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Using State Equation to Prove Non-Reachability in Timed Petrinets*

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Abstract. Non-reachability proofs in Timed Petrinets were usually done by proving the non-reachability within the underlying timeless net. However, in many cases this approach fails.

In this paper, we present an approach to prove non-reachability within the actual Timed Petrinet. For this purpose, we introduce a state equation for Timed Petrinets in analogy to timeless nets. Using this state equation, we can express reachability as a system of equations and inequations, which is solvable in polynomial time.

Keywords: Timed Petrinet, Duration net, state equation, non-reachability

1. Introduction

1.1. Motivation

The complexity of todays computer systems is increasing every year due to increasing computing power and due to increasing demands of users. This is not only valid for general purpose systems used in offices or at home, but also for embedded systems used inside technical systems such as cars or airplanes. The correct function of such systems does not only depend on functional aspects such as the correctness of a result, but also on nonfunctional properties such as performance, timeliness, configurability or reliability. In most cases (especially if a failure of the system results in danger for humans) it is not sufficient to argue that these properties are fulfilled, it is necessary to prove them.

In order to develop such a proof it is necessary to use a formal description technique to describe the system, and to express the desired properties.

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As an example for a composable architecture for embedded real-time systems we have developed a system architecture called *Message Scheduled System* (MSS) [2]. MSS supports composability, which means that one can build a system out of components and put them together in a way that nonfunctional properties such as timeliness are still holding and system properties such as end-to-end-delays can be precomputed.

MSS uses scheduling algorithms at three different levels to reach that goal of composability. In order to prove that we meet our objectives, we developed a formal specification of MSS's behavior based on Timed Petrinets [4, 3].

This specification allows model checking using the tool INA[6], but proofs for general cases are still an open question. To solve this, we need proving technologies to prove non-reachability in a Timed Petrinet – this will allow us to prove that forbidden (dangerous) states in the MSS-model will never be reached.

A common approach to prove non-reachability in a Timed Petrinet (also called Duration Net, DPN) is to prove this property in the underlying timeless Petrinet, called skeleton of the DPN. However, since non-reachability in the timeless skeleton is a sufficient but not necessary condition for non-reachability in the DPN, this approach has limits.

In this paper, we present an approach that allows us to prove non-reachability in the actual DPN. For that purpose, we have developed a state equation for the DPN that allows us to determine the marking of a DPN in analogy to the state equation of timeless nets.

1.2. Related Work

The question of reachability and non-reachability raises for several problems which can be described by Petrinets. In general, the state space of a Petrinet is infinite. Different methods have been developed in order to prove certain properties without computing resp. knowing the whole state space of the net. One of the first approaches here is the computing of invariants (comp. [5]). In a classical Petrinet it is relatively easy to obtain place or transition invariants using the state equation of the net (comp. [5]).

But, when the Petrinet is a time dependent one, the state space becomes smaller. In this case, the use of the state equation of the skeleton is not very reasonable, because this state equation does not take into account the time constrains of the net. Because of the last, in the time dependent net new invariants can be arised, i.e., a reachable state in the skeleton can become unreachable.

Till now, there is no comparable approach for time dependent Petrinets as for the classical nets. In this work we consider Timed Petrinets, defined by Ramchandani (comp. [1]).

The remainder of this paper is organized as follows: Section 2 introduces the needed definitions. In Section 3, we present and prove the state equation for a DPN. In Section 4, we give an example for a non-reachability proof, using the state equation. Finally, the paper ends with some conclusions in Section 5.

2. Basics

2.1. Notation

This section introduces the basic notations we use in our paper. $\mathbb{N}$ and $\mathbb{R}$ denote the set of natural numbers and rational numbers, respectively. $\mathbb{N}^+ = \mathbb{N}\backslash\{0\}$ denotes the set of natural numbers without 0, and $\mathbb{Q}_0^+$ denotes the set of nonnegative rational numbers.

A matrix $A \in \mathcal{M}(m,n)$ is a matrix with m rows and n columns. A superindex in parentheses distinguishes different matrices or vectors, and their elements, respectively. $A_i^{(k)}$ is the i-th row and $A_{.j}^{(k)}$ is the j-th column of a matrix $A^{(k)}$.

Let M be a finite set. $|M|$ is the number of elements of M. $E_n = (e_{i,j}) \in \mathcal{M}(n,n)$ denotes the (unit-)matrix with $e_{i,j} = \begin{cases} 1 & i = j \\ 0 & \text{else} \end{cases}$ and $\mathcal{O}_n = (o_{i,j}) \in \mathcal{M}(n,n)$ the (zero-)matrix with $o_{i,j} = 0$ $\forall i,j$ The relation $r^{(1)} \not\geq r^{(2)}$ of the two vectors $r^{(1)}, r^{(2)} \in \mathcal{M}(m,1)$ means, that there exists at least one $i,\ i \in \{1,\ldots,m\}$ with $r_i^{(1)} < r_i^{(2)}$.

