mathbf{1.4} \\ \mathbf{1.4} \\ \mathbf{1.4} \\ \mathbf{4.2} \\ \sharp \end{pmatrix}\right)}_{z_0 \xrightarrow{\sigma(\tau)} z} \qquad \underbrace{\left(m_\sigma, \begin{pmatrix} \mathbf{2.0} \\ \mathbf{2.0} \\ \mathbf{2.0} \\ \mathbf{2.0} \\ \mathbf{5.0} \\ \sharp \end{pmatrix}\right)}_{z_0 \xrightarrow{\sigma(?)} \lceil z \rceil}$$

Example - Continuation

The runs

$$\sigma(\tau_1^*) := z_0 \xrightarrow{\mathbf{1}} \xrightarrow{t_1} \xrightarrow{\mathbf{0}} \xrightarrow{t_3} \xrightarrow{\mathbf{1}} \xrightarrow{t_4} \xrightarrow{\mathbf{1}} \xrightarrow{t_2} \xrightarrow{\mathbf{0}} \xrightarrow{t_3} \xrightarrow{\mathbf{1}} \lfloor z \rfloor$$

and

$$\sigma(\tau_2^*) := z_0 \xrightarrow{\mathbf{1}} \xrightarrow{t_1} \xrightarrow{\mathbf{0}} \xrightarrow{t_3} \xrightarrow{\mathbf{0}} \xrightarrow{t_4} \xrightarrow{\mathbf{2}} \xrightarrow{t_2} \xrightarrow{\mathbf{0}} \xrightarrow{t_3} \xrightarrow{\mathbf{2}} \lceil z \rceil$$

are also feasible in $\mathcal{Z}$.

Example - Continuation

The runs

$$\sigma(\tau_1^*) := z_0 \xrightarrow{\mathbf{1}} \xrightarrow{t_1} \xrightarrow{\mathbf{0}} \xrightarrow{t_3} \xrightarrow{\mathbf{1}} \xrightarrow{t_4} \xrightarrow{\mathbf{1}} \xrightarrow{t_2} \xrightarrow{\mathbf{0}} \xrightarrow{t_3} \xrightarrow{\mathbf{1}} \lfloor z \rfloor$$

$$\sigma(\tau) = z_0 \xrightarrow{\mathbf{0.7}} \xrightarrow{t_1} \xrightarrow{\mathbf{0.0}} \xrightarrow{t_3} \xrightarrow{\mathbf{0.4}} \xrightarrow{t_4} \xrightarrow{\mathbf{1.2}} \xrightarrow{t_2} \xrightarrow{\mathbf{0.5}} \xrightarrow{t_3} \xrightarrow{\mathbf{1.4}} z$$

$$\sigma(\tau_2^*) := z_0 \xrightarrow{\mathbf{1}} \xrightarrow{t_1} \xrightarrow{\mathbf{0}} \xrightarrow{t_3} \xrightarrow{\mathbf{0}} \xrightarrow{t_4} \xrightarrow{\mathbf{2}} \xrightarrow{t_2} \xrightarrow{\mathbf{0}} \xrightarrow{t_3} \xrightarrow{\mathbf{2}} \lceil z \rceil$$

are also feasible in $\mathcal{Z}$.

Main Property

Theorem 1:

Let $\mathcal{Z}$ be a TPN and $\sigma = t_1 \cdots t_n$ be a feasible transition sequence in $\mathcal{Z}$ with a feasable run $\sigma(\tau)$ of σ $\big(\tau = \tau_0 \ldots \tau_n\big)$ i.e.

$$z_0 \xrightarrow{\tau_0}\xrightarrow{t_1} \cdots \xrightarrow{t_n}\xrightarrow{\tau_n} z_n = (m_n, h_n),$$

and all $\tau_i \in \mathbb{R}_0^+$.
Then, there exists a further feasible run $\sigma(\tau^*)$, $\tau^* = \tau_0^* \ldots \tau_n^*$ of σ with

$$z_0 \xrightarrow{\tau_0^*}\xrightarrow{t_1} \cdots \xrightarrow{t_n}\xrightarrow{\tau_n^*} z_n^* = (m_n^*, h_n^*).$$

such that

Main Property

Theorem 1 – Continuation:

$$z_0 \xrightarrow{\tau_0} \xrightarrow{t_1} \cdots \xrightarrow{t_n} \xrightarrow{\tau_n} z_n = (m_n, h_n), \ \tau_i \in \mathbb{R}_0^+.$$
$$z_0 \xrightarrow{\tau_0^*} \xrightarrow{t_1} \cdots \xrightarrow{t_n} \xrightarrow{\tau_n^*} z_n^* = (m_n^*, h_n^*)$$

1. For each $i, 0 \leq i \leq n$ the time τ_i^* is a natural number.
2. For each enabled transition t at marking $m_n (= m_n^*)$ it holds:
   1. $h_n^*(t) = \lfloor h_n(t) \rfloor$.
   2. $\sum_{i=1}^{n} \tau_i^* = \lfloor \sum_{i=1}^{n} \tau_i \rfloor$
3. For each transition $t \in T$ it holds:
   t is ready to fire in z_n iff t is also ready to fire in $\lfloor z_n \rfloor$.

Main Property

Theorem 1 – Continuation:

$$z_0 \xrightarrow{\tau_0}\xrightarrow{t_1} \cdots \xrightarrow{t_n}\xrightarrow{\tau_n} z_n = (m_n, h_n),\ \tau_i \in \mathbb{R}_0^+.$$
$$z_0 \xrightarrow{\tau_0^*}\xrightarrow{t_1} \cdots \xrightarrow{t_n}\xrightarrow{\tau_n^*} z_n^* = (m_n^*, h_n^*)\ ,\ \tau_i^* \in \mathbb{N}.$$

1. For each $i, 0 \leq i \leq n$ the time τ_i^* is a natural number.
2. For each enabled transition t at marking $m_n (= m_n^*)$ it holds:
   1. $h_n^*(t) = \lfloor h_n(t) \rfloor$.
   2. $\sum_{i=1}^{n} \tau_i^* = \lfloor \sum_{i=1}^{n} \tau_i \rfloor$
3. For each transition $t \in T$ it holds:
   t is ready to fire in z_n iff t is also ready to fire in $\lfloor z_n \rfloor$.

Idea of the proof:

Let $X_\sigma := \{x_0, x_1, \ldots, x_n\}$ and $\beta_0 : X_\sigma \longrightarrow \mathbb{R}_0^+$ with $\beta_0(x_i) := \tau_i$ for each $i \in \{0, \ldots, n\}$.

β	$\beta(x_0)$	$\beta(x_1)$	$\cdots$	$\beta(x_{n-i})$	$\beta(x_{n-(i-1)})$	$\cdots$	$\beta(x_{n-1})$	$\beta(x_n)$
β_0	r	r	$\cdots$	r	r	$\cdots$	r	r

The successive construction of the assignment β^* from β_0.

Idea of the proof:

Let $X_\sigma := \{x_0, x_1, \ldots, x_n\}$ and $\beta_0 : X_\sigma \longrightarrow \mathbb{R}_0^+$ with $\beta_0(x_i) := \tau_i$ for each $i \in \{0, \ldots, n\}$.

