Humboldt Universität zu Berlin
Institut für Informatik

Computational universes

Oswald Berthold
<oberthol@informatik.hu-berlin.de>

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1 Preamble

In this text I want to investigate the relations that can be drawn between information, computation and physics. The view is centered on Konrad Zuse's 1969 key text "Calculating space" [32, 33], which builds on the notion of cellular automata (CA). Looking at these we will find ourselves within *digital physics* (DP) which we will explore a bit. From there we will consider other concepts and their relations, such as quantum computers and cosmology, the many worlds interpretation (MWI) of quantum mechanics (QM), our universe, the mathematical universe and computable universes.

This work is the written elaboration of my talk in PD Dr.-Ing. Horst Zuse's seminar "History of computer development" held in the summer term 2009 at Technical University Berlin. The talk was originally in german and titled "Ist das Universum ein Computer?" (Is the universe a computer?). During the additional work I undertook in fleshing out this paper it soon became apparent that this Q&A format was inappropriate here.

2 Introduction

The question of the ultimate nature of that which is has certainly for millenia attracted and provoked philosophical and scientific consideration and debate. It continues to do so. One possible answer is rooted in numbers and mechanism.
Armed with the relatively young theories of computation on the one hand and that of the quantum on the other, a variety of refined yet tentative answers in the above spirit have been constructed. Approaches taken are still diverse and i shall try to illuminate some of them in what follows. As a starting point I used German computer pioneer Konrad Zuse's text published in 1969, "Rechnender Raum" (Calculating Space) [32] and worked my way from there through variants of and alternatives to his early proposal of the physical laws being implemented on some kind of discrete information processing substrate, ultimately identical with the spacetime fabric itself. In the course of this work, I have arranged detailed discussion of Zuse's contribution within a conceptual context after first introducing digital physics and cellular automata.
Besides Zuse a number of physicists, around that time and later, notably Carl Friedrich von Weizsäcker, John A. Wheeler and Edward Fredkin, came to consider information as the most basic stuff out of which forces, matter and everything else (reality) is made. Wheeler equates information with discreteness, regarding a single bit as the elementary unit of information and arrives at the following result in his search for links between information, physics and the quantum [28].

> Otherwise stated, every physical quantity, every it, derives its ultimate significance from bits, binary yes—or-no indications, a conclusion which we epitomize in the phrase, it from bit [28].

The discreteness stems from the quantum $\hbar$ and as such lends itself well to discrete information processing models later to be named digital physics by Fredkin (who was also the initiator of the English translation of Zuse's text [24, comments on NKS]).
Zuse started from inside informatics and went on to see whether this science had anything to contribute conceptually to basic physical theory. He thought that generally the exchange between physicists and information processing specialists could experience a boost [32, p.2], see also Fig. 1. He developed the concept of a "digital particle" - a structure in a memory bank or group of cells in a cellular automaton and devises rules so they can display behaviour similar to that of classical physical particles.
The CA idea is most prominently pursued by Stephen Wolfram, culminating in the publication of NKS (a New Kind of Science) in 2002 which constitutes a huge collection of empirical investigations into the the behaviour of various cellular automata. However I will not be examining NKS any further in this paper.
Classical physics has come through the centuries on the paradigm of continuousness but it is unclear what the exact relation is between information, computability and the continuum. Unfortunately, further discussion of these issues is also beyond the scope of this document.
Taking a de-differentiated view again and possibly subsuming a good part of a current view in physics, Seth Lloyd states that "to a physicist, all physical systems are computers" [12, p.53], storing and processing information (Not only to a physicist, clearly). He sets out to describe certain "subsets" of the physical universe such as black holes or arbitrary volumes of matter in terms of quantum computation systems. The difference here is the direction, in that the physics based approach sees quantum objects first, performing elementary computations whereas the informatics

view focusses on the elementary abstract operations, how they can lead to the emergence of known physical laws and phenomena.
Finally, there is a more intricate strand of theorising about the latter that is centered on the theory of computation, algorithmic information theory and takes recourse in different shades of mathematical realism to explain reality as the "inside" views numbers have of themselves and their relations. "The notion of fundamental matter loose its meaning. Matter becomes a Moiré effect lived by the number when they infer relations from their point of views" [sic], as Bruno Marchal puts it [16]. The work of people like David Deutsch, Max Tegmark, Jürgen Schmidhuber and Bruno Marchal provides some valuable clues in this respect.
Either way a lot seems to be in store with toy universes and theories of everything. Whether it can support world-change technology [20] remains to be seen.

3 Digital Physics

Digital physics is an overarching term for some of the substance being discussed here. There are other related ones like Fredkin's later adopted digital philosophy, computationalism or pancomputationalism.
DP contains the following theses [30], in a condensed form of what we have seen above:

- the universe is basically informational <u>and</u> computable
- the universe is digital (some reals are not computable in finite time)
- the universe is itself a computer
- we are living in a giant simulation [1]

This field has already more than 30 years of history. In 1957 Edwin Jaynes published two papers connecting information theory, thermodynamics and QM and prior to that, also cybernetic attitudes might well be considered part of this lineage. In 1967 Konrad Zuse puts forward his conception of the calculating space, refering to a lecture by Weizsäcker in 1966 [34, p.15]. In 1971 Weizsäcker published this lecture as "Unity of Nature" where he introduced the concept of ur-alternatives, irreducible units of information. Edward Fredkin later coined the term of digital mechanics [7] from which the DPs in turn derive [8].
The 1980s saw additional important contributions. Stephen Wolfram started working on CA and published some important papers on the topic in 1983 and onwards [31]. In 1985 David Deutsch presented a universal quantum turing machine and showed the connection with the Church-Turing principle. This is a stronger version of the Church-Turing thesis, declaring that

> 'every finitely realizable physical system can be perfectly simulated by a universal model computing machine operating by finite means'

