The Zoology of Petri Nets with Time

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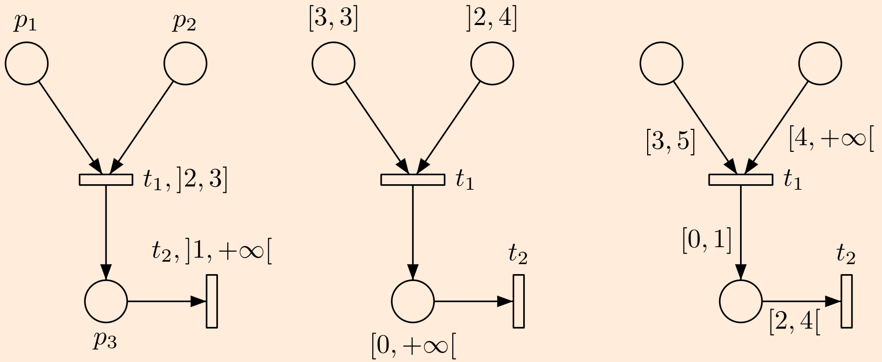
Where to add time?

From an untimed model given as a safe, ordinary, colored Petri net, etc.,

Syntax

Time intervals can be associated with transitions, or places,
or (the various kinds of) arcs.



How to add time?

Time elapsing can be associated with

- Firing duration
- Firing delay

Choice policy

- Firing a transition with earliest delay
- Non deterministic choice

Server policy

- One delay per instance of firing (w.r.t. firing degree)
- A single delay per enabled transition

Memory policy

- Resetting the delay of all transitions
- Memorizing the remaining delay of still enabled transitions
- Memorizing the remaining delay of all transitions

In addition, time may be discrete or dense.

Timed semantics

as Timed Transition System

Act alphabet of actions, $\mathbb{T}$ time domain contained in $\mathbb{R}_{\geq 0}$,

$\mathcal{T} = (S, s_0, E)$ timed transition system

- S set of configurations, s_0 initial configuration,
- $E \subseteq S \times (Act \cup \{\varepsilon\} \cup \mathbb{T}) \times S$ contains

  action transitions: $s \xrightarrow{a} s'$, execution of a

  delay transitions: $s \xrightarrow{d} s'$, time elapsing for d time units.

Variant: labels in $(Act \cup \{\varepsilon\}) \times \mathbb{T}$, with combined steps of the form $s \xRightarrow{(d,a)} s'$.

Timed observations

are sequences of the form: $d_1 a_1 d_2 a_2 \ldots$ or $(a_1, \tau_1)\ (a_2, \tau_2) \ldots$
where $\tau_i = \sum_{j \leq i} d_j$ is the "date" of a_i.

According to duration or delay semantics, d_i can be seen as

- waiting time followed by instantaneous action a_i,
- duration of action a_i.

Time intervals on transitions

Configuration specification

A configuration is a pair $s = (M, v)$ like $s = (3p_1 + p_2, (2.5, \bot))$
$M \in Bag(P)$: marking, $v \in (\mathbb{R}_{\geq 0} \cup \{\bot\})^T$: valuation of transition clocks.

Time elapsing

$v(t)$ represents the time elapsed since last time t was enabled.

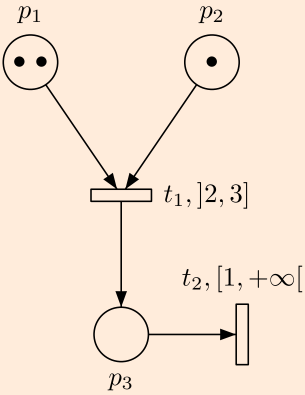
$$(M, v) \xrightarrow{d} (M, v + d)$$

with $(v + d)(t) = v(t) + d$ if t is enabled.
Strong semantics: transitions cannot become dead
(when $v(t)$ exceeds the upper bound of $I(t)$).