β	$\beta(x_0)$	$\beta(x_1)$	$\cdots$	$\beta(x_{n-i})$	$\beta(x_{n-(i-1)})$	$\cdots$	$\beta(x_{n-1})$	$\beta(x_n)$
β_0	r	r	$\cdots$	r	r	$\cdots$	r	r
β_1	r	r	$\cdots$	r	r	$\cdots$	r	$\boldsymbol{k}$

The successive construction of the assignment β^* from β_0.

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β	$\beta(x_0)$	$\beta(x_1)$	$\cdots$	$\beta(x_{n-i})$	$\beta(x_{n-(i-1)})$	$\cdots$	$\beta(x_{n-1})$	$\beta(x_n)$
β_0	r	r	$\cdots$	r	r	$\cdots$	r	r
β_1	r	r	$\cdots$	r	r	$\cdots$	r	$\boldsymbol{k}$
β_2	r	r	$\cdots$	r	r	$\cdots$	$\boldsymbol{k}$	k

The successive construction of the assignment β^* from β_0.

Idea of the proof:

Let $X_\sigma := \{x_0, x_1, \ldots, x_n\}$ and $\beta_0 : X_\sigma \longrightarrow \mathbb{R}_0^+$ with $\beta_0(x_i) := \tau_i$ for each $i \in \{0, \ldots, n\}$.

β	$\beta(x_0)$	$\beta(x_1)$	$\cdots$	$\beta(x_{n-i})$	$\beta(x_{n-(i-1)})$	$\cdots$	$\beta(x_{n-1})$	$\beta(x_n)$
β_0	r	r	$\cdots$	r	r	$\cdots$	r	r
β_1	r	r	$\cdots$	r	r	$\cdots$	r	$\boldsymbol{k}$
β_2	r	r	$\cdots$	r	r	$\cdots$	$\boldsymbol{k}$	k
$\vdots$			$\vdots$			$\vdots$		

The successive construction of the assignment β^* from β_0.

Idea of the proof:

Let $X_\sigma := \{x_0, x_1, \ldots, x_n\}$ and $\beta_0 : X_\sigma \longrightarrow \mathbb{R}_0^+$ with $\beta_0(x_i) := \tau_i$ for each $i \in \{0, \ldots, n\}$.

β	$\beta(x_0)$	$\beta(x_1)$	$\cdots$	$\beta(x_{n-i})$	$\beta(x_{n-(i-1)})$	$\cdots$	$\beta(x_{n-1})$	$\beta(x_n)$
β_0	r	r	$\cdots$	r	r	$\cdots$	r	r
β_1	r	r	$\cdots$	r	r	$\cdots$	r	$\boldsymbol{k}$
β_2	r	r	$\cdots$	r	r	$\cdots$	$\boldsymbol{k}$	k
$\vdots$			$\vdots$			$\vdots$		
β_i	r	r	$\cdots$	r	$\boldsymbol{k}$	$\cdots$	k	k

The successive construction of the assignment β^* from β_0.

Idea of the proof:

Let $X_\sigma := \{x_0, x_1, \ldots, x_n\}$ and $\beta_0 : X_\sigma \longrightarrow \mathbb{R}_0^+$ with $\beta_0(x_i) := \tau_i$ for each $i \in \{0, \ldots, n\}$.

β	$\beta(x_0)$	$\beta(x_1)$	$\cdots$	$\beta(x_{n-i})$	$\beta(x_{n-(i-1)})$	$\cdots$	$\beta(x_{n-1})$	$\beta(x_n)$
β_0	r	r	$\cdots$	r	r	$\cdots$	r	r
β_1	r	r	$\cdots$	r	r	$\cdots$	r	$\boldsymbol{k}$
β_2	r	r	$\cdots$	r	r	$\cdots$	$\boldsymbol{k}$	k
$\vdots$			$\vdots$			$\vdots$		
β_i	r	r	$\cdots$	r	$\boldsymbol{k}$	$\cdots$	k	k
$\vdots$			$\vdots$			$\vdots$		

The successive construction of the assignment β^* from β_0.

Idea of the proof:

Let $X_\sigma := \{x_0, x_1, \ldots, x_n\}$ and $\beta_0 : X_\sigma \longrightarrow \mathbb{R}_0^+$ with $\beta_0(x_i) := \tau_i$ for each $i \in \{0, \ldots, n\}$.

β	$\beta(x_0)$	$\beta(x_1)$	$\cdots$	$\beta(x_{n-i})$	$\beta(x_{n-(i-1)})$	$\cdots$	$\beta(x_{n-1})$	$\beta(x_n)$
β_0	r	r	$\cdots$	r	r	$\cdots$	r	r
β_1	r	r	$\cdots$	r	r	$\cdots$	r	$\boldsymbol{k}$
β_2	r	r	$\cdots$	r	r	$\cdots$	$\boldsymbol{k}$	k
$\vdots$			$\vdots$			$\vdots$		
β_i	r	r	$\cdots$	r	$\boldsymbol{k}$	$\cdots$	k	k
$\vdots$			$\vdots$			$\vdots$		
β_n	r	$\boldsymbol{k}$	$\cdots$	k	k	$\cdots$	k	k

The successive construction of the assignment β^* from β_0.

Idea of the proof:

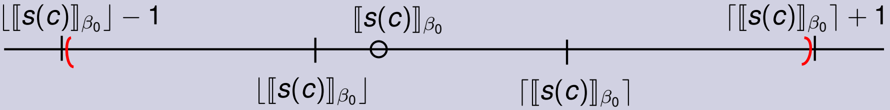
Let $X_\sigma := \{x_0, x_1, \ldots, x_n\}$ and $\beta_0 : X_\sigma \longrightarrow \mathbb{R}_0^+$ with $\beta_0(x_i) := \tau_i$ for each $i \in \{0, \ldots, n\}$.

β	$\beta(x_0)$	$\beta(x_1)$	$\cdots$	$\beta(x_{n-i})$	$\beta(x_{n-(i-1)})$	$\cdots$	$\beta(x_{n-1})$	$\beta(x_n)$
β_0	r	r	$\cdots$	r	r	$\cdots$	r	r
β_1	r	r	$\cdots$	r	r	$\cdots$	r	$\boldsymbol{k}$
β_2	r	r	$\cdots$	r	r	$\cdots$	$\boldsymbol{k}$	k
$\vdots$			$\vdots$			$\vdots$		
β_i	r	r	$\cdots$	r	$\boldsymbol{k}$	$\cdots$	k	k
$\vdots$			$\vdots$			$\vdots$		
β_n	r	$\boldsymbol{k}$	$\cdots$	k	k	$\cdots$	k	k
$\beta^* := \beta_{n+1}$	$\boldsymbol{k}$	k	$\cdots$	k	k	$\cdots$	k	k

The successive construction of the assignment β^* from β_0.

Idea of the proof:

For each inequality $c \in B_\sigma$ let $s(c)$ be the sum of variables.