[1] There are many recurrences to this subthesis in the literature. Examples can be found (non-exclusively) in Descartes' Medidations, Hilary Putnam's "brain in a vat", Nick Bostrom simulation argument or in popularised versions in Galoye's story Simulacron-3, adapted into a TV-play by Rainer Werner Fassbinder (Welt am Draht) and the more recent Matrix series of movies.

which is a more than strong assertion about physics and the continuum. He explained the way a quantum computer performs its calculations in terms of the MWI. Later, he elaborates more verbosely on this, among two other strands, as being needed to explain The Fabric of Reality [4].
In the 1990s there followed the work of Jürgen Schmidhuber, Bruno Marchal and others that leant even more towards algorithmic information theory (AIT), as founded by Solomonov, Kolmogorov and Chaitin.
In parallel Seth Lloyd started publishing on the subject. For example, while more ordinary aggregate objects pose no fundamental problems in the informational view, the role of black holes is not entirely clear which opens up many interesting challenges.
Common among all stances of DP is the assertion that the basis of physics is information, discretised information. This statement builds on the assumption of the existence of a program running on a universal Turing-machine, that calculates the next state of the universe. This is just the CT-principle again. As a consequence of this, the continuum is strictly denied physical existence.
As we will see again when looking at Zuse's work, there are some problems in reconciling this view with some of the assumptions of relativistic and quantum physics. The isotropy of space is violated by the discretisation of space. Continous physics can only remain a good approximation of the workings of the digital laws. It also violates continous symmetries important in standard physics and it has a problem with non-locality in QM. Obviously, any DP will have to explain in detail how both relativity and quantum mechanics emerge from the discrete rulesets.
On the other hand there is a plain and intuitive argument against the continuum courtesy of Richard Feymann [2, 5].

> It always bothers me that, according to the laws as we understand them today, it takes a computing machine an infinite number of logical operations to figure out what goes on in no matter how tiny a region of space, and no matter how tiny a region of time. How can all that be going on in that tiny space? Why should it take an infinite amount of logic to figure out what one tiny piece of space/time is going to do? So I have often made the hypotheses that ultimately physics will not require a mathematical statement, that in the end the machinery will be revealed, and the laws will turn out to be simple, like the chequer board with all its apparent complexities.

3.1 Cellular automata

What Zuse is talking about, while being aware of it, is that the calculating space is a cellular automaton or put in reverse, cellular automata are caluclating spaces. The basics mathematical definition of a CA is a tuple (R, N, Q, δ) with

- R a space (cellular space)
- N a finite neighbourhood
- Q a set of states
- a local transition rule $\delta : Q^N \to Q$

The space R can be defined in various ways through the cellular arrangement. Likewise, there are many possible finite neighbourhoods N. Most commonly R is a one- or two-dimensional rectangular grid. Common neighbourhoods are n-nearest-neighbours in the one-dimensional case and the von-Neumann- and Moore neighbourhoods in the two-dimensional case. Every cell is in a state $q \in Q$. Most importantly, CA are known to be capable of universal computation and some such specific CA have been found.

CA have been investigated extensively throughout their 60 or so year history. The CA approach was first employed by John von Neumann when he was working on self-reproducing automata. The decisive hint came from Stanilaw Ulam [29], with whose framework von Neumann could more or less achieve his goal of formulating a universal constructor. Notably, this was done on paper only. In the 1950s CA-like structures were employed by Barricelli [6, p.58] in what were early experiments in artificial evolution. After going through a period of dormancy, CA were again repopularised in the early 1970s by John Conway, Christopher Langton and others, shortly after Zuse's work on the CS had been published. Quite likely this had to do with computers becoming available that were sufficiently fast for the simulation of cellular systems.

As mentioned earlier, Stephen Wolfram has worked a lot on cellular automata and, among other things came to postulate the principle of computational equivalence. This states that almost all non-trivial systems found in the natural world do indeed reach the equivalent Turing universality when viewed as computational systems. This is of course true for CA as well. From the various known universal CA, Wolfram claims to have found a universal one-dimensional CA with 2 states and 3 rules.

They provide a fruitful framework for experimentation and testing of model universes, as can be seen exemplified in the section on Zuse below and they provide suitable models for demonstrating emergent phenomena. Quite simple rules can give rise to very complex patterns.

While CA are most commonly seen as strictly discrete structures, the German Wikipedia entry for analog computer describes how to set up an analog cellular network for the simulation of groundwater currents. This gives a structurally discrete system with continous state values and corresponds to a coupled map lattice (CML) model.

Since CAs are so thouroughly described in many places, further discussion is brought to a halt here. For further discussion on the properties of CA you are referred to the literature, for example Cellular Automata by Andrew Ilachinski [9] which includes a final section discussing the question

> "Is nature, at its core, a CA?"

3.2 Calculating space

Konrad Zuse published the article "Rechnender Raum" in 1967 in which he developed rules for a discretised version of an artificial physics that enables the creation, propagation, interaction and annihilation of particles. In 1969 followed the publication of an eponymous book. Simple rules computing these particles are given for the cases of 1- and 2-dimensional lattices of special design, enabled by the derivation of simple laws of motion from difference equations.

He proceeds very cautiously and starts off by justifying his interference with the foundations of physics from his information processing background [33, p.1], for which he sees a need.

> The question therefore appears justified whether data processing can have no more than an effectuating part in the interplay or whether it can also be the source of fruitful ideas which themselves influence the physical theories.

Soon after he poses a similar question and considers the consequence of doing so [33, p.22].

