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Int. J. Production Economics 71 (2001) 417–428

international journal of production economics

www.elsevier.com/locate/dsw

Strategic safety stocks in reverse logistics supply chains

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Abstract

In the last few years growing interest has been dedicated to supply chain management. Modeling complexity is added to the supply chain coordination problem by accounting for reverse logistics activities. An increasing number of ecological constraints, together with economic incentives, allows product recovery become an interesting field in supply chain management. Limitations, enormous waste and by-product disposal cost, the duty for manufacturers to take back used products from customers and the fact that returned products might have a positive economic value are some of the reasons. The objective of this paper is to combine the problem of safety stock planning in a general supply chain with the integration of external and internal product return and reuse. © 2001 Elsevier Science B.V. All rights reserved.

Keywords: Safety stock; Supply chain management; Reverse logistics

1. Introduction

The presence of substantial cost reduction and service improvement potential provides an incentive for designing planning techniques and to coordinate the various partners that are involved in the logistic chain. The presence of several sources of original uncertainty, i.e. timing and size of customer orders, variable processing times, yield of production processes, timing, and quantity of replenishment order deliveries, requires for an adequate implementation of buffers in order to realize sufficient cost and service performance results. Safety stock being generated as an additional inventory in order to smooth delivery and demand variations coupled with timing and quantity deviations is one (beside others) buffering technique. The challenging research question is where to place which amount of safety stocks in a complex supply chain.

Modeling complexity is added to the supply chain coordination problem by accounting for reverse logistics activities. An increasing number of ecological constraints together with economic incentives allows researchers and practitioners put more interest towards return and reverse logistics. The reasons are limitations or enormous waste and by-product disposal costs, the duty for many manufacturers to take back used products from the customers and the fact that the returned used products might have a positive economic value to be unlocked by recovery operations. After disassembly and remanufacturing, parts of the returned product components can be used in new products (e.g. copiers) or be sold as spare parts in service logistics (e.g. motor engines).

The objective of this paper is to combine the two problem areas mentioned above. It provides a safety stock planning approach for general network supply chains with integrated reverse logistics return and reuse materials flows. Safety stocks are regarded from a strategic planning perspective. The buffers established by the approach are meant to

PII: S0925-5273(00)00138-9

cover against a maximum reasonable amount of variation. From an operative point of view, the size of these stocks may be insufficient because extraordinarily large variations, that exceed the maximum reasonable level, are not taken into account. Nevertheless, from a practical point of view not all variations are covered by stocks. There are additional opportunities summarized under the term operating flexibility, e.g. expediting orders, external supply, assumed to be available if such extraordinary variations occur. As a first step, only safety stock requirements with respect to uncertain customer demand and returns are considered. Other sources of uncertainty in a supply chain are a matter of necessary extensions.

Integrated reverse logistics material flows can be classified into external product returns and internal by-products. As a matter of legislation or marketing strategy for selling new products, a manufacturer may be obliged to take back used products. Instead of disposing them, the products are disassembled where some parts can – after undergoing some remanufacturing activities – be used as substitutes for produced or procured materials in the regular manufacturing process. A second topic is provided by merchandise returns where final products are sent back by customers due to several reasons. In the catalogue seller business an amount of up to 30% of sold products are returned. These products undergo some quality check within the take-back process and can afterwards be sold again as regular products. Internal reverse flows occur if a production process yields several outcomes. Especially in chemical and pharmaceutical industry, processes yield one major desired output and one or several jointly generated by-products. After some processing, these by-products can either be used as a substitute for materials at an upstream processing stage (which creates a cyclic backward reuse flow in the supply network) or they can be used as input material substitutes in different product lines which will be regarded as a forward reuse activity. Besides the effect of saving disposal and raw material purchasing cost by investing in remanufacturing operations, the main interest of this analysis is how these additional external and internal material flows impact the required amount of supply chain safety stock. The contribution of this paper is that it provides a safety stock planning methodology that applies for general networks and in addition allows for cycles that are induced by return and recovery flows.