2.2. Definitions

Definition 2.1. (Petrinet)
The structure $N = (P,T,F,V,m_o)$ is called a Petrinet (PN) iff

1. P,T,F are finite sets with $P \cap T = \emptyset,\ \ P \cup T \neq \emptyset,\ \ F \subseteq (P \times T) \cup (T \times P)$ and $dom(F) \cup cod(F) = P \cup T$
2. $V : F \longrightarrow \mathbb{N}^+$ (weight of the arcs)
3. $m_o : P \longrightarrow \mathbb{N}$ (initial marking)

A marking of a PN is a function $m : P \longrightarrow \mathbb{N}$, such that $m(p)$ denotes the number of tokens at the place p. The pre- and post-sets of a transition t are given by $Ft = \{p \mid p \in P \wedge pFt\}$ and $tF = \{p \mid p \in P \wedge tFp\}$, respectively. Each transition $t \in T$ induces the markings t^- and t^+, defined as follows:

$$t^-(p) = \begin{cases} V(p,t) & \text{iff} \quad (p,t) \in F \\ 0 & \text{iff} \quad (p,t) \notin F \end{cases} \qquad t^+(p) = \begin{cases} V(t,p) & \text{iff} \quad (t,p) \in F \\ 0 & \text{iff} \quad (t,p) \notin F \end{cases}$$

Furthermore, Δt denotes $t^+ - t^-$. A transition $t \in T$ is enabled (may fire) at a marking m iff $t^- \leq m$ (e.g., $t^-(p) \leq m(p)$ for every place $p \in P$). When an enabled transition t at a marking m fires, this yields a new marking m' given by $m'(p) := m(p) + \Delta t(p)$. The firing is denoted by $m \xrightarrow{t} m'$.

Definition 2.2. (Timed Petrinet, Duration Petrinet, DPN)
The structure $Z = (N,D)$ is called a Timed Petrinet (DPN) iff:

1. $S(Z) = N$ is a PN (Skeleton of Z).
2. $D : T \longrightarrow \mathbb{Q}_0^+$ (duration function) so that $D(t)$ denotes the delay of transition t.

$D(t_i) = d_i$ is the duration of the firing transition t_i. It is easy to see, that considering DPNs with $D : T \longrightarrow \mathbb{N}$ will not result in a loss of generality. Therefore, only such time functions D will be considered subsequently. In the rest of this paper, we are only considering nets with no zero-delays, i.e., $d_i > 0$.

Example 2.1. The Timed Petrinet Z_1 which is used for illustration is shown in Figure 2.2.

Let $d := \max\{d_i | i \in \{1 \ldots |T|\}\} + 1$. In the net Z_1: $d = 3$.

Definition 2.3. (time marking)
Let Z be a DPN. A matrix m with $m \in \mathcal{M}(|P|, d)$ is a time marking in Z.

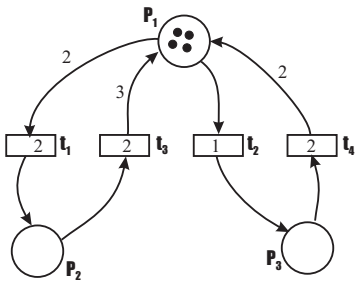

Figure 1. Net from Example 2.1

Definition 2.4. (initial time marking)
$m^{(0)}$ is an initial time marking, iff $m^{(0)}_{.1} = m_0$ and $m^{(0)}_{i,j} = 0$ for $i = 1 \ldots |P|$ and $j = 2 \ldots d$.

Each column of the time marking matrix represents a (partial) marking of a place for different time steps. The first column represents the present, the second one represents potential[1] changes in the marking one time unit in the future, the third two time units, etc.

Example 2.2. The net from Example 2.1 has the following initial time marking:

$$m^{(0)} = \begin{pmatrix} 4 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Definition 2.5. (transition clock vector)
Let $Z = (N, D)$ be a DPN. Then, the vector $h : T \longrightarrow \mathbb{N}$ is called transition clock vector of Z, iff $\forall t\,(t \in T \rightarrow h(t) \leq D(t))$

An element h_i of h is non-zero if the related transition is firing, and zero otherwise. The so-called initial transition clock vector is the zero-vector.

Definition 2.6. (maximal step)
Let $Z = (N, D)$ be an DPN. $\mathcal{B} \subset T$ is called a maximal step on the time marking m with the transition clock vector h iff

1. $\sum_{t \in \mathcal{B}} t^- \leq m_{.1}$
2. $\forall t(t \in \mathcal{B} \rightarrow h(t) = 0)$
3. $\neg\exists \mathcal{B}^*\,((\mathcal{B}^* \supset \mathcal{B}) \wedge (\mathcal{B}^* \text{ satisfies 1. and 2.}))$

Example 2.3. In the net from Example 2.1, $\mathcal{B} = \{t_1, t_2\}$ is the only maximal step on $m^{(0)}$ with a initial transition clock vector $h^{(0)}$.

Definition 2.7. (state change[2])
Let $Z = (N, D)$ be a DPN and $m^{(1)}$ and $m^{(2)}$ time markings in Z.

[1] Please note, that a row does not give a full description of future changes, since a transition with a shorter delay may fire later.
[2] The pair (m, h) can be considered as a state of a DPN.

The time marking $m^{(2)}$ is yield from $m^{(1)}$

(a) by firing maximal step $\mathcal{B} = \{t_{i_1}, \ldots, t_\kappa\}$ ($m^{(1)} \overset{\mathcal{B}}{\mapsto} m^{(2)}$), iff

$$m^{(2)}_{i,j} := \begin{cases} m^{(1)}_{i,j} - \sum\limits_{t_s \in \mathcal{B}} V(P_i, t_s) + \sum\limits_{\substack{t_s \in \mathcal{B}, \\ d_s = j-1 (=0)}} V(t_s, P_i) & , j = 1 \\ m^{(1)}_{i,j} + \sum\limits_{\substack{t_s \in \mathcal{B}, \\ d_s = j-1}} V(T_s, P_i) & , j > 1 \end{cases} \text{ and } h^{(2)}_i := \begin{cases} D(t_i) & , t_i \in \mathcal{B} \\ h^{(1)}_i & , else \end{cases}$$

(b) By elapsing one time unit ($m^{(1)} \underset{1}{\mapsto} m^{(2)}$), iff

$$m^{(2)}_{i,j} := \begin{cases} m^{(1)}_{i,1} + m^{(1)}_{i,2} & , j = 1 \\ m^{(1)}_{i,j+1} & , 2 \le j \le d-1 \\ 0 & , j = d \end{cases} \quad \text{and } h^{(2)}_i := \max(0, h^{(1)}_i - 1)$$

(a) and (b) happen strictly alternating, beginning with (a) from an initial time marking. But of course, the maximal step $\mathcal{B}$ may be the empty set.