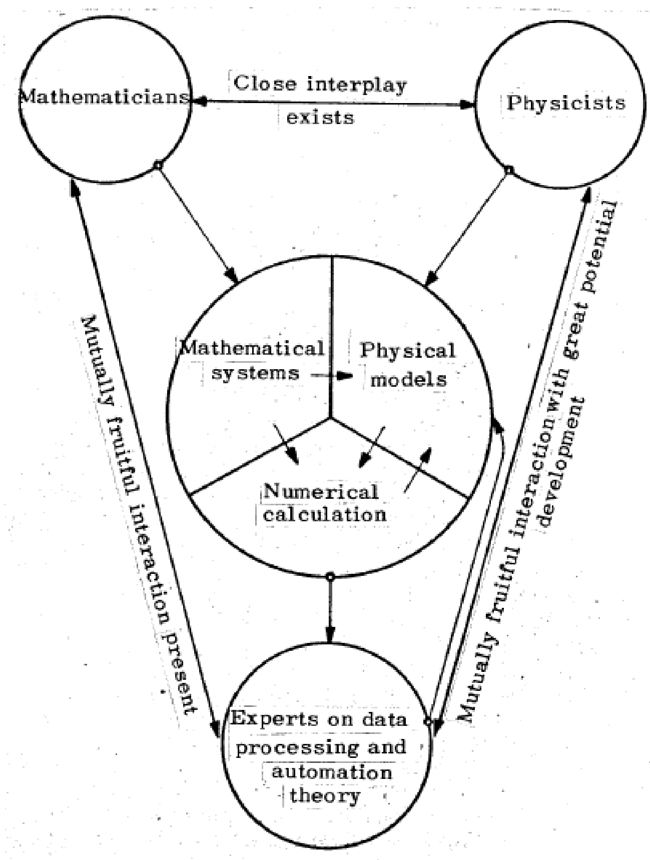

Fig. 1

Figure 1: Relationship between the mathematical, physical and informatical disciplines.

> To what extent are the realisations gained from study of calculable functions useful when applied directly to the physical models? *Is nature analog, digital or hybrid?* And is there essentially any justification for asking such a question?

Evidently he believed so. Since the employment of numerical algorithms in the simulation of physical processes is well established and tightly intertwined with the beginnings of computing itself, he has to make a point for the plausibility of transporting automata-theoretic considerations into physics. He does so by constructing working examples.

Before he does so, he introduces automata theory, differentiable and discrete machine models with their respective properties and amongst others the important concepts of autonomous and cellular automata with reference to von Neumanns work on self-reproducing automata. All of classical physics is based on the assumption of continuity of nature. The mention of hybrid systems shows the possibilities of mutual inclusion of the two paradigms. Discrete systems can be seen as special cases of continous ones, an example being the frequency coding of neural spikes in stimulus transmission, and conversely continous systems can be seen as being approximated by discrete ones.

Now we are equipped to start discretising systems of differential equations that describe an ideal frictionless gas in a straight tube. Our supply of values can be binary or ternary, for example -1,0,1 corresponding to, for example, -e, 0, +e, the possible elementary electrical charges, or elementary velocities in the case of our gas. These values populate a lattice structure of triangular, orthogonal or yet different character.

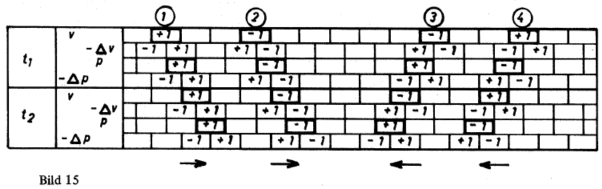

Figure 2: The first four basic digital particles

3.2.1 Digital particles

We consider the quantities p (pressure) which are fixed at points 1, 2, 3, ... and v (velocity) which are in turn fixed at points 1', 2', 3', ...

```
p 1 | 2 | 3 | 4 | 5 ...
v   1'| 2'| 3'| 4'  ...
```

$\triangle_p^s$ and $\triangle_v^s$ then are the differences between neighbouring points and $\triangle_p^t$ and $\triangle_v^t$ those between incidental times. From this the following rules can be extracted.

$$v - \triangle_p^s \rightarrow v$$
$$p - \triangle_v^s \rightarrow p$$

Here, we obtain v_{t+1} on the right-hand side, velocity in the next time-step from the difference between the current velocity and the pressure difference to the neighbouring cell.
While normally programmers aim for difference equations that approximate the differential system to a sufficent degree, here we are faced with the question of which is the most coarse discretisation allowed with respect to the initial ideal gas behaviour. Zuse answers this by incrementally modifying his initial model until it displays all of the desired properties.
By way of paper or computer simulation [1] we convince ourselves that this model indeed works. There are different initial configurations [2] that affect overall behaviour. Specifically, these contain four stable (energy-conserving) basic particles. These are identified with *digital particles* and can be regarded as propagating disturbances in a cellular automaton. The grid can be seen as an in principle linearily extended infinite automaton. The v and p are the states and dv, dp can be obtained from the states. The state-transition rules implement the difference equations. Unstable configurations can also be easily found in, for example, a single isolated pressure impulse [3]. Two of those same impulses are stable again.
In the initial model, particles are transparent to each other due to the principle of superposition. With the inclusion of nonlinear elements such as limiting the value range of p, we can force reactions

[2] Around line 37 in the code given in [1].
[3] Line 52 in [1].

to occur upon encounter with other particles. With this limitation, different modes of interaction are possible. These depend on the relative phase-shift. The phase-shift is indiscernible from the outside without knowledge about the fine-structure of space.

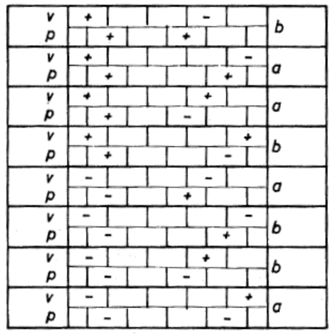

Figure 3: Collision of 2 digital particles in different phases of incidence.

Another extension is to allow for fractional propagation speeds and so obtain a new system with particles of periods $n\triangle t$. Switching speed in this case is greater than particle propagation speed, but switching is only locally relevant and signal transmission speed remains within respective limits (e.g. as required by relativity). The particle's location is only well defined for entire periods and not for the single phases of its progression which is reminiscent of Heisenberg type uncertainties.

