1. Aufg: $y'' - \frac{2}{x+1} y' = 7$

oder $(x+1)\, y'' - 2y' = 7(x+1)$

(a) DGL

(b) AWP: $y'(0) = 2$, $y(0) = \frac{1}{2}$

Lsg: (a) Sei $z := y'$ $\Rightarrow$

$z' = y''$

$$\underbrace{z' - \frac{2}{x+1}}_{a(x)} z = \underbrace{7}_{f(x)} \quad \Rightarrow \quad A(x) = -2\int \frac{1}{x+1}\,dx = \ln (x+1)^{-2}$$

$$\Rightarrow \quad e^{A(x)} = \frac{1}{(x+1)^2} \qquad e^{-A(x)} = (x+1)^2$$

$$\Rightarrow y'(x) = z(x) = \left(\int 7\cdot(x+1)^{-2}\,dx + c\right)(x+1)^2$$

$$= \left(-7\cdot(x+1)^{-1} + c\right)\cdot(x+1)^2 \quad \text{- allg. Lsg f. } y'$$

$$= -7\cdot(x+1) + c\,(x+1)^2$$

$$\Rightarrow y(x) = \int\left(-7(x+1) + c(x+1)^2\right)dx$$

$$= -\frac{7}{2}(x+1)^2 + \frac{c}{3}(x+1)^3 + D \quad \text{allg. Lsg.}$$

(b) $2 = y'(0) = -7 + c \Leftrightarrow c = 9$

$$\frac{1}{2} = y(0) = -\frac{7}{2} + 3 + D = -\frac{1}{2} + D \Leftrightarrow D = 1$$

$$\Rightarrow y(x) = -\frac{7}{2}(x+1)^2 + 3(x+1)^3 + 1 \quad \text{part. Lsg für (b)}$$

2. Aufg: $\frac{1}{2}y' + (x^3+\frac{1}{2})y = \frac{1}{2}(e^{-\frac{x^4}{2}} + \frac{1}{2}x^3e^{-x})$

(a) DGL lösen

(b) AWP = DGL (a) + $y(0) = 3$

Lsg: (a) $y' + \underbrace{(2x^3+1)}_{a(x)}y = \underbrace{e^{-\frac{x^4}{2}} + \frac{1}{2}x^3e^{-x}}_{f(x)}$

$$\Rightarrow A(x) = \int(2x^3+1)\,dx = \frac{x^4}{2} + x$$

$$\Rightarrow y(x) = \left(\int\left(e^{-\frac{x^4}{2}} + \frac{1}{2}x^3e^{-x}\right)e^{\frac{x^4}{2}+x}\,dx + c\right)e^{-\frac{x^4}{2}-x}$$

$$= \left(\int\left(e^{x} + \frac{1}{2}x^3e^{\frac{x^4}{2}}\right)dx + c\right)e^{-\frac{x^4}{2}-x}$$

$$= \left(e^{x} + \int\frac{1}{2}x^3e^{\frac{x^4}{2}}\,dx + c\right)e^{-\frac{x^4}{2}-x}$$

oder $u := \frac{x^4}{2} \Rightarrow u' = 2x^3$

$$= \left(e^{x} + \frac{1}{4}\int e^{\frac{x^4}{2}}\,d\,\frac{x^4}{2} + c\right)e^{-\frac{x^4}{2}-x}$$

$$= \left(e^{x} + \frac{1}{4}e^{\frac{x^4}{2}} + c\right)e^{-\frac{x^4}{2}-x} = e^{-\frac{x^4}{2}} + \frac{1}{4}e^{-x} + c\cdot e^{-\frac{x^4}{2}-x}$$

allg. Lsg

(b) $3 = y(0) = e^0 + \frac{1}{4}e^0 + c\cdot e^0 \Leftrightarrow 3 = \frac{5}{4} + c$

$\Leftrightarrow c = \frac{7}{4}$

$$\Rightarrow y(x) = e^{-\frac{x^4}{2}} + \frac{1}{4}e^{-x} + \frac{7}{4}e^{-\frac{x^4}{2}-x}$$

part. Lsg von (b)