The number of tokens $m_{i,j}$ arrives at place i after $j-1$ time steps.

Remark 2.1. Let $m^{(1)} \overset{\mathcal{B}}{\mapsto} m^{(2)}$. Then, it is easy to see from Definition 2.6.1 that the following holds

$$m^{(2)}_{.1} \not\geq t^- \text{ for each } t \in T \backslash \mathcal{B} \tag{1}$$

In case of self concurrency of transitions (1) holds for each $t \in T$.

Remark 2.2. Let $m^{(1)} \overset{\mathcal{B}}{\mapsto} m^{(2)}$ and $\forall t(t \in \mathcal{B} \longrightarrow d(t) > 0)$. Then, it follows from Definition 2.7 that

$$m^{(2)}_{.1} + \sum_{t \in \mathcal{B}} t^- \le m^{(1)}_{.1}. \tag{2}$$

For the reader's comfort, we will mark the results state changes (Definition 2.7) of kind (a) and kind (b) in different ways, where they need to be distinguished. Variables that are connected with a state change of the type (a) will be denoted with a hat (e.g., $\hat{m}$), and variables that are a result of a state change (b) will be denoted with a tilde (e.g., $\tilde{m}$).

Example 2.4. Consider Z_1 again. $\mathcal{B} := \{t_1, t_2\}$ is a maximal step on $m^{(0)}$. Let $m^{(1)}$ be the time marking which the net Z_1 reaches after firing $\mathcal{B}$ at $m^{(0)}$, i.e. $m^{(0)} \overset{\mathcal{B}}{\longrightarrow} \hat{m}^{(1)}$. It is easy to see that

$$\hat{m}^{(1)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 2 \\ 0 & 1 & 0 \end{pmatrix} \text{ and } \hat{h}^{(1)} = \begin{pmatrix} 2 \\ 1 \\ 0 \\ 0 \end{pmatrix}.$$

Now, starting from $\hat{m}^{(1)}$ the net obtains the time marking $\tilde{m}^{(1)}$ after elapsing 1 time unit $\tilde{m}^{(1)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 1 & 0 & 0 \end{pmatrix}$ and the transition clock vector $\tilde{h}^{(1)} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$.

Please note, that the alternating sequence of state changing always starts from initital state (initial time marking, initial transition clock vector) with firing, but may end with one of both, time elapsing or firing.

In order to introduce a state equation for DPN we define an incidence matrix for DPN and a Parikh matrix for DPN.

Definition 2.8. (incidence matrix)
Let $Z = (N, D)$ be a DPN. The matrix $C \in \mathcal{M}(|P|, d \cdot |T|)$ is called the incidence matrix of Z, iff $C := (C^{(1)}, C^{(2)}, \ldots, C^{(|T|)})$ with $C^{(k)} \in \mathcal{M}(|P|, d)$ and $k \in \{1, \ldots, |T|\}$ and

$$c_{i,j}^{(k)} := \begin{cases} V(t_k, P_i) - V(P_i, t_k) & , d_k = 0, j = 1 \\ -V(P_i, t_k) & , d_k > 0, j = 1 \\ V(t_k, P_i) & , (d_k > 0), 0 < j - 1 = d_k \\ 0 & , else \end{cases}$$

The matrix C consists of submatrices $C^{(k)}$ representing the transitions t_k of the net. $c_{i,1}^{(k)}$ is the number of tokens that will be changed at place P_i immediately, if the transition t_k is an element of the firing bag. $c_{i,j}^{(k)}$ shows a number of tokens that will arrive at place P_i after $(j-1)$ time units, if the transition t_k is an element of the firing bag.

Example 2.5. In the Example 2.1 the incidence matrix of Z_1 is

$$C = \begin{pmatrix} -2 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \end{pmatrix}$$

Now we define a matrix $B \in \mathcal{M}(d \cdot |T|, d)$ in order to define a Parikh-matrix later.

Definition 2.9. (bag matrix)
The matrix $B \in \mathcal{M}(d \cdot |T|, d)$ is called the bag matrix of $\mathcal{B} = \{t_{i_1}, \ldots, t_{i_\kappa}\}$, $t_{i_j} \in T$ iff

$$B = \begin{pmatrix} B^{(1)} \\ B^{(2)} \\ \vdots \\ B^{(|T|)} \end{pmatrix} \text{ and } B^{(s)} := \begin{cases} E_d & \text{if } s \in \{i_1, \ldots, i_\kappa\} \\ \mathcal{O}_d & \text{else (i.e., } s \in \{1, \ldots, |T|\} \backslash \{i_1, \ldots, i_\kappa\}) \end{cases}$$

Example 2.6. Let us consider the step $\mathcal{B} = \{t_1, t_2\}$ in Z_1:

$$B = \begin{pmatrix} B^{(1)} \\ B^{(2)} \\ B^{(3)} \\ B^{(4)} \end{pmatrix} \text{ with } B^{(1)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad B^{(2)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$B^{(3)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad B^{(4)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\text{i.e.: } B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

Next, we define a progress matrix R in order to compute the progress of time.