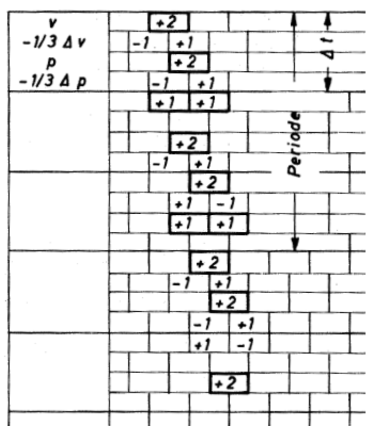

Figure 4: Propagation of digital particles with period $3\triangle t$.

On closer inspection, the events "passing through" and "repulsion" lose their meaning with digital particles because they are indiscernible. Again, this relates phenomenally to how particle identity is lost in quantum physics.

3.2.2 Two-dimensional systems

So far, we have seen one-dimensional systems but extending the model into two dimension is only a minor investment. In a first attempt to go on, Zuse poses a disjunctive law. Let $\varphi_{x,y}$ be the state of the system at point (x, y), then we get

$$\varphi_{x-1,y} \vee \varphi_{x+1,y} \vee \varphi_{x,y-1} \vee \varphi_{x,y+1} \rightarrow \varphi_{x,y}$$

with which we fill all of space with 1's. Regardless of the specific laws we use, the system obviously has preferred directions of propagation. Propagation in parallel to the axes is faster than along diagonals. This is in contradiction to the isotropy of space but it could be remedied by postulating a grid that is sufficiently finegrained. An alternative law has the following form:

$$K(\varphi_{x-1,y} + \varphi_{x+1,y} + \varphi_{x,y-1} + \varphi_{x,y+1}) \rightarrow \varphi_{x,y}$$

so that new values can be multiples of 1 and can additionally be multiplied by a factor K. This second rule together with $K < 1$ leads to an extending front of ever lower values which soon will drop below the smallest possible value and die out. Clearly, to obtain stable particles different laws are needed.

Zuse at this point imports the interleaved grids of v's and p's from the one-dimensional case. Then this setup is structurally reduced. Velocities are dropped and the grid becomes a pure p-grid populated by arrows. The p-arrows can travel on orthogonal axes. There are already two cases of the encounter of two arrows on the same orthogonal, both passing through or annihilating each other, in again a phase-dependent manner. For the case of the mutual encounter of two arrows with orthogonal directions, each of them propagates along its initial direction but then this direction is rotated by $\pi/2$. In this way we obtain one stable particle shifting across the grid diagonally with a period of $2\triangle t$. Also, stable single and double nests (unit square trajectories) become possible.
By extending this ruleset to allow for arrows of variable parametrisation, particles are constructed of varying rational propagation directions. Tying up with phase dependence through non unit speeds, such particles can interact with others only when they meet in zero-phase points. This means that the interaction capacity of particles is modified as they propagate through the space. Phase adherence is also the reason for different velocities in different discrete directions depicted in Fig. 5.

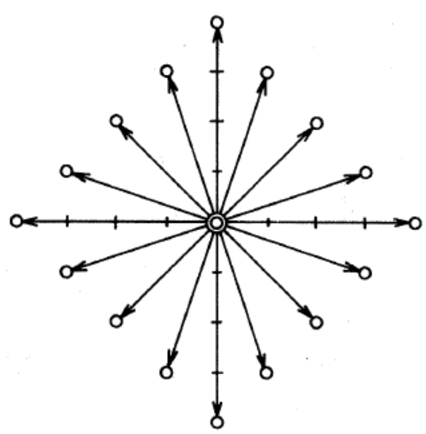

Figure 5: Propagation directions of digital particles with perdiod $4\triangle t$.

3.2.3 Summing up

At this point it is clear, that "interesting" toy physics can be recovered from the CS. Stable particles can be created that can move in almost arbitrary directions. When they meet other particles, they can pass through or annihilate each other, be repelled or deflected. Oscillations, nests and much more can be realised. This and much more is now known because of the ensuing research on CA by Conway, Langton and others from the early 1970s onwards. An extension into the third dimension is not discussed further in Zuse's text but is deemed possible without fundamental complications.
Some immediate objections against discrete nature were treated by Zuse himself. Traditional physics at that time had already come to considering spacetime quanta although not within the context of automata theory. One strong objection is the dismissal of isotropy of space, that has been mentioned already. Nothing had been observed experimentally that indicated preferred discrete directions. This of course depends on the order of the grid-constant and the energies involved. Zuse mentions that the grid-constant has to be smaller than the size of elementary particles (10^{-15}m for a proton) by some orders of magnitude. Today, Planck-sized spacetime cells are on the table.
The dependence of energy and wavelength in relation to grid-size allows the conjecture, that calculation errors could appear when the energies involved are sufficiently high.
There are problems with computing non-euclidean geometries, which could be remedied in terms of self-modifying or growing automata. Specifically though, the assumption of a fundamental grid implies a distinguished reference frame, which contradicts relativity. Lorentz transforms, which are essential in relativity can still be arbitrarily approximated and finitely many distinguished inertial systems could be generated in the framework.
Zuse also introduces the concept of *switching volume* (or shifting volume in [33]) as

$$V_S = \#\text{switching elements} \cdot \#\text{switching cycles}$$

which we will find later again in the work of Lloyd 4. This straightforward defintion is the amount of possible bit-flips within a given volume of spacetime. It remains invariant under the approximate Lorentz transforms in the simulated relativistic frames.
Another interesting problem arises from within automata theory. Since the cosmos is assumed finite, c.f. Fredkin's *finite nature* [7], the CS must be a finite automaton. Since it is also assumed as closed (receiving no input from the outside) it is thus autonomous. As such, autonomous finite automata are known to end in periodic cycles [33, p.71]. This can be regarded quantitatively. A lower bound approximation for the possible states of the cosmic automaton is $2^{10^{123}}$, the volume of a body of 10^{41} elementary lengths (10^{-15}m) edge length. Zuse further approximates the number of clock pulses since the beginning of the universe on the order of its spatial extension, 10^{41}. The conclusion is that only a miniscule fraction of possible states could have been reached, or in other words [33, p.73],

> Of what value is the realisation that the evolution of the universe follows a periodic cycle, when even within the already very large range of time being considered one single period at most can pass, and most likely not even that [4]?