Position of the real number $[\![s(c)]\!]_{\beta_0}$ and the integers $\lfloor [\![s(c)]\!]_{\beta_0} \rfloor - 1$, $\lfloor [\![s(c)]\!]_{\beta_0} \rfloor$, $\lceil [\![s(c)]\!]_{\beta_0} \rceil$ and $\lceil [\![s(c)]\!]_{\beta_0} \rceil + 1$.

$$z_0 \xrightarrow{0.7} \xrightarrow{t_1} \xrightarrow{0.0} \xrightarrow{t_3} \xrightarrow{0.4} \xrightarrow{t_4} \xrightarrow{1.2} \xrightarrow{t_2} \xrightarrow{0.5} \xrightarrow{t_3} \xrightarrow{1.4} (m_\sigma, (1.9, 1.4, 1.4, 1.4, 4.2, \sharp))$$

I	x_0	x_1	x_2	x_3	x_4	x_5	$h_\sigma(t_1)$	$h_\sigma(t_2)$	$h_\sigma(t_5)$
$\hat{\beta} = \beta_0$	0.7	0.0	0.4	1.2	0.5	1.4	1.9	1.4	4.2

$h_\sigma(t_1) = x_4 + x_5$, $h_\sigma(t_2) = h_\sigma(t_3) = h_\sigma(t_4) = x_5$

$h_\sigma(t_5) = x_1 + x_2 + x_3 + x_4 + x_5$

$z_0 \xrightarrow{0.7} \xrightarrow{t_1} \xrightarrow{0.0} \xrightarrow{t_3} \xrightarrow{0.4} \xrightarrow{t_4} \xrightarrow{1.2} \xrightarrow{t_2} \xrightarrow{0.5} \xrightarrow{t_3} \xrightarrow{1.4} (m_\sigma, (1.9, 1.4, 1.4, 1.4, 4.2, \sharp))$

I			x_0	x_1	x_2	x_3	x_4	x_5	$h_\sigma(t_1)$	$h_\sigma(t_2)$	$h_\sigma(t_5)$
$\hat{\beta}$	=	β_0	0.7	0.0	0.4	1.2	0.5	1.4	1.9	1.4	4.2
		β_1	0.7	0.0	0.4	1.2	0.5				

$h_\sigma(t_1) = x_4 + x_5,$ $\quad h_\sigma(t_2) = h_\sigma(t_3) = h_\sigma(t_4) = x_5$

$h_\sigma(t_5) = x_1 + x_2 + x_3 + x_4 + x_5$

$$z_0 \xrightarrow{0.7} \xrightarrow{t_1} \xrightarrow{0.0} \xrightarrow{t_3} \xrightarrow{0.4} \xrightarrow{t_4} \xrightarrow{1.2} \xrightarrow{t_2} \xrightarrow{0.5} \xrightarrow{t_3} \xrightarrow{1} (m_\sigma, (1.9, 1.4, 1.4, 1.4, 4.2, \sharp))$$

I			x_0	x_1	x_2	x_3	x_4	x_5	$h_\sigma(t_1)$	$h_\sigma(t_2)$	$h_\sigma(t_5)$
$\hat{\beta}$	=	β_0	0.7	0.0	0.4	1.2	0.5	1.4	1.9	1.4	4.2
		β_1	0.7	0.0	0.4	1.2	0.5	**1**			

$h_\sigma(t_1) = x_4 + x_5$, $\quad h_\sigma(t_2) = h_\sigma(t_3) = h_\sigma(t_4) = x_5$

$h_\sigma(t_5) = x_1 + x_2 + x_3 + x_4 + x_5$

$$z_0 \xrightarrow{0.7} \xrightarrow{t_1} \xrightarrow{0.0} \xrightarrow{t_3} \xrightarrow{0.4} \xrightarrow{t_4} \xrightarrow{1.2} \xrightarrow{t_2} \xrightarrow{0.5} \xrightarrow{t_3} \xrightarrow{1} (m_\sigma, (1.5, 1.0, 1.0, 1.0, 3.8, \sharp))$$

I			x_0	x_1	x_2	x_3	x_4	x_5	$h_\sigma(t_1)$	$h_\sigma(t_2)$	$h_\sigma(t_5)$
$\hat{\beta}$	$=$	β_0	0.7	0.0	0.4	1.2	0.5	1.4	1.9	1.4	4.2
		β_1	0.7	0.0	0.4	1.2	0.5	**1**	1.5	**1.0**	3.8

$h_\sigma(t_1) = x_4 + x_5$, $\quad h_\sigma(t_2) = h_\sigma(t_3) = h_\sigma(t_4) = x_5$

$h_\sigma(t_5) = x_1 + x_2 + x_3 + x_4 + x_5$

$$z_0 \xrightarrow{0.7} \xrightarrow{t_1} \xrightarrow{0.0} \xrightarrow{t_3} \xrightarrow{0.4} \xrightarrow{t_4} \xrightarrow{1.2} \xrightarrow{t_2} \xrightarrow{0.5} \xrightarrow{t_3} \xrightarrow{1} (m_\sigma, (1.5, 1.0, 1.0, 1.0, 3.8, \sharp))$$

I			x_0	x_1	x_2	x_3	x_4	x_5	$h_\sigma(t_1)$	$h_\sigma(t_2)$	$h_\sigma(t_5)$
$\hat{\beta}$	$=$	β_0	0.7	0.0	0.4	1.2	0.5	1.4	1.9	1.4	4.2
		β_1	0.7	0.0	0.4	1.2	0.5	**1**	1.5	**1.0**	3.8
		β_2	0.7	0.0	0.4	1.2		1			

$h_\sigma(t_1) = x_4 + x_5$, $\quad h_\sigma(t_2) = h_\sigma(t_3) = h_\sigma(t_4) = x_5$

$h_\sigma(t_5) = x_1 + x_2 + x_3 + x_4 + x_5$

$$z_0 \xrightarrow{0.7} \xrightarrow{t_1} \xrightarrow{0.0} \xrightarrow{t_3} \xrightarrow{0.4} \xrightarrow{t_4} \xrightarrow{1.2} \xrightarrow{t_2} \xrightarrow{0} \xrightarrow{t_3} \xrightarrow{1} (m_\sigma, (1.5, 1.0, 1.0, 1.0, 3.8, \sharp))$$

I	x_0	x_1	x_2	x_3	x_4	x_5	$h_\sigma(t_1)$	$h_\sigma(t_2)$	$h_\sigma(t_5)$
$\hat{\beta} = \beta_0$	0.7	0.0	0.4	1.2	0.5	1.4	1.9	1.4	4.2
β_1	0.7	0.0	0.4	1.2	0.5	**1**	1.5	**1.0**	3.8
β_2	0.7	0.0	0.4	1.2	**0**	1			

$h_\sigma(t_1) = x_4 + x_5$, $\quad h_\sigma(t_2) = h_\sigma(t_3) = h_\sigma(t_4) = x_5$

$h_\sigma(t_5) = x_1 + x_2 + x_3 + x_4 + x_5$

$$z_0 \xrightarrow{0.7} \xrightarrow{t_1} \xrightarrow{0.0} \xrightarrow{t_3} \xrightarrow{0.4} \xrightarrow{t_4} \xrightarrow{1.2} \xrightarrow{t_2} \xrightarrow{0} \xrightarrow{t_3} \xrightarrow{1} (m_\sigma, (1.0, 1.0, 1.0, 1.0, 3.3, \sharp))$$