Definition 2.10. (progress matrix)
Let $Z = (N, D)$ be a DPN. The matrix $R \in \mathcal{M}(d, d)$ is called the progress matrix of Z, iff

$$r_{i,j} := \begin{cases} 1 & \text{if } (i = j = 1) \text{ or } (i = j + 1) \\ 0 & \text{else} \end{cases}$$

Example 2.7. In Example 2.1 the progress matrix R of the net Z_1 is $\begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$.

Definition 2.11. (reachability)
Let $Z = (N, D)$ be a DPN. A time marking m' is reachable in Z iff there exists a sequence $\sigma = (\mathcal{B}_1, \ldots, \mathcal{B}_\alpha)$ of maximal steps such that

$$m^{(0)} \xrightarrow{\mathcal{B}_1} \hat{m}^{(1)} \underset{1}{\longrightarrow} \tilde{m}^{(1)} \xrightarrow{\mathcal{B}_2} \hat{m}^{(2)} \underset{1}{\longrightarrow} \tilde{m}^{(2)} \ldots \longrightarrow m'$$

3. A State Equation

3.1. Elements of the State Equation

We want to derive a state equation for DPN in analogy to the state equation of timeless nets.

Lemma 3.1. Let $Z = (N, D)$ be a DPN, $m^{(1)}$ a reachable time marking in Z and $m^{(1)} \underset{1}{\longrightarrow} m^{(2)}$. Then it holds $m^{(2)} = m^{(1)} \cdot R$.

Proof:
$m^{(2)}_{i,j} = m^{(1)}_{i,j} \cdot R$

$$\textit{Case 1:} \quad j = 1 \quad \rightsquigarrow \quad \sum_{s=1}^{d} m^{(1)}_{i,s} \cdot r_{s,1} = m^{(1)}_{i,1} \cdot 1 + m^{(1)}_{i,2} \cdot 1 \underset{acc.def}{=} m^{(2)}_{i,1}$$

$$\textit{Case 2:} \quad d > j \geq 2 \quad \rightsquigarrow \quad \sum_{s=1}^{d} m^{(1)}_{i,s} \cdot r_{s,j} = m^{(1)}_{i,j+1} \cdot 1 \underset{acc.def}{=} m^{(2)}_{i,j}$$

$$\textit{Case 3:} \quad j = d \quad \rightsquigarrow \quad \sum_{s=1}^{d} m^{(1)}_{i,s} \cdot r_{s,j} = 0 = m^{(2)}_{i,j}$$

□

Example 3.1. We consider the Timed Petrinet Z_1 and time marking $\hat{m}^{(1)}$ and $\tilde{m}^{(1)}$ from Example 2.4.

It is easy to see that $\tilde{m}^{(1)} = m^{(1)} \cdot R$, i.e., $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 2 \\ 0 & 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$.

Lemma 3.2. Let $Z = (N, D)$ be a DPN and $m^{(1)} \xrightarrow{\mathcal{B}} m^{(2)}$, $\mathcal{B} = \{t_{i_1}, \ldots, t_{i_\kappa}\}$. Then holds: $m^{(2)} = m^{(1)} + C \cdot B$.

Proof:
We consider $m^{(2)}_{i,j}$.

Case 1: $j = 1$

According to Definition 2.7 the following is true:

$$m^{(2)}_{i,1} = m^{(1)}_{i,j} - \sum_{t_s \in \mathcal{B}} V(P_{i,j}, t_s) + \sum_{\substack{t_s \in \mathcal{B} \\ d_s = j-1=0}} V(t_s, P_i) \ .$$

Therefore, it is sufficient to show that

$$\sum_{j=1}^{d \cdot |T|} c_{i,j} b_{j,1} = - \sum_{t_s \in \mathcal{B}} V(P_i, t_s) + \sum_{\substack{t_s \in \mathcal{B}, \\ d_s = 0}} V(t_s, P_i) \ .$$

Now we consider the bag matrix B. The first column $B_{\cdot 1}$ of B is $B_{\cdot 1} = (b^{(1)}, b^{(2)}, \ldots, b^{(|T|)})^T$, where $b^{(k)}$ is a d-dimensional vector with

$$b^{(k)} = (b^{(k)}_1, \ldots, b^{(k)}_d) \quad k = 1, \ldots, |T| \text{ and } b^{(k)}_j = \begin{cases} 1 & , j = 1, k \in \{i_1, \ldots, i_\kappa\} \\ 0 & , else \end{cases}$$

That means, $b^{(k)}$ is the d-dimensional vector $(1, 0, \ldots, 0)^T$ if $t_k \in \mathcal{B}$ and the d-dimensional zero-vector else.

Hence,

$$\sum_{j=1}^{d \cdot |T|} c_{i,j} \cdot b_{j,1} = \sum_{t_k \in \mathcal{B}} c^{(k)}_{i,j} \cdot b^{(k)}_j = \sum_{t_k \in \mathcal{B}} c^{(k)}_{i,1} \underset{acc.def}{=} - \sum_{t_k \in \mathcal{B}} V(P_i, t_k) + \sum_{\substack{t_k \in \mathcal{B}, \\ d_k = 0}} V(t_k, P_i)$$

Case 2: $j \geq 2$

Obviously, now we have to show that $\sum_{s=1}^{d\cdot|T|} c_{i,s} \cdot b_{s,j} = \sum_{\substack{t_k \in \mathcal{B}, \\ d_k = j-1}} V(t_s, P_i)$. We consider the j-th column of the bag matrix B, the vector $B_{\cdot j}$:

$$B_{\cdot j} = (b^{(1)}, b^{(2)}, \ldots, b^{(|T|)}) \text{ with } b^{(k)} = (b_1^{(k)}, \ldots, b_d^{(k)})^T \quad \overset{|T|}{\underset{1}{\forall}} k \text{ and } b_s^{(k)} = \begin{cases} 1 & , s = k \\ 0 & , else \end{cases}$$

i.e. $b^{(k)} = \underbrace{(0, \ldots, 0)}_{d}$ if $t_k \notin \mathcal{B}$ and $b^{(k)} = \underbrace{(0, \ldots, \underset{\uparrow k.}{1}, 0, \ldots, 0)^T}_{d}$, if $t_k \in \mathcal{B}$.