What has to be dealt with as well is the issue of information content and turnover. Zuse is led to postulate a "conservation of complicatedness" although he is not conclusive in regards to a more

[4] The translation is a bit slack here and I would rather say: ... if within the very large ranges of time considered, such a cycle cannot be reached, not to speak of passing it only once.

strict definition of information, or complicatedness for that matter. One big step in developing DP since the time of Zuse's writing is the discovery of reversible logic and subsequently, reversible universal automata by Fredkin, Toffoli and others. Not surprisingly this is missing in the discussion of determinacy and causality. Justified by the preceding discussion, a preliminary table is presented relating different phenomena in the different physics [33, p.92]:

CLASSICAL PHYSICS	QUANTUM PHYSICS	CALCULATING SPACE
Point mechanics	Wave mechanics	Automaton theory Counter algebra
Particles	Wave - particle	Counter state, digital particle
Analog	Hybrid	Digital
Analysis	Differential equations	Difference equations and logical operations
All values continuous	A number of values quantized	All values are discrete
No limiting values	With the exception of the speed of light, no limiting values	Minimum and maximum values for every possible magnitude
Infinitely accurate	Probability relation	Limits on calculation accuracy
Causality in both time directions	Only static causality, division into probabilities	Causality only in the positive time direction, introduction of probability terms possible, but not necessary

We have seen that we can create a program that can compute a wide range of toy physics. Without taking resources into account, the program could compute toy universes the size of ours. The crucial question is, whether our universe is indeed computable. The Church-Turing-Deutsch principle states exactly this. A lot has been accomplished in addition to what Zuse was able to show. Obviously, by universality, we are not restricted to considering only cellular automata but any kind of formalism capable of universal computation. We will see more of this below but before that, we will examine what current physics is making of the results.

4 Quantum computation

An information processing based view of reality can be extracted from QM that is quantum computation. Indeed, if we stick with Deutsch's explicit Church-Turing physicalisation [3], a universal quantum computer is all that is needed to perfectly simulate any finite physical process.

4.1 Quantum computer universe

Seth Lloyd has written prolificly in recent years on the issues of quantum computation in relation to cosmology and fundamental physics, specifically on the idea of the universe being a quantum computer. Aphoristically put he also says that

The assertion that the cosmos is a computer is literally true [11].

According to his view, elementary particles represent nothing else than units of information. Every physical interaction of these particles is an instance of information processing in which bits are being switched. This follows already from the work of Maxwell, Boltzmann and Gibbs. The entropy of a system is proportional to the number of information bits that can be stored in the movements of its particles, in other words $S = k_B \ln \Omega$. Shannon later provided an important substantiation for this relation [11].
Quantum mechanics implies the discrete behaviour of Nature in regard to available energies and time. The switching of a single quantum information bit requires a minimum amount of time. The Margolus-Levitin theorem states that switching time is inversely proportional to the energy expended: $\triangle t \geq h/4E$. This theorem has consequences for the geometry of spacetime and the computing power of the universe. To investigate the latter, Lloyd then constructs two model computers:

1. An arbitrary mass of 1 kg within the volume of one liter (the Ultimate Laptop (UL))
2. a Black Hole (BH)

The UL uses its mass as energy according to $E = mc^2$. If all of its energy is being used, 10^{51} ops / s are facilitated. With increasing time the computation slows down. If the mass is converted to energy, a temperature of 10^9 Kelvin is obtained. Since entropy is proportional to E/T, the number of bits that can be stored within the system is about 10^{31}. One single bit can switch 10^{20} times per second. Since switching time is close to signal propagation time, this models a parallel computing system[12].
If any kind of physical system is a computer, a black hole is a computer of minimal size. A black hole of a mass of 1 kg has a radius of ca. 10^{-27} m. A proton in comparison is huge with a radius of 10^{-15} m. Through the immense compression of the particles within the BH gravity starts to affect them by manner of which they become coupled. This reduces the storage capacity. The information capacity of a BH is according to Hawking / Bekenstein and the holographic principle proportional to its surface, ca. 10^{16} bit. Switching time is reduced to 10^{-35} seconds, which is the same amount of time that light needs to cross the inside of the computer.
So, for the black hole computer we have input enformed by matter/energy that goes into the hole and output as Hawking radiation off the surface of the hole. This radiation has a wavelength on the order of the size of the BH and lies in the high energy gamma portion of the spectrum. The emission rate is inversely proportional to its size.
Not only the existence of Hawking radiation is disputed but also its character: randomised or not. The mechanism producing it is thought to be based on entangled particles that are created at the event horizon. The annihilation of one of the particles within the BH corresponds to its measurement and the result is transferred onto the outside element of the pair. This would explain how it is possible to communicate the result of the computation performed by the black hole.
The properties of black holes are tied to the properties of spacetime. On the smallest scales, spacetime is, across different conceptual frameworks, foamy (Wheeler) or cellular. These cells' size is on the order of the planck length of 10^{-35}m. On this scale, quantum fluctuations and quantum gravity, about which nothing definite is known, come into play. Spacetime is not directly observable anymore as this (current) fundamental limit is hit. Practically speaking, long before that. It has also been suggested, that those cells could be varying in size and be proportional to the volumes

that are being observed [12]. The energy of the measurement device within a volume must not lead to collapse of that volume.
Lloyd then goes on to examining the universe as a whole in terms of it's computational capacity. It exists for a finite time or 10^{10} yr by standard big-bang theory. The amount of energy contained within the universe is 10^{72} Joule and by the Margolus-Levitin theorem this corresponds to 10^{106} ops / s. The age of the universe in seconds is $1.4 \cdot 10^{10} \cdot 365 \cdot 24 \cdot 60^2 = 4.41504 \cdot 10^{17}$, which yields about 10^{123} operations. The number of bits in the universe is 10^{92} by what statistical mechanics and cosmology tell us, maximally 10^{124} according to the holographic principle (t'Hooft, Susskind). With this configuration the universe is very close to a critical density and the number of operations is maximal. According to Lloyd [10],

> These numbers of ops and bits can be interpreted in three distinct ways: (i) They give upper bounds to the amount of computation that can have been performed by all the matter in the Universe since the Universe began. (ii) They give lower bounds to the number of ops and bits required to simulate the entire universe on a quantum computer. (iii) If one chooses to regard the Universe as performing a computation, these numbers give the numbers of ops and bits in that computation.