I			x_0	x_1	x_2	x_3	x_4	x_5	$h_\sigma(t_1)$	$h_\sigma(t_2)$	$h_\sigma(t_5)$
$\hat{\beta}$	$=$	β_0	0.7	0.0	0.4	1.2	0.5	1.4	1.9	1.4	4.2
		β_1	0.7	0.0	0.4	1.2	0.5	**1**	1.5	**1.0**	3.8
		β_2	0.7	0.0	0.4	1.2	**0**	1	**1.0**		3.3

$h_\sigma(t_1) = x_4 + x_5$, $h_\sigma(t_2) = h_\sigma(t_3) = h_\sigma(t_4) = x_5$

$h_\sigma(t_5) = x_1 + x_2 + x_3 + x_4 + x_5$

$$z_0 \xrightarrow{0.7} \xrightarrow{t_1} \xrightarrow{0.0} \xrightarrow{t_3} \xrightarrow{0.4} \xrightarrow{t_4} \xrightarrow{1.2} \xrightarrow{t_2} \xrightarrow{0} \xrightarrow{t_3} \xrightarrow{1} (m_\sigma, (1.0, 1.0, 1.0, 1.0, 3.3, \sharp))$$

I			x_0	x_1	x_2	x_3	x_4	x_5	$h_\sigma(t_1)$	$h_\sigma(t_2)$	$h_\sigma(t_5)$
$\hat{\beta}$	=	β_0	0.7	0.0	0.4	1.2	0.5	1.4	1.9	1.4	4.2
		β_1	0.7	0.0	0.4	1.2	0.5	**1**	1.5	**1.0**	3.8
		β_2	0.7	0.0	0.4	1.2	**0**	1	**1.0**		3.3
		β_3	0.7	0.0	0.4		0	1			

$h_\sigma(t_1) = x_4 + x_5,$ $\quad h_\sigma(t_2) = h_\sigma(t_3) = h_\sigma(t_4) = x_5$

$h_\sigma(t_5) = x_1 + x_2 + x_3 + x_4 + x_5$

$$z_0 \xrightarrow{0.7} \xrightarrow{t_1} \xrightarrow{0.0} \xrightarrow{t_3} \xrightarrow{0.4} \xrightarrow{t_4} \xrightarrow{1} \xrightarrow{t_2} \xrightarrow{0} \xrightarrow{t_3} \xrightarrow{1} (m_\sigma, (1.0, 1.0, 1.0, 1.0, 3.3, \sharp))$$

I			x_0	x_1	x_2	x_3	x_4	x_5	$h_\sigma(t_1)$	$h_\sigma(t_2)$	$h_\sigma(t_5)$
$\hat{\beta}$	$=$	β_0	0.7	0.0	0.4	1.2	0.5	1.4	1.9	1.4	4.2
		β_1	0.7	0.0	0.4	1.2	0.5	**1**	1.5	**1.0**	3.8
		β_2	0.7	0.0	0.4	1.2	**0**	1	**1.0**		3.3
		β_3	0.7	0.0	0.4	**1**	0	1			

$h_\sigma(t_1) = x_4 + x_5$, $\quad h_\sigma(t_2) = h_\sigma(t_3) = h_\sigma(t_4) = x_5$

$h_\sigma(t_5) = x_1 + x_2 + x_3 + x_4 + x_5$

$$z_0 \xrightarrow{0.7} \xrightarrow{t_1} \xrightarrow{0.0} \xrightarrow{t_3} \xrightarrow{0.4} \xrightarrow{t_4} \xrightarrow{1} \xrightarrow{t_2} \xrightarrow{0} \xrightarrow{t_3} \xrightarrow{1} (m_\sigma, (1.0, 1.0, 1.0, 1.0, 3.1, \sharp))$$

I			x_0	x_1	x_2	x_3	x_4	x_5	$h_\sigma(t_1)$	$h_\sigma(t_2)$	$h_\sigma(t_5)$
$\hat{\beta}$	=	β_0	0.7	0.0	0.4	1.2	0.5	1.4	1.9	1.4	4.2
		β_1	0.7	0.0	0.4	1.2	0.5	**1**	1.5	**1.0**	3.8
		β_2	0.7	0.0	0.4	1.2	**0**	1	**1.0**		3.3
		β_3	0.7	0.0	0.4	**1**	0	1			3.1

$h_\sigma(t_1) = x_4 + x_5$, $\quad h_\sigma(t_2) = h_\sigma(t_3) = h_\sigma(t_4) = x_5$

$h_\sigma(t_5) = x_1 + x_2 + x_3 + x_4 + x_5$

$$z_0 \xrightarrow{0.7} \xrightarrow{t_1} \xrightarrow{0.0} \xrightarrow{t_3} \xrightarrow{0.4} \xrightarrow{t_4} \xrightarrow{1} \xrightarrow{t_2} \xrightarrow{0} \xrightarrow{t_3} \xrightarrow{1} (m_\sigma, (1.0, 1.0, 1.0, 1.0, 3.1, \sharp))$$

I	x_0	x_1	x_2	x_3	x_4	x_5	$h_\sigma(t_1)$	$h_\sigma(t_2)$	$h_\sigma(t_5)$
$\hat{\beta} = \beta_0$	0.7	0.0	0.4	1.2	0.5	1.4	1.9	1.4	4.2
β_1	0.7	0.0	0.4	1.2	0.5	**1**	1.5	**1.0**	3.8
β_2	0.7	0.0	0.4	1.2	**0**	1	**1.0**		3.3
β_3	0.7	0.0	0.4	**1**	0	1			3.1
β_4	0.7	0.0		1	0	1			

$h_\sigma(t_1) = x_4 + x_5$, $\quad h_\sigma(t_2) = h_\sigma(t_3) = h_\sigma(t_4) = x_5$

$h_\sigma(t_5) = x_1 + x_2 + x_3 + x_4 + x_5$

$z_0 \xrightarrow{0.7} \xrightarrow{t_1} \xrightarrow{0.0} \xrightarrow{t_3} \xrightarrow{1} \xrightarrow{t_4} \xrightarrow{1} \xrightarrow{t_2} \xrightarrow{0} \xrightarrow{t_3} \xrightarrow{1} (m_\sigma, (1.0, 1.0, 1.0, 1.0, 3.1, \sharp))$

I			x_0	x_1	x_2	x_3	x_4	x_5	$h_\sigma(t_1)$	$h_\sigma(t_2)$	$h_\sigma(t_5)$
$\hat{\beta}$	=	β_0	0.7	0.0	0.4	1.2	0.5	1.4	1.9	1.4	4.2
		β_1	0.7	0.0	0.4	1.2	0.5	**1**	1.5	**1.0**	3.8
		β_2	0.7	0.0	0.4	1.2	**0**	1	**1.0**		3.3
		β_3	0.7	0.0	0.4	**1**	0	1			3.1
		β_4	0.7	0.0	**1**	1	0	1			

$h_\sigma(t_1) = x_4 + x_5,$ $h_\sigma(t_2) = h_\sigma(t_3) = h_\sigma(t_4) = x_5$