Now we can compute

$$\sum_{s=1}^{d\cdot|T|} c_{i,s} \cdot b_{s,j} = \sum_{\substack{t_k \in \mathcal{B}, \\ j \geq 2}} c_{i,j}^{(k)} \underset{acc.def}{=} \sum_{\substack{t_k \in \mathcal{B}, \\ d_k = j-1}} V(t_k, P_i)$$

□

Example 3.2. Considering Z_1, $m^{(0)}$, $m^{(1)}$ and $\mathcal{B}$ from Example 2.3 again, we can verify that

$$\underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 2 \\ 0 & 1 & 0 \end{pmatrix}}_{=m^{(1)}} = \underbrace{\begin{pmatrix} 4 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}}_{m^0} + \underbrace{\begin{pmatrix} -2 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \end{pmatrix}}_{=C} \cdot \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}}_{=B}$$

Definition 3.1. (Parikh-matrix)
The matrix $\Psi \in \mathcal{M}(|P|, d)$ is called Parikh-matrix of the sequence $\sigma = (\mathcal{B}_1, \ldots, \mathcal{B}_n)$, iff

$$\Psi_\sigma := \sum_{i=1}^{n} B^{(i)} \cdot R^{n-i}$$

Theorem 3.1. (state equation)
Let $Z = (N, D)$ be a DPN, all $d_i > 0$, $\sigma = (\mathcal{B}_1, \ldots, \mathcal{B}_n)$ a firing sequence in Z, $m^{(0)}$ the initial time marking of Z and $m^{(0)} \xrightarrow{\mathcal{B}_1} \hat{m}^{(1)} \underset{1}{\longrightarrow} \tilde{m}^{(1)} \xrightarrow{\mathcal{B}_2} \hat{m}^{(2)} \underset{1}{\longrightarrow} \ldots \xrightarrow{\mathcal{B}_n} m^{(n)}$. Then it holds:

$$m^{(n)} = m^{(0)} \cdot R^{n-1} + C \cdot \Psi_\sigma \tag{3}$$

Proof:
Proof by induction on n.

Basis: $n = 1$.

We have to show that for $m^{(0)} \xrightarrow{\mathcal{B}_1} m^{(1)}$ holds:

$$m^{(1)} = m^{(0)} \cdot R^0 + C \cdot \Psi_\sigma = m^{(0)} + C \cdot \Psi_\sigma \text{ with } \Psi_\sigma = \sum_{i=1}^{1} B^{(i)} \cdot R^{n-i} = B^{(1)} \cdot R^0 = B^{(1)}$$

Following, we have to proof that $m^{(1)} = m^{(0)} + C \cdot B^{(1)}$. That is true because of Lemma 3.2.

Step: We consider the firing sequence $\sigma' = (\sigma, \mathcal{B}_{n+1})$ with $m^{(0)} \xrightarrow{\sigma} \hat{m}^{(n)} \underset{1}{\longrightarrow} \tilde{m}^{(n)} \xrightarrow{\mathcal{B}_{n+1}} \hat{m}^{(n+1)}$. We have to show that

$$\hat{m}^{(n+1)} = m^{(0)} \cdot R^n + C \cdot \Psi_{\sigma'} \text{ and } \Psi_{\sigma'} = \sum_{i=1}^{n+1} B^{(i)} \cdot R^{n+1-i}$$

Because of the induction hypothesis it holds:

$$m^{(n)} \quad = \quad m^{(0)} \cdot R^{n-1} + C \cdot \Psi_\sigma \quad = \quad m^{(0)} \cdot R^{n-1} + C \cdot \left(\sum_{i=1}^{n} B^{(i)} \cdot R^{n-1} \right) \tag{4}$$

Because of Lemma 3.1 and resp. Lemma 3.2 it holds too:

$$\tilde{m}^{(n)} = \hat{m}^{(n)} \cdot R \tag{5}$$

resp.

$$\hat{m}^{(n+1)} = \tilde{m}^{(n)} + C \cdot B^{(n+1)} \tag{6}$$

From (4), (5) and (6) it follows:

$$\begin{aligned}
\hat{m}^{(n+1)} &= \tilde{m}^{(n)} + C \cdot B^{(n+1)} \underset{(5)}{=} \hat{m}^{(n)} \cdot R + C \cdot B^{(n+1)} \\
&\underset{(4)}{=} \left(m^{(0)} \cdot R^{n-1} + C \cdot \left(\sum_{i=1}^{n} B^{(i)} R^{n-1} \right) \right) \cdot R + C \cdot B^{(n+1)} \\
&= m^{(0)} \cdot R^n + C \cdot \left(\sum_{i=1}^{n} B^{(i)} R^{n+1-i} \right) + C \cdot B^{(n+1)} \cdot \underbrace{E_d}_{=R^0} \\
&= m^{(0)} \cdot R^n + C \cdot \left(\sum_{i=1}^{n} B^{(i)} R^{n+1-i} + B^{(n+1)} \cdot R^{(n+1)-(n-1)} \right) \\
&= m^{(0)} \cdot R^n + C \cdot \left(\sum_{i=1}^{n+1} B^{(i)} R^{n+1-i} \right) \\
&= m^{(0)} R^n + C \cdot \Psi_{\sigma'}
\end{aligned}$$

□

3.2. Discussion

We call Equation 3 a *state equation* in analogy to timeless Petrinets. However, in difference to timeless Petrinet, the time marking does not constitute the whole state of the DPN. To get the full state, h would have to be considered.