3. Aufg.:

$$(1-x^2)y'' = 4x - 2xy' \quad , x \in \mathbb{R}$$

(a) DGL (b) AWP $y'(1)=4$, $y(1)=1/3$ $-1 < x < 1$

Lsg.: (a) $z := y'$

$$\Rightarrow z' = \frac{4x}{1-x^2} - \frac{2x}{1-x^2} z \Leftrightarrow z' + \underbrace{\frac{2x}{1-x^2}}_{a(x)} z = \underbrace{\frac{4x}{1-x^2}}_{f(x)}$$

$$A(x) = \int \frac{2x}{1-x^2} dx = -\int \frac{1}{1-x^2} d(1-x^2)$$

$$= -\ln(1-x^2) = \ln(1-x^2)^{-1}$$

$$\Rightarrow e^{A(x)} = \frac{1}{1-x^2} \quad e^{-A(x)} = (1-x^2)$$

$$\Rightarrow y'(x) = z(x) = \left(\int \frac{4x}{1-x^2} \cdot \frac{1}{1-x^2} dx + c\right)(1-x^2)$$

Subst.: u:=1-x^2
u'= -2•x

$$= \left(-\int \frac{2d(1-x^2)}{(1-x^2)^2} + c\right)(1-x^2)$$

$$= \left(- 2 \cdot (-1)(1-x^2)^{-1} + c\right)(1+x^2)$$

$$= \left(\frac{2}{1-x^2} + c\right)(1+x^2) = 2 + c \cdot (1+x^2)$$

$$\Rightarrow y(x) = \int (2 + c(1+x^2)) dx =$$

$$= 2x + c\left(x + \frac{x^3}{3}\right) + D \quad \text{allg Lsg}$$

(b) $4 = y'(1) = 2 + c \cdot 2 \Leftrightarrow 2 = 2c \Leftrightarrow c = 1$

$$\frac{1}{3} = y(1) = 2 + \left(1 + \frac{1}{3}\right) + D = 3 + \frac{1}{3} + D \Leftrightarrow D = -3$$

$$\Rightarrow y(x) = 3x + \frac{x^3}{3} - 3 \quad \text{part. Lsg für (b)}$$

4. Aufgabe:

$$\sqrt{2+x^4}\cdot y' + \frac{x^3}{\sqrt{2+x^4}}\cdot y = x^3\sqrt{2+x^4} \qquad y(1)=\frac{3}{5}$$

Lsg:

$$\Rightarrow y' + \underbrace{\frac{x^3}{2+x^4}}_{a(x)}\cdot y = \underbrace{x^3}_{f(x)}$$

$$\Rightarrow A(x) = \int\frac{x^3}{2+x^4}\,dx = \frac{1}{4}\int\frac{d(x^4+2)}{2+x^4} = \frac{1}{4}\ln(x^4+2) = \ln(x^4+2)^{\frac{1}{4}}$$

$$\Rightarrow y(x) = \left(\int x^3\cdot(x^4+2)^{\frac{1}{4}}\,dx + C\right)\cdot(x^4+2)^{-\frac{1}{4}}$$

$$e^{A(x)} = e^{\ln(x^4+2)^{\frac{1}{4}}} = (x^4+2)^{\frac{1}{4}}$$

$$e^{-A(x)} = e^{-\ln(\dots)^{\frac{1}{4}}} = e^{\ln(\dots)^{-\frac{1}{4}}} = (x^4+2)^{-\frac{1}{4}}$$

$$= \left(\frac{1}{4}\int(x^4+2)^{\frac{1}{4}}\,d(x^4+2) + C\right)\cdot(x^4+2)^{-\frac{1}{4}}$$

$$= \left(\frac{1}{\cancel{4}}(x^4+2)^{\frac{5}{4}}\cdot\frac{\cancel{4}}{5} + C\right)(x^4+2)^{-\frac{1}{4}} = \underbrace{\frac{1}{5}(2+x^4) + C\cdot(2+x^4)^{\frac{1}{4}}}_{\text{allg.}}$$

AWP:

$$\frac{3}{5} = y(1) = \frac{1}{5}(2+1) + C\cdot(2+1)^{\frac{1}{4}} = \frac{3}{5} + C\cdot\sqrt[4]{3}$$

$$\Leftrightarrow 0 = C\cdot\sqrt[4]{3} \Leftrightarrow C = 0$$

$$\Rightarrow y(x) = \frac{1}{5}(2+x^4)$$ Lsg d. AWPs.