$h_\sigma(t_5) = x_1 + x_2 + x_3 + x_4 + x_5$

$$z_0 \xrightarrow{0.7} \xrightarrow{t_1} \xrightarrow{0.0} \xrightarrow{t_3} \xrightarrow{1} \xrightarrow{t_4} \xrightarrow{1} \xrightarrow{t_2} \xrightarrow{0} \xrightarrow{t_3} \xrightarrow{1} (m_\sigma, (1.0, 1.0, 1.0, 1.0, 3.7, \sharp))$$

I	x_0	x_1	x_2	x_3	x_4	x_5	$h_\sigma(t_1)$	$h_\sigma(t_2)$	$h_\sigma(t_5)$
$\hat{\beta} = \beta_0$	0.7	0.0	0.4	1.2	0.5	1.4	1.9	1.4	4.2
β_1	0.7	0.0	0.4	1.2	0.5	**1**	1.5	**1.0**	3.8
β_2	0.7	0.0	0.4	1.2	**0**	1	**1.0**		3.3
β_3	0.7	0.0	0.4	**1**	0	1			3.1
β_4	0.7	0.0	**1**	1	0	1			3.7

$h_\sigma(t_1) = x_4 + x_5,$ $\quad h_\sigma(t_2) = h_\sigma(t_3) = h_\sigma(t_4) = x_5$

$h_\sigma(t_5) = x_1 + x_2 + x_3 + x_4 + x_5$

$z_0 \xrightarrow{0.7} \xrightarrow{t_1} \xrightarrow{0.0} \xrightarrow{t_3} \xrightarrow{1} \xrightarrow{t_4} \xrightarrow{1} \xrightarrow{t_2} \xrightarrow{0} \xrightarrow{t_3} \xrightarrow{1} (m_\sigma, (1.0, 1.0, 1.0, 1.0, 3.7, \sharp))$

I			x_0	x_1	x_2	x_3	x_4	x_5	$h_\sigma(t_1)$	$h_\sigma(t_2)$	$h_\sigma(t_5)$
$\hat{\beta}$	=	β_0	0.7	0.0	0.4	1.2	0.5	1.4	1.9	1.4	4.2
		β_1	0.7	0.0	0.4	1.2	0.5	**1**	1.5	**1.0**	3.8
		β_2	0.7	0.0	0.4	1.2	**0**	1	**1.0**		3.3
		β_3	0.7	0.0	0.4	**1**	0	1			3.1
		β_4	0.7	0.0	**1**	1	0	1			3.7
		β_5	0.7		1	1	0	1			

$h_\sigma(t_1) = x_4 + x_5,$ $\quad h_\sigma(t_2) = h_\sigma(t_3) = h_\sigma(t_4) = x_5$

$h_\sigma(t_5) = x_1 + x_2 + x_3 + x_4 + x_5$

$$z_0 \xrightarrow{0.7} \xrightarrow{t_1} \xrightarrow{0} \xrightarrow{t_3} \xrightarrow{1} \xrightarrow{t_4} \xrightarrow{1} \xrightarrow{t_2} \xrightarrow{0} \xrightarrow{t_3} \xrightarrow{1} (m_\sigma, (1.0, 1.0, 1.0, 1.0, 3.7, \sharp))$$

I			x_0	x_1	x_2	x_3	x_4	x_5	$h_\sigma(t_1)$	$h_\sigma(t_2)$	$h_\sigma(t_5)$
$\hat{\beta}$	$=$	β_0	0.7	0.0	0.4	1.2	0.5	1.4	1.9	1.4	4.2
		β_1	0.7	0.0	0.4	1.2	0.5	**1**	1.5	**1.0**	3.8
		β_2	0.7	0.0	0.4	1.2	**0**	1	**1.0**		3.3
		β_3	0.7	0.0	0.4	**1**	0	1			3.1
		β_4	0.7	0.0	**1**	1	0	1			3.7
		β_5	0.7	**0**	1	1	0	1			

$h_\sigma(t_1) = x_4 + x_5$, $h_\sigma(t_2) = h_\sigma(t_3) = h_\sigma(t_4) = x_5$

$h_\sigma(t_5) = x_1 + x_2 + x_3 + x_4 + x_5$

$$z_0 \xrightarrow{0.7} \xrightarrow{t_1} \xrightarrow{0} \xrightarrow{t_3} \xrightarrow{1} \xrightarrow{t_4} \xrightarrow{1} \xrightarrow{t_2} \xrightarrow{0} \xrightarrow{t_3} \xrightarrow{1} (m_\sigma, (1.0, 1.0, 1.0, 1.0, 3.7, \sharp))$$

I	x_0	x_1	x_2	x_3	x_4	x_5	$h_\sigma(t_1)$	$h_\sigma(t_2)$	$h_\sigma(t_5)$
$\hat{\beta} = \beta_0$	0.7	0.0	0.4	1.2	0.5	1.4	1.9	1.4	4.2
β_1	0.7	0.0	0.4	1.2	0.5	**1**	1.5	**1.0**	3.8
β_2	0.7	0.0	0.4	1.2	**0**	1	**1.0**		3.3
β_3	0.7	0.0	0.4	**1**	0	1			3.1
β_4	0.7	0.0	**1**	1	0	1			3.7
β_5	0.7	**0**	1	1	0	1			3.7

$h_\sigma(t_1) = x_4 + x_5$, $\quad h_\sigma(t_2) = h_\sigma(t_3) = h_\sigma(t_4) = x_5$

$h_\sigma(t_5) = x_1 + x_2 + x_3 + x_4 + x_5$

$$z_0 \xrightarrow{0.7} \xrightarrow{t_1} \xrightarrow{0} \xrightarrow{t_3} \xrightarrow{1} \xrightarrow{t_4} \xrightarrow{1} \xrightarrow{t_2} \xrightarrow{0} \xrightarrow{t_3} \xrightarrow{1} (m_\sigma, (1.0, 1.0, 1.0, 1.0, 3.7, \sharp))$$

I			x_0	x_1	x_2	x_3	x_4	x_5	$h_\sigma(t_1)$	$h_\sigma(t_2)$	$h_\sigma(t_5)$
$\hat{\beta}$	=	β_0	0.7	0.0	0.4	1.2	0.5	1.4	1.9	1.4	4.2
		β_1	0.7	0.0	0.4	1.2	0.5	**1**	1.5	**1.0**	3.8
		β_2	0.7	0.0	0.4	1.2	**0**	1	**1.0**		3.3
		β_3	0.7	0.0	0.4	**1**	0	1			3.1
		β_4	0.7	0.0	**1**	1	0	1			3.7
		β_5	0.7	**0**	1	1	0	1			3.7
β^*	=	β_6		0	1	1	0	1			

$h_\sigma(t_1) = x_4 + x_5,$ $h_\sigma(t_2) = h_\sigma(t_3) = h_\sigma(t_4) = x_5$

$h_\sigma(t_5) = x_1 + x_2 + x_3 + x_4 + x_5$

$$z_0 \xrightarrow{1} \xrightarrow{t_1} \xrightarrow{0} \xrightarrow{t_3} \xrightarrow{1} \xrightarrow{t_4} \xrightarrow{1} \xrightarrow{t_2} \xrightarrow{0} \xrightarrow{t_3} \xrightarrow{1} (m_\sigma, (1.0, 1.0, 1.0, 1.0, 3.7, \sharp))$$