Please note the consistency of our state equation with the state equation of timeless Petrinets. Consider a net without delays, i.e., $d = 1$. Applied to Equation 3 we gain the following implications:

- All $m^{(i)}$ are vectors.
- The incidence matrix C is the same as in the timeless equation.
- R is a scalar with the value 1.

- The Parikh matrix Ψ is reduced to the legacy Parikh vector.

Now, our state equation has the same format as the equation for a timeless nets: $m^{(n)} = m^{(0)} + C \cdot \Psi$. Thus, the state equation for a timeless Petrinets may be considered as a special case of Equation 3. Please note, that this does not impliy that timeless Petrinets may be considered as special cases of Timed Petrinets, since the latter have the additional maximal step semantics. This fact is reflected by the maximal step inequalities given by (1) and (2), respectively.

For all considerations till now, we have assumed nets without zero-delays (cp. Section 2.2). However, if there exists at least one duration $d_i = 0$, our state equation can be adapted. In this case, we have to consider that we lose the strict alternation between maximal step and time elapsing. Now, firing may comprise a sequence of maximal steps before a time elapse becomes possible.

4. Non-reachability of Time Markings — An Example

We proved already that in a given Timed Petrinet each reachable time marking fulfills the state equation and the maximal step inequalities, i.e. they are necessary conditions for reachability of the time marking. As expected, the condition is not sufficient.

In this section we consider an example using the necessity of completion the conditions mentioned above in order to prove non-reachability of a certain time marking in a given net.

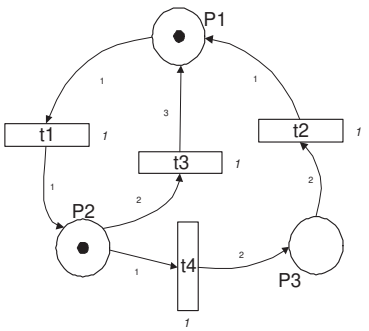

Figure 2. Example net Z_2

Consider Z_2 as in Figure 4. The time marking $m^{(0)}$ is the matrix $m^{(0)} = \begin{pmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 0 \end{pmatrix}$, the incidence matrix is $C = \begin{pmatrix} -1 & 0 & 0 & 1 & 0 & 3 & 0 & 0 \\ 0 & 1 & 0 & 0 & -2 & 0 & -1 & 0 \\ 0 & 0 & -2 & 0 & 0 & 0 & 0 & 2 \end{pmatrix}$ and $R = \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}$ is the progress matrix.

Obviously it is true that $R^n = R$ for each $n \geq 1$ ($R^0 = E_2$).

We want to show that the time marking $m^* = \begin{pmatrix} 0 & p^* \\ 2 & q^* \\ 0 & r^* \end{pmatrix}$ is unreachable in Z_2 for each $p^*, q^*, r^* \in \mathbb{R}$. Please note, that this is not true for the timeless net. In the skeleton of Z_2, a firing of $\mathcal{B}_0 = \{t_1\}$ leads immediately to m^*.

Proof:
Assume that m^* is reachable in Z_2, so it has to fulfill the state equation:

$$m^* = m^{(0)} \cdot R^{n-1} + C \cdot \Psi_\sigma$$

for some firing sequence σ and the maximal step inequalities:

$$m_{.1} \not\geq \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad (7a) \qquad m_{.1} \not\geq \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix} \quad (7b) \qquad m_{.1} \not\geq \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix} \quad (7c) \qquad m_{.1} \not\geq \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad (7d)$$

for each reachable time marking, obtained after firing a maximal step, unless the related transition has fired within the last step.

We do not need to consider the whole sequence σ, but the last marking change. There are two cases: Changing by firing a maximal step and changing by elapsing time. In both cases we consider the last two reached time markings.

Case 1: Last change by time elapsing,

i.e.: $m^{(0)} \xrightarrow{\mathcal{B}_1} \ldots \xrightarrow{\mathcal{B}_n} \hat{m} \underset{1}{\longrightarrow} m^*$. Then it holds for $\hat{m}$:

$$\begin{aligned}
\hat{m} &= m^{(0)} \cdot R^{n-1} + C \cdot \left(\sum_{i=1}^{n} B^{(i)} R^{n-i} \right) \\
&= m^{(0)} \cdot \underbrace{R^{n-1}}_{=R} + C \cdot \sum_{i=1}^{n-1} B^{(i)} \cdot \underbrace{R^{n-i}}_{=R} + C \cdot B^{(n)} \cdot \underbrace{R^0}_{=E_2} \\
&= \begin{pmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix} + C \cdot \sum_{i=1}^{n-1} \left(\begin{pmatrix} \begin{pmatrix} \alpha_1^i & 0 \\ 0 & \alpha_1^i \end{pmatrix} \\ \begin{pmatrix} \alpha_2^i & 0 \\ 0 & \alpha_2^i \end{pmatrix} \\ \begin{pmatrix} \alpha_3^i & 0 \\ 0 & \alpha_3^i \end{pmatrix} \\ \begin{pmatrix} \alpha_4^i & 0 \\ 0 & \alpha_4^i \end{pmatrix} \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix} \right) + C \cdot \begin{pmatrix} \begin{pmatrix} \kappa & 0 \\ 0 & \kappa \end{pmatrix} \\ \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} \\ \begin{pmatrix} \mu & 0 \\ 0 & \mu \end{pmatrix} \\ \begin{pmatrix} \nu & 0 \\ 0 & \nu \end{pmatrix} \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}
\end{aligned}$$