In short, the universe is perfectly busy perfectly simulating itself, compute its own existence. Amazing but is has a touch of circularity. Now we want to move over things slowly to attain another perspective.

4.2 Complexity

Complexity is a concept that's as important as it is hard to define. It is relevant to physics, informatics, biology and artificial life, cosmology and certainly several more fields. Most often it is defined as being present when higher-level phenomena cannot readily be explained by low-level business within a system. Here we are concerned with the complexity of bitstrings that describe universes. It is related to information content. Somehow, we feel that the observable universe displays a fair amount of complexity, as in galaxies, stars, more exotic radiation sources, solar systems, ecosystems, life, laws of microscopic physics and so on.
The computing universe offers an elegant explanation of why we do observe what we observe. The most poetic picture is that of those infinite monkeys typing away at their typewriters probabilistically reproducing known major works of literature or matching substrings thereof.
The mean of the length of the longest overlapping partial sequence of a given bitstring s occuring in a random bitstring r is the logarithm of the length of s. If that bitstring r is instead read and interpreted by a universal computing structure, we can expect “interesting” things to appear with much higher probability. This is an important result of algorithmic information theory (AIT). The “monkeys” could be other sources of randomness. According to Lloyd [13] it is decoherence that introduces randomness into computations. Thereby, determinacy becomes superposition. And here we can set the anthropic principle to work. This states that the amount of complexity observed results from the fact, that the ability of a system to observe itself presupposes a certain degree of complexity. We will return to this question further below.

4.3 Subquantum concepts

There is a related development in the physics of quantum gravity worth briefly mentioning here. It is Loop Quantum Gravity (LQG), a competitor to string theory, supporting a non-big-bang cosmology.
Gravity is a phenomenon that cannot be quantised as can be done with other basic forces in physics. Gravity is an inherent property of spacetime itself. As such LQG is a theory that cannot be acting on the stage of spacetime but instead must create that stage. The basic postulate are spacetime cells of variable geometries (quanta) on the Planck-scale [27] that can store a state. At every Planck-time cycle cells with the lowest state are destroyed and created. The others change their state according to the laws of the spin-network established among the spacetime cells. An idea strongly reminiscent of CA.

5 Computable universes

There is another line of thinking involved with the reality-from-computation problem. A commonality in these works is the assumption of a strong kind of mathematical realism. From there on, views start to diverge although still many of them also share the view that the mathematics needed for a complete description of reality is discrete. The phenomena actually turning up within reality are assumed to be computable. The Church-Turing principle is strongly manifest here as well. There is an algorithm that computes all possible universes, that is, all possible bitstrings. A strong relation is established between these possible computational universes and the infinity of Everett worlds in the MWI. These theories are termed by what they aim to explain as theories of everything (TOE).

5.1 Mathematical universe

Early in this line lies the work of Max Tegmark who asked in 1997 whether an ensemble theory of everything is a good fit. According to this view, the basic entities existing a priori are mathematical structures. Mathematical structures are abstract entities, in particular sets and the relations (functions) between them. Examples are Boolean algebgra, Newtonian theory of gravity, general relativity or quantum field theory [26, p.27]). This is called the mathematical universe hypothesis (MUH): "our external physical reality is a mathematical structure". This defintion sits on the external reality hypothesis (ERH) according to which there exists an oberserver-independent external reality. So in this "ontic" stance of universal structural realism [26, p.4] external reality *is* mathematics, rather than being merely, no matter how well, described by it.
These mathematical structures can be defined in two ways. In the finite case, relations can simply be tabulated. In the infinite case, relations have to be given as algorithms that compute the relations. This leads Tegmark to argue for an augmented version of the MUH, the computable universe hypothesis (CUH): "the mathematical structure that is our external physical reality is defined by computable functions" [26]. Computable functions that are garantueed to halt, to be more specific.
Observers in the MU are so-called self-aware substructures (SAS). These SAS, like us, are experiencing a "frog-view" of the world. There is also the "bird's view", a full identification of the structure from the outside. This realisation corresponds with the exo/endo distinction or 1/3-person experiences in the work of Rössler and Marchal [21, 14] respectively. Any such SAS will

experience the relations within the structure as physically real.
In regard to the simulation argument, Tegmark points out that time in the CUH does not need to proceed in any way and time need not be associated with consecutive steps of a computation. It is sufficient for the description of the relations to be computable. The exact grade of structure has to be specified, Tegmark asks "how well-defined do mathematical structures need to be to be real" [26]? With the CUH, this range is limited to "computable structures (whose relations are defined by halting computations)". Finite structures and computations not garantueed to halt lie beyond this limit. This position overlaps with that of mathematical intuitionism. One question remaining is that if there is no need for actual execution, how halting may come to matter.