I			x_0	x_1	x_2	x_3	x_4	x_5	$h_\sigma(t_1)$	$h_\sigma(t_2)$	$h_\sigma(t_5)$
$\hat{\beta}$	$=$	β_0	0.7	0.0	0.4	1.2	0.5	1.4	1.9	1.4	4.2
		β_1	0.7	0.0	0.4	1.2	0.5	**1**	1.5	**1.0**	3.8
		β_2	0.7	0.0	0.4	1.2	**0**	1	**1.0**		3.3
		β_3	0.7	0.0	0.4	**1**	0	1			3.1
		β_4	0.7	0.0	**1**	1	0	1			3.7
		β_5	0.7	**0**	1	1	0	1			3.7
β^*	$=$	β_6	**1**	0	1	1	0	1			

$h_\sigma(t_1) = x_4 + x_5$, $\quad h_\sigma(t_2) = h_\sigma(t_3) = h_\sigma(t_4) = x_5$

$h_\sigma(t_5) = x_1 + x_2 + x_3 + x_4 + x_5$

$$z_0 \xrightarrow{1} \xrightarrow{t_1} \xrightarrow{0} \xrightarrow{t_3} \xrightarrow{1} \xrightarrow{t_4} \xrightarrow{1} \xrightarrow{t_2} \xrightarrow{0} \xrightarrow{t_3} \xrightarrow{1} (m_\sigma, (1.0, 1.0, 1.0, 1.0, 4.0, \sharp))$$

I	x_0	x_1	x_2	x_3	x_4	x_5	$h_\sigma(t_1)$	$h_\sigma(t_2)$	$h_\sigma(t_5)$
$\hat{\beta} = \beta_0$	0.7	0.0	0.4	1.2	0.5	1.4	1.9	1.4	4.2
β_1	0.7	0.0	0.4	1.2	0.5	**1**	1.5	**1.0**	3.8
β_2	0.7	0.0	0.4	1.2	**0**	1	**1.0**		3.3
β_3	0.7	0.0	0.4	**1**	0	1			3.1
β_4	0.7	0.0	**1**	1	0	1			3.7
β_5	0.7	**0**	1	1	0	1			3.7
$\beta^* = \beta_6$	**1**	0	1	1	0	1			**4.0**

$h_\sigma(t_1) = x_4 + x_5$, $h_\sigma(t_2) = h_\sigma(t_3) = h_\sigma(t_4) = x_5$

$h_\sigma(t_5) = x_1 + x_2 + x_3 + x_4 + x_5$

Main Property

Theorem 2:

Let $\mathcal{Z}$ be a TPN and $\sigma = t_1 \ \cdots \ t_n)$ be a feasible transition sequence in $\mathcal{Z}$, with feasable run $\sigma(\tau)$ of σ $\left(\tau = \tau_0 \ldots \tau_n\right)$ i.e.

$$z_0 \xrightarrow{\tau_0}\xrightarrow{t_1} \cdots \xrightarrow{t_n}\xrightarrow{\tau_n} z_n = (m_n, h_n),$$

and all $\tau_i \in \mathbb{R}_0^+$. Then, there exists a further feasible run $\sigma(\tau^*)$ of σ with

$$z_0 \xrightarrow{\tau_0^*}\xrightarrow{t_1} \cdots \xrightarrow{t_n}\xrightarrow{\tau_n^*} z_n^* = (m_n^*, h_n^*).$$

such that

Main Property

Theorem 2 – Continuation:

1. For each $i, 0 \leq i \leq n$ the time τ_i^* is a natural number.
2. For each enabled transition t at marking $m_n (= m_n^*)$ it holds:
   1. $h_n(t)^* = \lceil h_n(t) \rceil$.
   2. $\sum_{i=1}^{n} \tau_i^* = \lceil \sum_{i=1}^{n} \tau_i \rceil$
3. For each transition $t \in T$ holds:
   t is ready to fire in z_n iff t is also ready to fire in $\lceil z_n \rceil$.

$$z_0 \xrightarrow{0.7} \xrightarrow{t_1} \xrightarrow{0.0} \xrightarrow{t_3} \xrightarrow{0.4} \xrightarrow{t_4} \xrightarrow{1.2} \xrightarrow{t_2} \xrightarrow{0.5} \xrightarrow{t_3} \xrightarrow{1.4} (m_\sigma, (1.9, 1.4, 1.4, 1.4, 4.2, \sharp))$$

∥			x_0	x_1	x_2	x_3	x_4	x_5	$h_\sigma(t_1)$	$h_\sigma(t_2)$	$h_\sigma(t_5)$
$\hat{\beta}$	$=$	β_0	0.7	0.0	0.4	1.2	0.5	1.4	1.9	1.4	4.2
		β_1	0.7	0.0	0.4	1.2	0.5	**2**	2.5	**2.0**	4.8
		β_2	0.7	0.0	0.4	1.2	**0**	2	**2.0**		4.3
		β_3	0.7	0.0	0.4	**2**	0	2			5.1
		β_4	0.7	0.0	**0**	2	0	2			4.7
		β_5	0.7	**0**	0	2	0	2			4.7
		β_6	**1**	0	0	2	0	2			**5.0**

$$z_0 \xrightarrow{1} \xrightarrow{t_1} \xrightarrow{0} \xrightarrow{t_3} \xrightarrow{0} \xrightarrow{t_4} \xrightarrow{2} \xrightarrow{t_2} \xrightarrow{0} \xrightarrow{t_3} \xrightarrow{2} (m_\sigma, (2,\ 2,\ 2,\ 2,\ 5,\ \sharp))$$

$$z_0 \xrightarrow{0.7} \xrightarrow{t_1} \xrightarrow{0.0} \xrightarrow{t_3} \xrightarrow{0.4} \xrightarrow{t_4} \xrightarrow{1.2} \xrightarrow{t_2} \xrightarrow{0.5} \xrightarrow{t_3} \xrightarrow{1.4} (m_\sigma, (1.9, 1.4, 1.4, 1.4, 4.2, \sharp))$$

The time length of the run $\sigma(\tau)$ is

$$\hat{\beta}(x_0) + \hat{\beta}(x_1) + \hat{\beta}(x_2) + \hat{\beta}(x_3) + \hat{\beta}(x_4) + \hat{\beta}(x_5) = 4.2$$

$$z_0 \xrightarrow{0.7} \xrightarrow{t_1} \xrightarrow{0.0} \xrightarrow{t_3} \xrightarrow{0.4} \xrightarrow{t_4} \xrightarrow{1.2} \xrightarrow{t_2} \xrightarrow{0.5} \xrightarrow{t_3} \xrightarrow{1.4} (m_\sigma, (1.9, 1.4, 1.4, 1.4, 4.2, \sharp))$$