Transition firing

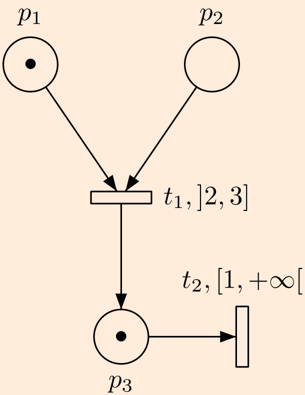
Transition t can be fired if $v(t)$ belongs to $I(t)$ and this firing causes some transitions to be newly enabled (several possible definitions).

Example



$$(2p_1 + p_2, (0, \perp)) \xrightarrow{2.5} (2p_1 + p_2, (2.5, \perp))$$

Example



$$(2p_1+p_2,(0,\bot)) \xrightarrow{2.5} (2p_1+p_2,(2.5,\bot)) \xrightarrow{t_1} (p_1+p_3,(\bot,0)) \xrightarrow{1.5} (p_1+p_3,(\bot,1.5))$$

Time intervals on places or arcs

Configuration specification

A configuration is a timed marking $s \in Bag(P \times \mathbb{R}_{\geq 0})$
like $s = 2(p_1, 0) + (p_1, 0.3) + (p_2, 0.7)$.

Time elapsing

The age of tokens increases at the rate of time:

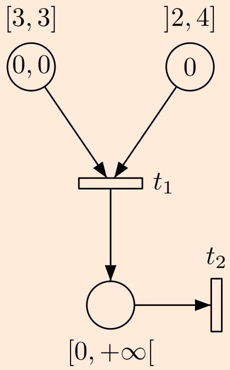
$$(p, \tau) \xrightarrow{d} (p, \tau + d)$$

Strong semantics: tokens cannot become dead (when age exceeds the upper bound of interval associated with place or input arc).

Transition firing

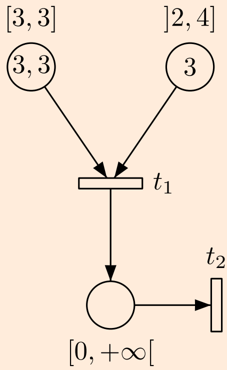
A token can be used in a transition firing if its age belongs to the interval of the place or input arc.

Examples



$$2(p_1, 0) + (p_2, 0) \xrightarrow{3} 2(p_1, 3) + (p_2, 3) \xrightarrow{t_1} (p_1, 3) + (p_3, 0) \xrightarrow{2.5} (p_1, 5.5) + (p_3, 2.5)$$

Examples



$$2(p_1,0)+(p_2,0)\xrightarrow{3}2(p_1,3)+(p_2,3)\xrightarrow{t_1}(p_1,3)+(p_3,0)\xrightarrow{2.5}(p_1,5.5)+(p_3,2.5)$$

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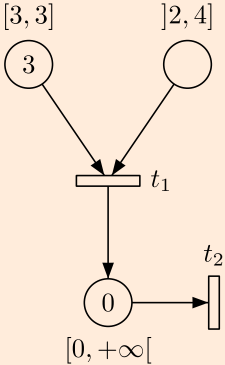

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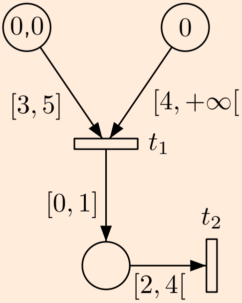

$$2(p_1, 0) + (p_2, 0) \xrightarrow{4} 2(p_1, 4) + (p_2, 4) \xrightarrow{t_1} (p_1, 4) + (p_3, 0.9) \xrightarrow{1.1} (p_1, 5.1) + (p_3, 2)$$

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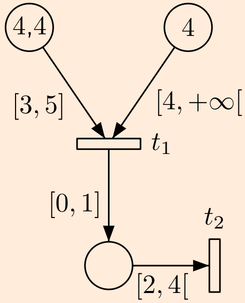

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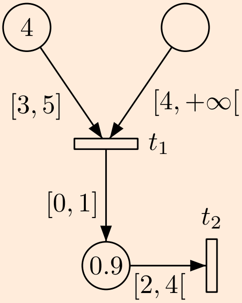

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How to compare models?