5.2 All possible universes

All possible universes, according to Jürgen Schmidhuber are initiated by the Great Programmer [22]. Possible meaning computable. They evolve in discrete time and any of these universes is describable by a finite bitstring. We can also approach the matter by asking: Is there a program that can compute the exact progression of events of our universe and that of all other possible universes [23]? Since computable universes are simpler than noncomputable ones and there is no clear contradiction with QM, computable ones are to be preferred. Schmidhuber examines the workings of a short and optimally fast algorithm that computes all possible universes in a parallelised fashion, as proposed earlier by Bruno Marchal [25].
We have seen above already, that the algorithm computing all possible universes can be simpler than one computing a particular universe. Indeed, it is a result of AIT that informs us so.
AIT is grounded in the work of Solomonov, Kolmogorov and Chaitin from the early 1960s and could laxly be considered as an elevated cocktail of Shannon and Turing. The central concept is Kolmogorov(-Solomonov-Chaitin) complexity $K(-SC)$ which is a different measure of information content than the one given by Shannon. If s is an arbitrary bitstring and $d(s)$ is the minimally short description of an algorithm that outputs s, then $K(s) = |d(s)|$. Randomness implies high complexity. For a maximally random bitstring we will have $|s| = |d(s)|$. $d(s)$ generally does not depend on any particular language (a particular TM) because the Compiler-theorem [22, p.1] states, that the process of compilation only adds a constant to the length, $K_1(s) \leq K_2(s) + c$.
So, if there exist uncomputable numbers in the universe, then the universe itself is uncomputable. Conversely, classical differential equation based models might be a good approximation of microscopically discrete behaviour. Schmidhuber cites Gerard t'Hooft as in support of such a deterministic universe concept.
As a consequence, primary chance in QM is not indeterminate but the output of a pseudo random number generator (PRNG) contained within the universe generating program. Observers (SAS) are substrings in the string describing particular universes. They are subject to uncertainties of the Heisenberg type and cannot decode the underlying determinacy. This determinacy is not ruled out by uncertainty, because any inside laws need not be valid on the outside. A bird-observer might halt the computation and inspect the current state entirely. A frog-observer cannot even notice if the computation is stopped from the outside (but see Tegmark on the "time misconception" [26, p.18]).
After considering this Schmidhuber sketches out the setting for the *shortest and fastest program*:

- A particular class of output strings of a universal machine can only be described by a short program if the output bits may be changed later. The output string need only converge to its final form in finite time.

- All possible programs are systematically enumerated (by length and lexicographic order).
- There exists a program that computes the enumeration and the parallel execution that is required. This is achieved through dovetailing [5].
- The shorter the program, the more frequently it will be run. Wait time for a program of length N is given as 2^{N+1} cycles on average.
- The short universal program is of the same order in speed as a program computing any specific bitstring history.

There are many equivalent programs but those with shorter descriptions will be farther progressed. Additionally, the probability of observing any string whatsoever is dominated by the probability of guessing its shortest programs. A task for science, Schmidhuber concludes, is the search for our universe's program, for which there might even exist an algorithm. Although, as he remarks, its predictive power might not be too grand, given that the prediction might only be possible by simulation, which cannot be faster than the evolution of the universe itself. Another implication for physics that Schmidhuber stresses, and as is present in the work of other authors is the association of computational parallelisation with the MWI [22, p.205].

> An automatic by-product of the Great Programmer's set-up is the well-known "many world hypothesis", (c)Everett III. According to it, whenever our universe's quantum mechanics allows for alternative next paths, all are taken and the world splits into separate universes. From the Great Programmer's view, however, there are no real splits - there are just a bunch of different algorithms which yield identical results for some time, until they start computing different outputs corresponding to different noise in different universes.

5.3 Universal Dovetailer Argument

Brussels-based philosopher and computer scientist Bruno Marchal has worked and published extensively on this subject for over a good decade. Over this time he has developed an informal argument that places the origin of physics within the natural numbers and their relations, when certain assumptions are met. It is called the universal dovetailer argument (UDA) [14, 15].

> If only number exists, then it would be like I am proposing a new theory. I could have done that, but this is not what I have done. What I give is a constructive proof that if I am machine, and CT is correct, then the laws of physics have to be reduced and derived from the laws of number (or any recursively isomorphic structure). The movie graph argument is what makes this obligatory. The notion of fundamental matter loose its meaning. Matter becomes a Moiré effect lived by the number when they infer relations from their point of views. You can still believe in matter if you want too, but

[5] Dove-tailing in algorithm design is a technique for executing programs in parallel in order to evade the possibility of getting stuck with any one particular non-halting program. If the programs to be executed by the dovetailer are arranged by length and lexicographical order and assigned a numbering of 1,2,3,... then each of these programs will be given one operation cycle in for example this order: 1, 2, 1, 3, 2, 1, 4, 3, 2, 1, ...

you cannot use it to explain even the physical observation, which have to emerge from special number's points of view.

That is what UDA shows.

As a hopefully worthy ending of the discussion of computational models I will try and summarize the argument in the following. As a starting point, Marchal defines computationalism (comp) or digital mechanism as the aggregate of three sub-theses.

1. The 'yes doctor'-hypothesis which asserts the substitutability of the brain on *some* level of description.
2. Church thesis (CT): all universal machine models are equivalent regarding the class of functions that they can compute. This in turn is equivalent to what is calculable at all. See also the definition of the Church-Turing-Deutsch principle above.
3. Arithmetical realism (AR): arithmetic statements are true independent of humans or universes or anything else.