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In tableau I: The time length of the run $\sigma(\tau_1^*)$ is 4

$$z_0 \xrightarrow{0.7} \xrightarrow{t_1} \xrightarrow{0.0} \xrightarrow{t_3} \xrightarrow{0.4} \xrightarrow{t_4} \xrightarrow{1.2} \xrightarrow{t_2} \xrightarrow{0.5} \xrightarrow{t_3} \xrightarrow{1.4} (m_\sigma, (1.9, 1.4, 1.4, 1.4, 4.2, \sharp))$$

The time length of the run $\sigma(\tau)$ is

$$\hat{\beta}(x_0) + \hat{\beta}(x_1) + \hat{\beta}(x_2) + \hat{\beta}(x_3) + \hat{\beta}(x_4) + \hat{\beta}(x_5) = 4.2$$

In tableau I: The time length of the run $\sigma(\tau_1^*)$ is 4

In tableau II: The time length of the run $\sigma(\tau_2^*)$ is 5

Some Conclusions

- Each feasible transitions sequence σ in $\mathcal{Z}$ can be realized with an **integer** run.
- Each reachable *p*-marking in $\mathcal{Z}$ can be reached using **integer** runs only.
- If z is reachable in $\mathcal{Z}$, then $\lfloor z \rfloor$ and $\lceil z \rceil$ are reachable in $\mathcal{Z}$ as well.
- The length of the shortest and longest time path (if this is finite) between two arbitrary *p*-markings are natural numbers.

A run $\sigma(\tau) = \tau_0 \; t_1 \; \tau_1 \ldots t_n \; \tau_n$ is an **integer** one, if $\tau_i \in \mathbb{N}$ for each $i = 0 \ldots n$.

Integer States

A state $z = (m, h)$ is an **integer** one, if $h(t) \in \mathbb{N}$ for each in m enabled transition t.

Theorem 3:

Let $\mathcal{Z}$ be a finite TPN, i.e. $lft(t) \neq \infty$ for all $t \in T$.
The set of all reachable integer states in $\mathcal{Z}$ is finite
if and only if
the set of all reachable p-markings in $\mathcal{Z}$ is finite.

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Remark:

Theorem 3 can be generalized for all TPNs (applying a further reduction of the state space).

Modified Rule

Let $\mathcal{Z}$ be an arbitrary TPN. The state change **by time elapsing** can be slightly **modified** for each transition t with $lft(t) = \infty$, because in order to fire such a transition t

- it is important to know whether t is old enough or not, i.e. whether t has been enabled last for $eft(t)$ (or more) time units or t is younger.
- Thus, the time $h(t)$ increases **until** $eft(t)$. After that, the clock of t remains in this position (although the time is elapsing), unless t becomes disabled.

Essential States

Theorem 4:

In an arbitrary TPN a p-marking is reachable using the non-modified definition iff it is reachable using the modified one.

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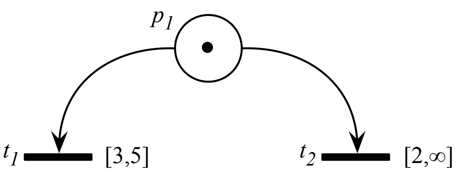
Theorem 5:

An arbitrary TPN is bounded iff the set of its reachable essential states is finite.

Standard Rule vs. Modified Rule



It holds that:

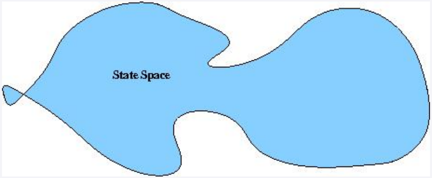
$$RIS_{\mathcal{Z}} := \{(1, \begin{pmatrix} 0 \\ 0 \end{pmatrix}), (1, \begin{pmatrix} 1 \\ 1 \end{pmatrix}), (1, \begin{pmatrix} 2 \\ 2 \end{pmatrix}), (1, \begin{pmatrix} 3 \\ 3 \end{pmatrix}), (1, \begin{pmatrix} 4 \\ 4 \end{pmatrix}), (1, \begin{pmatrix} 5 \\ 5 \end{pmatrix}), (0, \begin{pmatrix} \sharp \\ \sharp \end{pmatrix})\}$$

and

$$REIS_{\mathcal{Z}} := \{(1, \begin{pmatrix} 0 \\ 0 \end{pmatrix}), (1, \begin{pmatrix} 1 \\ 1 \end{pmatrix}), (1, \begin{pmatrix} 2 \\ 2 \end{pmatrix}), (1, \begin{pmatrix} 3 \\ 2 \end{pmatrix}), (1, \begin{pmatrix} 4 \\ 2 \end{pmatrix}), (1, \begin{pmatrix} 5 \\ 2 \end{pmatrix}), (0, \begin{pmatrix} \sharp \\ \sharp \end{pmatrix})\}.$$

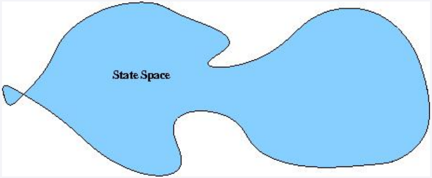
Clearly, neither $REIS_{\mathcal{Z}} \subseteq RIS_{\mathcal{Z}}$ nor $REIS_{\mathcal{Z}} \supseteq RIS_{\mathcal{Z}}$.

Discrete Reduction of the State Space

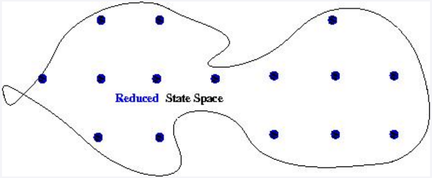


The set of all reachable states

Discrete Reduction of the State Space

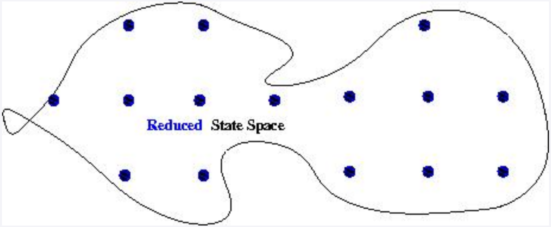


The set of all reachable states

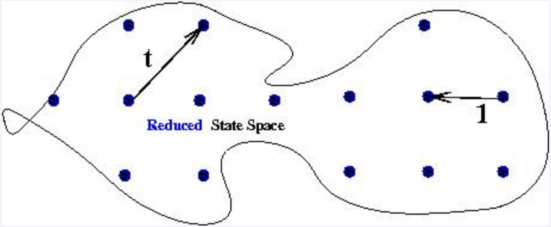


The set of all essential states

(Reduced) Reachability Graph

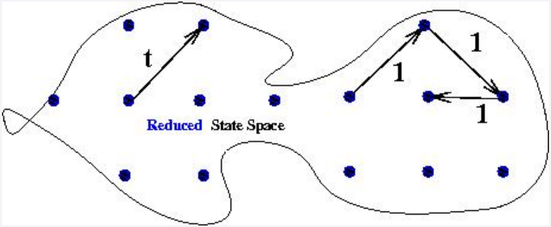


(Reduced) Reachability Graph



The reachability graph is a weighted directed graph, including the time explicit.