Expressiveness

- equality of accepted languages (for some suitable acceptance condition),
- weak timed bisimilarity,
- ...

associating labels with transitions, including ε to represent an internal action.

Computational complexity

for decidability of verification or comparison problems

Example: The covering problem is

- undecidable for strong Time-Transition Petri nets,
- decidable and $F_{\omega^{\omega^{\omega}}}$-complete [Haddad-Schmitz-Schnoebelen-LICS-12] for weak Timed-Arc Petri nets,
- EXPSPACE-complete for standard PN and weak Time-Transition Petri nets.

[Sangnier-Reynier-Concur-09]

An example for language equivalence

Question:

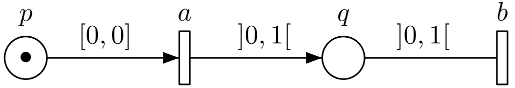
For weak Timed-Arc PN accepting infinite words, what is the power of general reset compared to 0-reset?

- Considering or not Zeno words
- Considering bounds in $\mathbb{Q}_{\geq 0}$ or in $\mathbb{N}$
- Considering or not read arcs.

With integer bounds and read arcs, general reset is strictly more powerful than 0-reset [Bouyer-Haddad-Reynier-IC-08]

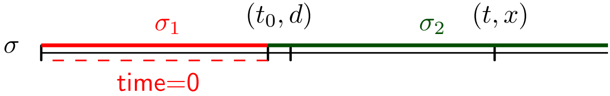
The language of infinite words
$L = \{(a,0)(b,\tau_1)\ldots(b,\tau_n)\ldots \mid \exists \tau < 1,\ 0 \leq \tau_1 \leq \ldots \leq \tau_n \leq \ldots < \tau\}$
cannot be accepted by any Timed-Arc PN with integral bounds producing only tokens with age 0.

A Timed-Arc PN with read arcs and general reset for L with Acc $= \{q \geq 1\}$:



Proof

Assume $\mathcal{N}$ as above accepts L and consider a firing sequence σ with label $w = (a, 0)(b, \tau_1) \ldots (b, \tau_n)$ for some $0 < \tau < 1$ such that $(\tau_n)_n$ is strictly increasing and converges to τ.



In σ, let (t_0, d) be the first transition fired at some non nul time $d > 0$, hence $\sigma = \sigma_1 \sigma_2$ with (t_0, d) the first transition in σ_2.

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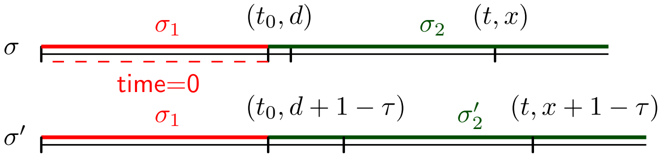


In σ, let (t_0, d) be the first transition fired at some non nul time $d > 0$, hence $\sigma = \sigma_1\sigma_2$ with (t_0, d) the first transition in σ_2.

Then $\sigma' = \sigma_1\sigma'_2$ where σ'_2 is obtained by delaying σ_2 by $1-\tau$ can still be fired:

- A token produced in σ_2 has the same age in σ_2 and σ'_2 when tested or consumed;
- A token initially present or produced by σ_1 with age 0 is tested or consumed by some transition t in σ_2 at some age y such that $0 < d \leq y < \tau < 1$, hence any incoming arc of t contains $]0,1[$. In σ'_2, the age of this token is $y' = y + 1 - \tau$, with $d + 1 - \tau \leq y' < 1$, hence t can also be fired.

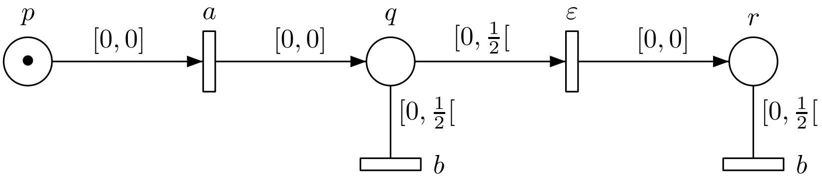
But since $(\tau_n + 1 - \tau)_n$ converges to 1, the label w' of σ' does not belong to L.