Based on these, the UDA is set to unfold in eight steps. Step 1-5 are successively more intricate variants of a basic teleportation thought experiment. That is, you are read in one location (information on the substitution level is scanned), annihilated and then reconstituted in someplace else. The variants consist in introducing reconstitution delays, reconstituting multiple copies of you with a possible delay for any one copy and of not annihilating the original. This introduces first person indeterminacy (1-indeterminacy) by way of not being able to experience in first person as being in more than one place at once. From a 3-person view all is perfectly in order though. Step 6 then emphasises the indistinguishability of "classical" reality and virtual reality in the spirit of the *dream argument*. This point is corroborated by Deutsch's investigation of virtual reality [4].
In Step 7 things start to get more involved. Here the dovetailer is introduced. First, by CT, all computable functions are computed by algorithms expressible in a language L. There is no language, that could by design only compute all total computable functions. This is shown by a diagonal argument. So if we want to actually run the simulation / calculation of all possbile machines, we are forced to dovetail, otherwise we will get hung on the first non-halting program. If the physical universe is sufficiently robust so that the "concrete" UD can run forever then it will generate all possible TM states infinitely often.
Then in Step 8 (also referred to as the movie graph argument) it is considered what to do if we don't grant a sufficiently robust concrete physical universe? *comp* forbids to associate inner experiences with the physical processing related to the computations corresponding (with comp) to those experiences. These inner experience can only be associated with the type of computation and not with materialist supervenience. It can be shown that physical activity can be made arbitrarily low for any given inner experience. Physics is a measure on the consistent computational histories, or maximal consistent extensions as seen from some first person pov. Physics follows as an element of "machine psychology".
The argument is set out to elicit what follows from a given set of assumptions (comp). These assumptions are not out of the ordinary, in fact nothing new has been added compared to everything else that has been discussed in the sections above. Most interestingly the result is a contradiction between the computationalist assumption and materialism.

6 Summary and Discussion

The preceding survey of the literature is far from complete, nor is it intended to be so. It is merely supposed to indicate a particular trajectory through the discursive landscape.

6.1 Universality

The twentieth century has seen a few remarkable discoveries about the nature of Nature. The ones I am referring to are those mathematical and physical findings: the theories of relativity and the quantum theory. Others can be appended like chaos theory and the "physics from within", endophysics. All of these further emancipate the observer, gnawing away at the classic scientific ideal of objectivity. One is still missing and it is the notion of the universal machine. Bruno Marchal commented about it in this way [17].

> . . . , the discovery of the notion universal machine is one of the most astonishing and gigantic discovery made by the humans, and what I do is just an exploitation of that discovery. Universes, cells, brains and computers are example of universal machine, and the notion of universal machine are a key to understand why eventually, once we say "yes to the doctor", and believe we can survive "qua computatio", we have to redefine physics as an invariant for the permutation of all possible observers, and how physics can be recovered from an invariant among all universal machines point-of- views . . .

This universality makes its possible to abstract from any kind of concrete computational model. CA may be a good framework to talk about certain aspects of how a physics can be implemented in such a way but ultimately it can be simulated by any of the other equivalent formalisms.
Deciding whether the world is taken as computable or not is so far obviously a matter of inclination. Discrete machines clearly are realisable in our universe. Discrete machines clearly can also approximate continous systems to an arbitrary degree. Just how exactly arbitrary depends. If computation actually needs to be executed on some substrate, true universality is only possible in an infinite universe. If, on the other hand, the description of the computation is sufficient, this ceases to be a problem. If for the moment we assume this last position, a variety of surprising conclusions follow in connection with 3-person determinacy and reversibility, 1-person randomness and indeterminacy, the many worlds hypothesis and the emergence of time. These fit well with results from QM.
The realisation that if the universe is computable, everything else must be as well could act eventually as a guide for the development and verification of models in other fields. The gap to be filled between a computational physics and even phenomena on a microscale is huge though. Clearly there are implications not only for physics but also for artificial intelligence.

6.2 Approaches

Within the computability-camp we were able to make out two main approaches. Many physicists have arrived at considering information as the fundamental principle in physics but, probably by habit, these considerations are centered on the idea that the relevant computations still have to be actually run on some kind of grid of tiny spacetime cells.

The mathematical structural realist's approach assumes only numbers and their relations to exist and this is seen as sufficient for all phenomena that we know of to emerge on the inside of these structures by self-reference.
However a large piece in the puzzle that is missing is the precise derivation of current physical theories within the computational physics framework.

6.3 Not discussed

A lot of issues have not even been mentioned so far. For instance, we have not described at all, how a deterministic picture based on a *continuous worldview* could be regained. This has to be left for another occasion.
We have not really touched the question of how *complexity* comes about, why we observe some structures at all out there. Clearly, in the collection of all possible universes, there are a) a lot of boring ones and b) we will only have to consider those rich enough to allow for SAS. This is the *anthropic principle* which states, in short, that the observable universe must be apt for the development of observers or intelligent life because otherwise we would not be here to observe and physically describe it [25]. This could imply the development of complexity as a precondition.
Then there is *consciousness*. It can be said that not much is known definitely about what it is and how it may really work. It is generally disputed whether the mind is computable or not. Again, if the universe is, then consciousness must be as well. It seems apparent that some very subtle microscopic interactions are taking place in order for the observer to elicit experienced reality from the rest of the world. Also, there arises some discrepancy between determinacy and our experience of free will. But to delve into these matters, even only on a shallow level, will burst out of the current frame.

6.4 Criticism

Finally, some words about criticism of the view adopted in this text. Gualtiero Piccinini takes a turn at it in his 2007 text [18], looking at computational modelling and computational explanation. Only the latter applies to what is discussed here. There are at least three things to say about his arguments against the validity of such views. First, he has not left the materialist frame and always assumes there is a physical substrate that executes algorithms. Clearly the work cited in the section on computable universes is not particularly vulnerable to arguments on this basis. Then, it is entirely unclear how the natural numbers or subquantum spacetime bubbles could be subject to miscomputation. Finally, the examples given of subsystems of the universe that do not perform computations, like planetary systems, stomachs and the weather are at least considered on an entirely inappropriate level of description.
There are other quite elaborate stances put forward by e.g. Roger Penrose or David Chalmers. These are focussed on the problem of consciousness, because assuming that consciousnesses are part of the universe, then what applies to one might apply to the other. The essential statement is that there indeed are uncomputable elements in reality. However Penrose's model is not overwhelmingly convincing either.
On a final note and by the dream argument enabling us to return to where we started, I quote Otto Rössler [20].

> Then if in the "big dream" of awakeness, which is being played to me without my being asked, a mathematical match of all its parts occurs, this does not mean that

this machinery would be the cause of my dream. It only means, that the dream is constructed “internally clean”. The dream is not the consequence of the parts within it.

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