Thierry et al. [1] present an overview on strategic product recovery and remanufacturing issues and their impact on the supply chain. The different product recovery options repair, refurbishing, remanufacturing, cannibalization, and recycling are discussed. A state of the art review on quantitative models developed for the different problems in reverse distribution, recovery production planning and inventory control is given by Fleischmann et al. [2]. For the purpose of materials coordination in product recovery systems the available research material follows the two well-known streams of stochastic inventory control (SIC) [3–5] and material requirements planning (MRP) [6,7].

In the following section, the strategic safety stock planning approach for general network supply chains without product recovery is introduced as a base for the integration of external and internal return reuse in the third chapter. Some implications resulting from the recovery integration are illustrated by a numerical example. The last section summarizes the approach with its managerial implications and discusses further extensions.

2. Safety stock placement in supply chains

The safety stock planning approach followed in this paper refers to the model of Simpson [8]. It was developed for a serial production/inventory environment to provide a framework for safety stock planning under random demand from a strategic buffering point of view. The model is quite robust and can be extended to more general divergent and convergent supply chains (see [9]) as well as to general networks as described in this section.

2.1. Model assumptions

The supply chain is given by a network. The nodes represent the stocking points i for purchased ($i \in A$), in-process ($i \in P$) and final product ($i \in E$) goods. Every process i yields a unique product outcome. Therefore, each stockpoint i characterizes one product as the single result of process i. The corresponding deterministic processing time denoted by λ_i is independent from the processed quantity. For final product stockpoints $i \in E$, λ_i includes the review period. The arcs of the network indicate the direct material requirements relationships. The arc weights represent the input coefficients $a_{i,j}$ that denote the number of items of i required to manufacture one item of j. The set $v(i)$ denotes all direct predecessors to i, cf. the set of material supplier stockpoints that deliver the ingredients to start process i. The set $n(i)$ denotes the stockpoints that are directly supplied by i, i.e. the direct successors to i. Let $V(i)$ denote the set of all predecessors and $N(i)$ the set of all successors of i (both including i). In order to characterize a cumulative supply process that results in the product represented by i, a set $w(i, j)$ defined as the set of consecutive stockpoints that are on a single path from point i to point j in the network is introduced. All paths that exist between i and j are collected in the set $W(i, j)$.

Customer demands for final products $i \in E$ are uncertain. The demand for product i within a base period $[t, t+1]$ is assumed to be stationary and normally distributed with known mean μ_i and standard deviation σ_i ($D_i \sim \mathrm{N}(\mu_i, \sigma_i^2)$, $\forall i \in E$). Serial correlation of demand in time is not incorporated into the model. The demand correlation between final products within a single period is denoted by $\rho_{i,j}$ ($-1 \leqslant \rho_{i,j} \leqslant 1 \; \forall i, j \in E$).

Supply chain coordination between the stockpoints is organized by following Kimball's base-stock concept (see [10] for a reprint of the original paper from 1955). Inventory review and ordering decision transmission are undertaken at discrete points of time $t, t+1, t+2, \ldots$. Each stockpoint observes the replenishment requests of its customers. For a final stage stockpoint this amount is given by the cumulative customer demands of the last period whereas for supply-side and in-process stockpoints the amount is given by the sum of all direct successors ordering quantities that are placed at the same review point of time. Kimball's control approach assumes that each stockpoint replenishes exactly the amount of stock that was requested by its customers, i.e. all its direct predecessors $j \in v(i)$ receive an order determined by the demand placed against i multiplied by the corresponding input coefficient $a_{j,i}$. Let $g_{i,j}$ denote the total quantity of product i required to process one item of final product j,

$$g_{i,j} = \sum_{l \in n(i)} a_{i,l}\, g_{l,j}, \quad g_{i,i} = 1.$$

Following this information flow approach, every supply chain installation gets full point-of-scale information concerning the final products it supplies. Therefore, the demands placed against internal stockpoints are normally distributed with parameters:

$$\mu_i = \sum_{j \in E} g_{i,j} \mu_j, \quad \sigma_i^2 = \sum_{j \in E} \sum_{l \in E} g_{i,j} g_{i,l} \sigma_j \sigma_l \rho_{j,l}.$$