Observations

Expressiveness remains the same:

- without read arcs,
- or if the integral bounds hypothesis is removed,
- or if Zeno words are excluded.

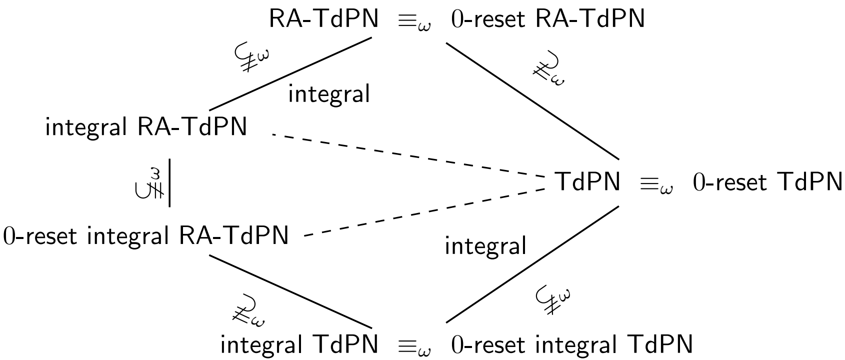
A Timed-Arc PN with read arcs, rational bounds and 0-reset for L with Acc $= \{q \geq 1 \text{ or } r \geq 1\}$:



Hierarchies for Timed-Arc PN

with read arcs and language equivalence [Bouyer-Haddad-Reynier-IC08]

For infinite words:



For finite words or infinite non Zeno words, the hierarchy collapses:

$$\text{RA-TdPN} \equiv_{*,\omega_{nz}} \text{TdPN} \equiv_{*,\omega_{nz}} 0\text{-reset}\,\text{TdPN}$$

Weak time bisimilarity

Given two timed transition systems $\mathcal{T}_1 = (S_1, s_1^0, E_1)$ and $\mathcal{T}_2 = (S_2, s_2^0, E_2)$

$\mathcal{T}_1$ and $\mathcal{T}_2$ are weaky timed bisimilar

if there is an equivalence relation $\approx$ on $S_1 \times S_2$ such that s_1^0 and s_2^0 are equivalent and:

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if $\quad s_1 \xrightarrow{a} s_1'$

$\quad\quad \approx$

$\quad\quad s_2$

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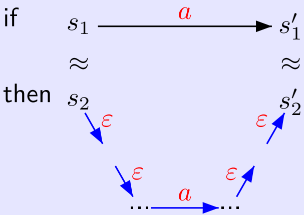

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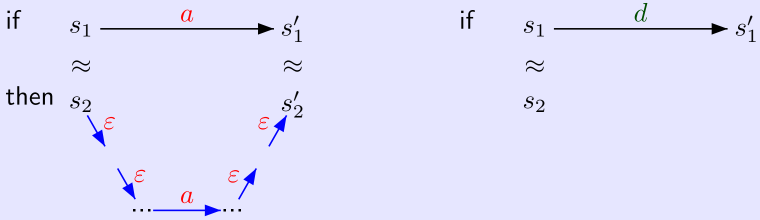

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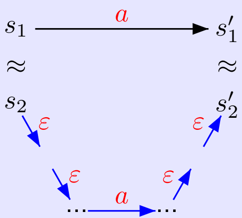
if there is an equivalence relation $\approx$ on $S_1 \times S_2$ such that s_1^0 and s_2^0 are equivalent and:

if / then



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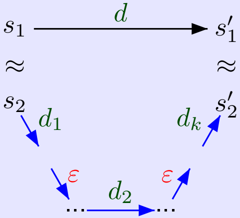


with $\Sigma d_i = d$

and vice versa.

An example for weak timed bisimilarity

Question :

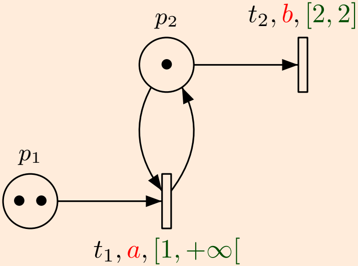
In the context of strong Time Transition PN, what is the impact of memory policies?