where

$$\alpha_j^i = \begin{cases} 1 & , t_j \in \mathcal{B}_i, i = 1, \ldots, n-1 \\ 0 & , else \end{cases}$$

and $\kappa = \begin{cases} 1 & , t_1 \in \mathcal{B}_n \\ 0 & , else \end{cases}$, $\lambda = \begin{cases} 1 & , t_2 \in \mathcal{B}_n \\ 0 & , else \end{cases}$, $\mu = \begin{cases} 1 & , t_3 \in \mathcal{B}_n \\ 0 & , else \end{cases}$, $\nu = \begin{cases} 1 & , t_4 \in \mathcal{B}_n \\ 0 & , else \end{cases}$

$$\hat{m} = \begin{pmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 0 \end{pmatrix} + C \cdot \begin{pmatrix} x_1 & 0 \\ x_1 & 0 \\ x_2 & 0 \\ x_2 & 0 \\ x_3 & 0 \\ x_3 & 0 \\ x_4 & 0 \\ x_4 & 0 \end{pmatrix} + C \cdot \begin{pmatrix} \kappa & 0 \\ 0 & \kappa \\ \lambda & 0 \\ 0 & \lambda \\ \mu & 0 \\ 0 & \mu \\ \nu & 0 \\ 0 & \nu \end{pmatrix}$$

x_i is the number of occurrences of the transition t_i in the sequence σ at large

$$\hat{m} \quad = \quad \begin{pmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} -x_1 + x_2 + 3x_3 & 0 \\ x_1 - 2x_3 - x_4 & 0 \\ -2x_2 + 2x_4 & 0 \end{pmatrix} + \begin{pmatrix} -\kappa & \lambda + 3\mu \\ -2\mu - \nu & \kappa \\ -2\lambda & 2\nu \end{pmatrix} \quad .$$

Hence, we obtain for $\hat{m} = \begin{pmatrix} \hat{\alpha} & \hat{p} \\ \hat{\beta} & \hat{q} \\ \hat{\gamma} & \hat{r} \end{pmatrix}$ for arbitrary $\hat{\alpha}$, $\hat{\beta}$, $\hat{\gamma}$, $\hat{p}$,$\hat{q}$,$\hat{r} \in \mathbb{N}$ that

$$\begin{aligned} &\hat{\alpha} = 1 - x_1 + x_2 + 3x_3 - \kappa && \hat{\beta} = 1 + x_1 - 2x_3 - x_4 - 2\mu - \nu \\ &\hat{\gamma} = -2x_2 + 2x_4 - 2\lambda && \hat{p} = \lambda + 3\mu \\ &\hat{q} = \kappa && \hat{r} = 2\nu \end{aligned} \tag{8}$$

Furthermore, because of $\hat{m} \underset{1}{\longrightarrow} m^*$ it holds $m^* = \hat{m} \cdot R$, i.e.:

$$\begin{pmatrix} 0 & p^* \\ 2 & q^* \\ 0 & r^* \end{pmatrix} = \begin{pmatrix} \hat{\alpha} + \hat{p} & 0 \\ \hat{\beta} + \hat{q} & 0 \\ \hat{\gamma} + \hat{r} & 0 \end{pmatrix} \quad .$$

Following, it is true: $\hat{\alpha} + \hat{p} = 0$, $\hat{\beta} + \hat{q} = 2$, $\hat{\gamma} + \hat{r} = 0$ and therefore $\hat{\alpha} = \hat{p} = \hat{\gamma} = \hat{r} = 0$

Last result, put in (8), leads to $\lambda = \mu = \nu = 0$ and $\kappa = \hat{q} = 1$ or $\kappa = \hat{q} = 0$.

That means:

$$\hat{\beta} = 2 \text{ or } \hat{\beta} = 1 \tag{9}$$

At least, $\tilde{m}$ has to fulfill (7a), (7b), (7c) and (7d), i.e.

$$\begin{pmatrix} 0 \\ \hat{\beta} \\ 0 \end{pmatrix} \not\geq \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} 0 \\ \hat{\beta} \\ 0 \end{pmatrix} \not\geq \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix}, \begin{pmatrix} 0 \\ \hat{\beta} \\ 0 \end{pmatrix} \not\geq \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} 0 \\ \hat{\beta} \\ 0 \end{pmatrix} \not\geq \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

except those conditions which belong to the transitions including in the step $\mathcal{B}_n$. In case that $\hat{\beta} = 2$ conditions (7c) and (7d) are not fulfilled. Assuming $\mathcal{B}_n$ includes t_3 and t_4 the conditions (7c) and

(7d) have not to be true. But then it follows that $\mu = \nu = 1$ and this is a contradiction to $\mu = \nu = 0$, computed above. Analogously, in the case that $\hat{\beta} = 1$ the condition (7d) is not fulfilled.

Assuming $\mathcal{B}_n$ includes t_4 it follows that $\nu = 1$ and this is a contradiction to $\nu = 0$, as already mentioned.

Therefore, Case 1 can not appear.

Case 2: Last change by firing a maximal step $\mathcal{B}_n$, i.e. $m^{(0)} \xrightarrow[n-2]{\mathcal{B}_1 \ldots \mathcal{B}_{n-1}} \hat{m} \xrightarrow[1]{} \tilde{m} \xrightarrow{\mathcal{B}_n} m^*$.