(Reduced) Reachability Graph



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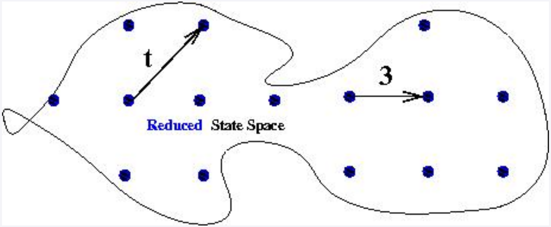

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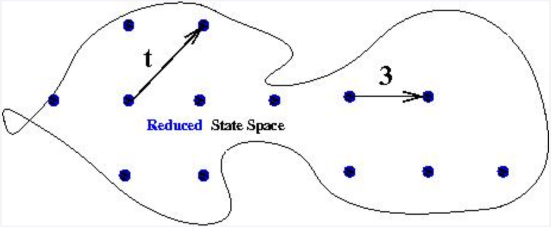
(Reduced) Reachability Graph



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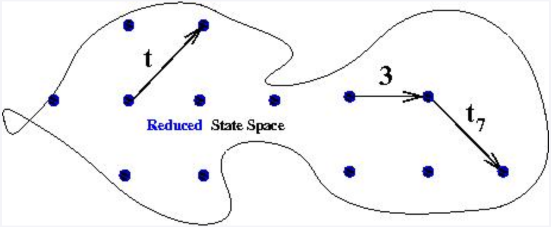
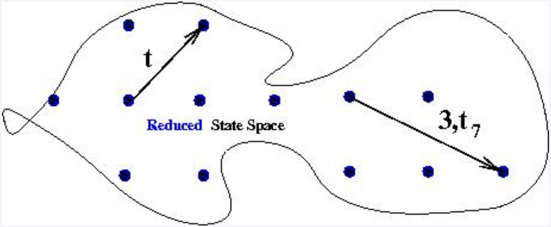


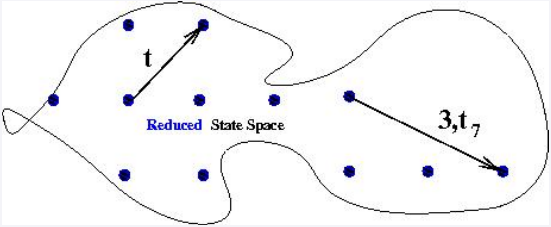
The reachability graph is a weighted directed graph, including the time explicit.

(Reduced) Reachability Graph



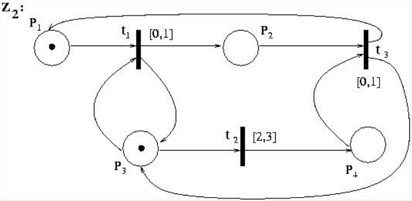
The reachability graph is a weighted directed graph, including the time explicit.

(Reduced) Reachability Graph



The reachability graph is a weighted directed graph, including the time explicit.

Example: A finite TPN and its reachability graph



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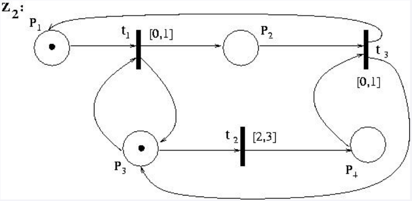

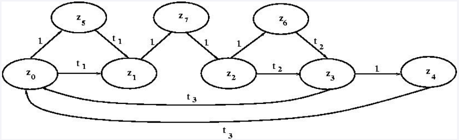

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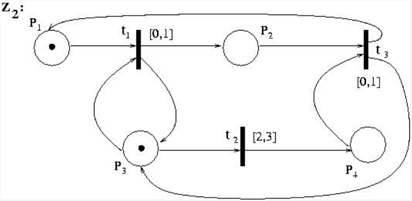

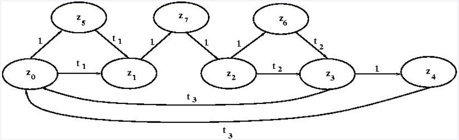

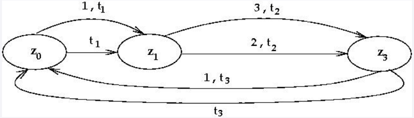

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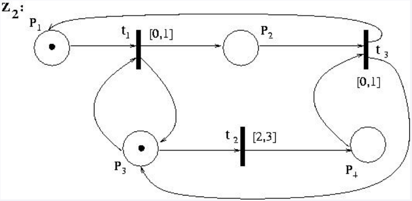
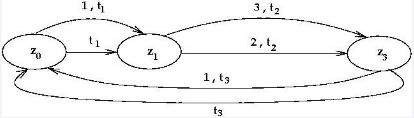
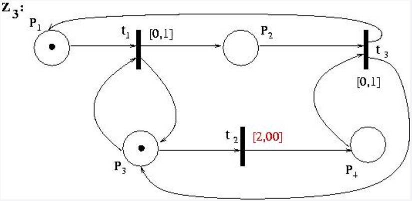
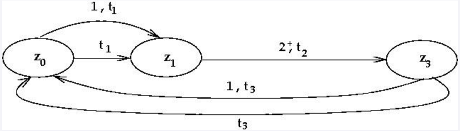


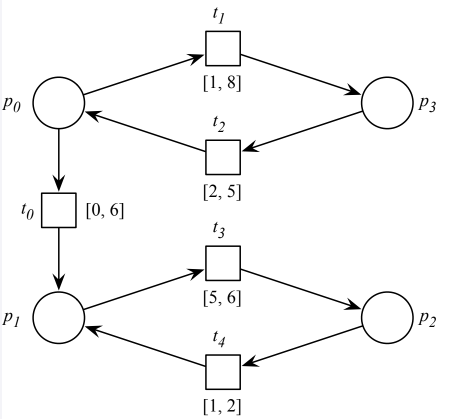


Example: A non-finite TPN and its reachability graph





State Classes Reachability Graphs vs. Essential States Reachability Graphs



number of tokens in p_0	essential-states algorithm			state class algorithm		
	number of vertices	number of edges	total number	number of vertices	number of edges	total number
0	1	0	1	1	0	1
1	4	21	25	4	5	9
2	63	310	373	81	157	238
3	250	1252	1502	258	574	832
4	692	3920	4612	1053	2979	4032
5	1367	8115	9482	2653	8119	10772
6	2265	13769	16034	5000	15884	20884
7	3386	20882	24268	8089	26315	34404
8	4730	29454	34184	11909	39371	51280
9	6297	39485	45782	16454	55023	71477
10	8087	50975	59062	21708	73210	94918

The firing rule is defined based on static conflict.

number of tokens in p_0	essential-states algorithm			state class algorithm		
	number of vertices	number of edges	total number	number of vertices	number of edges	total number
0	1	0	1	1	0	1
1	4	21	25	4	5	9
2	86	441	527	94	186	280
3	550	2740	3290	570	1354	1924
4	1916	9975	11891	2181	5907	8088
5	9167	50618	59785	16588	53781	70369
7	15152	84449	99601	34118	114249	148367
8	22862	127989	150851	61123	208195	269318
9	32165	180510	212675	97479	335218	432697
10	42989	241713	284702	142712	493602	636314

The firing rule is defined based on dynamic conflict.

Questions, Discussions ?