Different reset policies

For the clock value of a transition enabled after a firing:

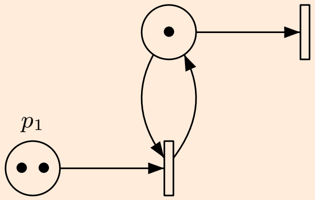
- Intermediate (classical) semantics (I): the transition is newly enabled if it was disabled after the consuming step or if it is the fired transition.
- Atomic semantics (A): the transition is newly enabled if it was disabled before the firing or if it is the fired transition.
- Persistent atomic semantics (PA): the transition is newly enabled if it was disabled before the firing.

Example



$$(2p_1 + p_2, [0,0]) \xrightarrow{1.3} (2p_1 + p_2, [1.3, 1.3]) \xrightarrow{a} \cdots$$

Example



$$(2p_1 + p_2, [0,0]) \xrightarrow{1.3} (2p_1 + p_2, [1.3, 1.3]) \xrightarrow{a} \cdots$$

(I): $(p_1 + p_2, [0,0]) \xrightarrow{2} (p_1 + p_2, [2,2])$

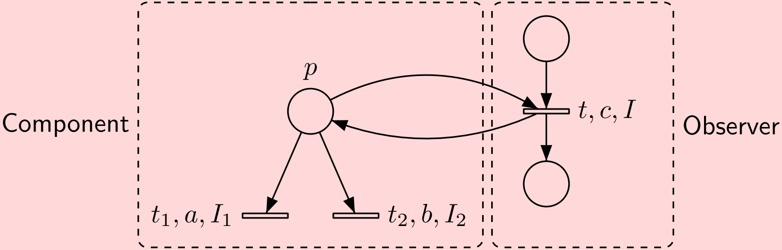
(A): $(p_1 + p_2, [0, 1.3]) \xrightarrow{0.7} (p_1 + p_2, [0.7, 2]) \xrightarrow{b} (p_1, [\bot, \bot])$

(PA): $(p_1 + p_2, [1.3, 1.3]) \xrightarrow{a} (p_2, [\bot, 1.3])$

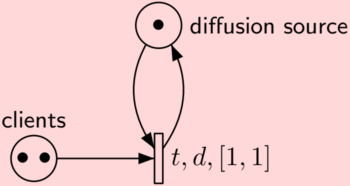
Motivation

Why alternative semantics ?

- (PA) is closer to the semantics of TA
- (A) or (PA) are sometimes more convenient than (I):



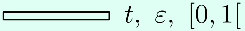
- For e.g. instantaneous multicast, (PA) is more convenient than (A) or (I):



Expressivity result

(PA) is strictly more expressive than (A) [BCHLR-ATVA05]

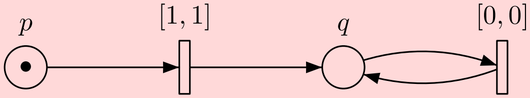
For the following TPN $\mathcal{N}_{[0,1[}$ with (PA) semantics, there is no TPN with (A) semantics bisimilar to $\mathcal{N}$.



This TPN lets time elapse without reaching 1 t.u.

The result does not hold for $[0,1]$

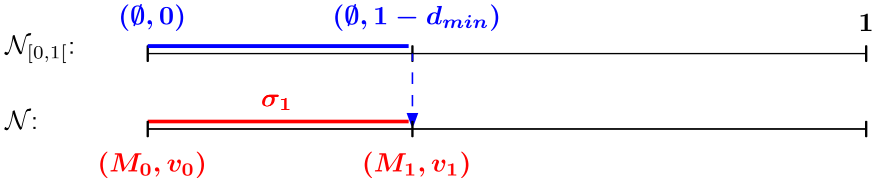
In this case, the following net with (A) or (I) semantics is bisimilar to $\mathcal{N}_{[0,1]}$:



For safe TPNs with upper-closed intervals, the three semantics are equivalent.

Proof

Assume some net $\mathcal{N}$ with atomic semantics is bisimilar to $\mathcal{N}_{[0,1[}$.
Define $d_{min} < 1$, with $d_{min} <$ min(non null upper bounds in $\mathcal{N}$).