Analogously to Case 1, we obtain for the time marking $\hat{m} = \begin{pmatrix} \hat{\alpha} & \hat{p} \\ \hat{\beta} & \hat{q} \\ \hat{\gamma} & \hat{r} \end{pmatrix}$ the following:

$$\begin{pmatrix} \hat{\alpha} & \hat{p} \\ \hat{\beta} & \hat{q} \\ \hat{\gamma} & \hat{r} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} -(x_1 - \hat{\kappa}) + (x_2 - \hat{\lambda}) + 3(x_3 - \hat{\mu}) & 0 \\ (x_1 - \hat{\kappa}) - 2(x_3 - \hat{\mu}) - (x_4 - \hat{\nu}) & 0 \\ -2(x_2 - \hat{\lambda}) + 2(x_4 - \hat{\nu}) & 0 \end{pmatrix} + \begin{pmatrix} -\hat{\kappa} & \hat{\lambda} + 3\hat{\mu} \\ -2\hat{\mu} - \hat{\nu} & \hat{\kappa} \\ -2\hat{\lambda} & 2\hat{\nu} \end{pmatrix} \tag{10}$$

where $\hat{\kappa}, \hat{\lambda}, \hat{\mu}, \hat{\nu} \in \{0, 1\}$ are defined similarly for the step $\mathcal{B}_{n-1}$ as $\kappa, \lambda, \mu, \nu$ in Case 1; x_1, x_2, x_3, x_4 are the same as in Case 1 and therefore $(x_1 - \hat{\kappa})$, resp. $(x_2 - \hat{\lambda})$, resp. $(x_3 - \hat{\mu})$, resp. $(x_4 - \hat{\nu})$ is the number of occurrences of the transition t_1, resp. t_2, resp. t_3, resp. t_4 in the sequence $(\mathcal{B}_1 \ldots \mathcal{B}_{n-2})$.

For the time marking $\tilde{m} = \begin{pmatrix} \tilde{\alpha} & \tilde{p} \\ \tilde{\beta} & \tilde{q} \\ \tilde{\gamma} & \tilde{r} \end{pmatrix}$ holds $\begin{pmatrix} \tilde{\alpha} & \tilde{p} \\ \tilde{\beta} & \tilde{q} \\ \tilde{\gamma} & \tilde{r} \end{pmatrix} = \begin{pmatrix} \hat{\alpha} + \hat{p} & 0 \\ \hat{\beta} + \hat{q} & 0 \\ \hat{\gamma} + \hat{r} & 0 \end{pmatrix}$.

Because $m^*_{.1} \not\geq t_3^-$ and $m^*_1 \not\geq t_4^-$ do not hold and because we consider nets without self-concurrency of transitions, the last step $\mathcal{B}_n$ includes at least the transitions t_3 and t_4, i.e. $\mathcal{B}_n \supseteq \{t_3, t_4\}$, and it holds (corr. remark 2.2 and $d(t) > 0$ for each $t \in T$) $m^*_{.1} + t_3^- + t_4^- \leq \tilde{m}_{.1}$, i.e.

$$\tilde{m}_{.1} \geq \begin{pmatrix} 0 \\ 5 \\ 0 \end{pmatrix} \tag{11}$$

have to be fulfilled.

From (10) and (11) it follows:

$$\begin{aligned} 5 \leq & \hat{\beta} + \hat{q} \\ = & (x_1 - \hat{\kappa}) - 2(x_3 - \hat{\mu}) - (x_4 - \hat{\nu}) - 2\hat{\mu} - \hat{\nu} \\ = & x_1 - 2x_3 - x_4 - \hat{\kappa} \end{aligned} \tag{12}$$

It is easy to see that for the time marking m^* the state equation leads to

$$\begin{pmatrix} 0 & p^* \\ 2 & q^* \\ 0 & r^* \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} -x_1 + x_2 + 3x_3 & 0 \\ x_1 - 2x_3 - x_4 & 0 \\ -2x_2 + 2x_4 & 0 \end{pmatrix} + \begin{pmatrix} -\kappa & \lambda + 3\mu \\ -2\mu - \nu & \kappa \\ -2\lambda & 2\nu \end{pmatrix} .$$

In particular, we obtain for

$$m^*_{2,1} = 2 = 1 + x_1 - 2x_3 - x_4 - 2\mu - \nu$$

$$\rightsquigarrow 1 = x_1 - 2x_3 - x_4 - 2\mu - \nu \ . \tag{13}$$

Furthermore, it is clear that $\mathcal{B} \supseteq \{t_3, t_4\}$ and hence $\mu = \nu = 1$. Thus, it follows from (13):

$$1 = x_1 - 2x_3 - x_4 - 3$$

$$\rightsquigarrow 4 = x_1 - 2x_3 - x_4 \tag{14}$$

Putting (14) in (12) leads to $5 \leq 4 - \hat{\kappa}$, i.e. $1 \leq -\hat{\kappa}$, and this is a contradiction to $\hat{\kappa} \in \{0, 1\}$.

Thus, Case 2 is not possible too and following the time marking m^* can not be reached in Z_2 by initial time marking $m^{(0)}$. □

Remark 4.1. In some models, self-concurency is allowed for TPN. In this case, our proof would be even more simple, since Case 2 is obviously trival: In nets with self-concurency, all maximum step inequalities (7a) - (7d) must hold in any case. However, (7c) and (7d) are already invalid by poof assumption.

5. Conclusions

In this paper, we have developed a state equation for DPN and have presented a proving approach that utilizes the state equation.

Obviously, the state equation is valid for each arbitary but finite sequence.

The presented proving technique is generally applicable. It leads to a system of equations and inequalities, and thus, it is always solvable in polynomial time. A contradiction in the equation and inequalities system proves non-reachability. Please note however, that we do not make any statement about the reverse case: The existence of a solution does not prove reachability.

In future work, we intend to extend the approach presented here to π-nets [4], allowing description of priorities within Time Petrinets.

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