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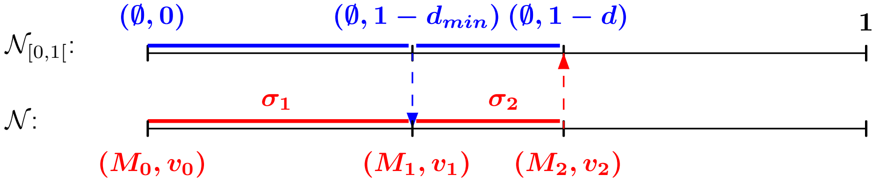

From (M_1, v_1), sequence σ_2 is built so that all transitions enabled in (M_1, v_1) are fired or disabled. Hence for all $t \in En(M_2)$, $v_2(t) \leq d_{min} - d$, with $0 < d \leq d_{min}$.

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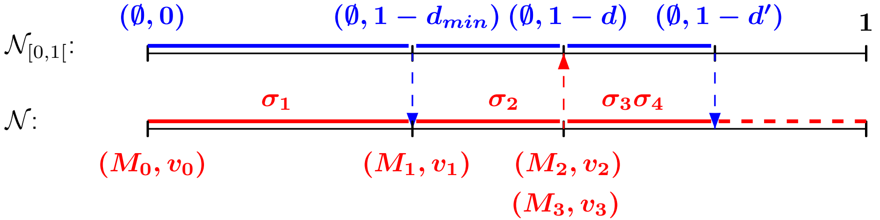

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Since time can still elapse in $\mathcal{N}_{[0,1[}$, with $0 < d' < d$, there is a sequence $\sigma_3\sigma_4$ from (M_2, v_2), such that σ_3 is maximal in null time, leading to (M_3, v_3). Hence, time can elapse in (M_3, v_3), and each enabled transition has an interval with non null upper bound $b > d_{min}$.

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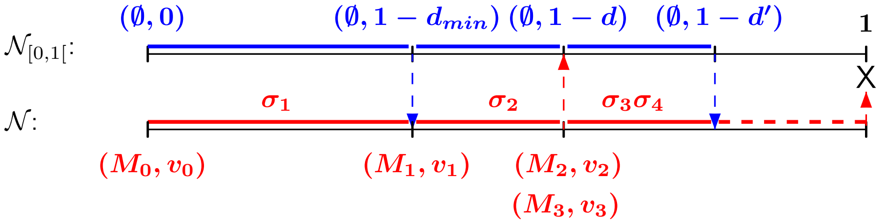

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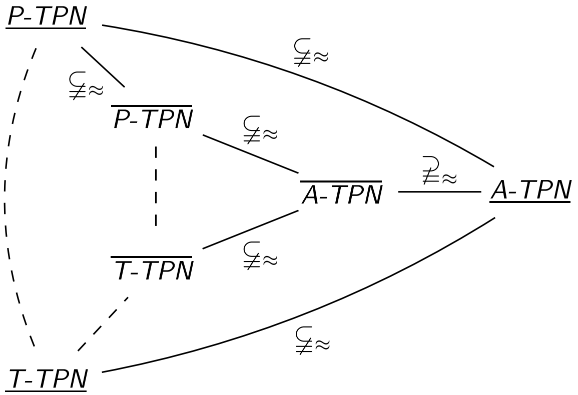
Then $(M_3, v_3) \xrightarrow{d}$, a contradiction to bisimilarity with $(\emptyset, 1 - d)$

Hierarchy for safe Time Petri nets

with intermediate semantics, for weak timed bisimulation
[Boyer-Roux-FI-08]



- With both weak and strong semantics, A-TPN are strictly more expressive than P-TPN and T-TPN, which in turn are incomparable;
- T-TPN with strong semantics and T-TPN with weak semantics are incomparable. Otherwise there is a strict inclusion.

Conclusion

A lot of open issues remain to be studied

- between the various classes of PN with time and
- between these classes and other important timed models like Timed Automata (or networks of TA).

Thank you