Regarding the materials flow that is initiated by the described information transmission, Kimball introduced the so-called service time concept. Each stockpoint i fills the replenishment requests after a service time S_i. These figures are regarded as a strategic control variable for supply chain coordination. Under a make-to-stock production strategy, customers are immediately served from inventory and the final-stage service times have to be zero. The opposite extreme of a pure make-to-order strategy implies that the service time equals the maximum cumulative processing time required to build the product. The decisions upon internal service times and the given processing times λ_i determine the replenishment lead times, i.e. the time span between an order release and the availability of processed goods. Therefore, the replenishment lead time L_i is determined by the maximum service time of all direct predecessors (the time it takes to have all component material available) plus processing time (the time to perform the processing):

$$L_i = \max_{j \in v(i)} \{S_j\} + \lambda_i. \tag{1}$$

The implementation of strategic service time decisions in a stochastic customer demand environment is strongly connected to the required amount of safety stock in order to guarantee these service times at some service level α_i. Suppose a stockpoint offers a zero service time. A required service level can only be attained if the available safety stock covers against demand variations over the replenishment lead time. If the service time to successors equals the replenishment lead time, no safety stock is necessary because all material is received from the predecessors and processed within the service time. The safety stock that is required at i therefore is influenced by the replenishment lead time L_i and by the service time announced to all successors. The time span T_i over which safety stock coverage against demand variations is necessary equals the difference of lead time and service time:

$$T_i = L_i - S_i. \tag{2}$$

Besides demand variability and coverage time, the required amount of safety stock depends on the service level. From several well known measures of service, the non-stockout probability (α-service level) is applied. If a stockpoint has to guarantee an α_i-service level, the probability that the inventory at the end of an arbitrary period is smaller than zero has to be smaller or equal to $1 - \alpha_i$. Equivalently, the probability that the cumulative demand over T_i time periods $D_i(T_i)$ is less or equal to a base-stock inventory level B_i has to be larger or equal to α_i:

$$P\{D_i(T_i) \leqslant B_i\} \geqslant \alpha_i.$$

Given $D_i(T_i)$ is normally distributed with mean $T_i\mu_i$ and standard deviation $\sigma_i\sqrt{T_i}$ the solution of the above probability constraint for the base-stock level yields

$$B_i(T_i) = \mu_i T_i + k(\alpha_i)\sigma_i\sqrt{T_i}. \tag{3}$$

The safety factor k is given by the α_i-quantile of the standard normal probability distribution function ($k = \Phi^{-1}(\alpha_i)$).

From an operative point of view, this inventory can be insufficient to meet all orders within the service time if extraordinarily large demands occur. In this case an additional time delay would occur until this shortfall quantity becomes available. Simpson's strategic safety stock approach operates under a no delay assumption, i.e. the additional shortfall delay is neglected. In a lot of practical applications, safety stock levels are implemented to cope with regular demand variations that do not exceed a maximum reasonable demand level. Larger variations are dealt with by using some operating flexibility provided by rescheduling or expediting orders. Therefore, the internal service levels $\alpha_i, i \in A \cup P$, represent the degree of available operating flexibility.

Physical stock holding and the invested capital in inventory are subject to holding cost h_i per item per period. The following two classes of inventory are considered.

(1) Pipeline inventory $\mathrm{PI}_i = \lambda_i \mu_i$.
Average work-in-process is not affected by the coverage and service time decisions and can therefore be omitted from the following holding cost optimization.
(2) Safety stock $\mathrm{SST}_i = k(\alpha_i)\sigma_i\sqrt{T_i}$.
The safety stock is defined as the average net inventory at a stockpoint just before the next order arrives and is influenced by the coverage decisions and valued with holding costs.

2.2. Optimization problem

The decision variables of the model presented in the previous subsection are the strategic service times that stockpoints quote to their successors. The objective is to place strategic safety stocks in the way that the resulting inventory holding cost are minimized. The feasible solution region is restricted by logic dependencies of the service times. In the following, it is assumed that a make to stock production strategy is followed. This implies that service times quoted to external customers are zero. For other stockpoints, service times cannot be negative at the left hand boundary and cannot exceed the replenishment lead time at the right hand boundary. This gives the following non-linear service time optimization problem (see [9]):

$$\min \quad C = \textstyle\sum_{i \in A \cup P \cup E} h_i \sigma_i k_i \sqrt{\max\limits_{j \in v(i)} \{S_j\} + \lambda_i - S_i}$$

$$\text{s.t.} \quad 0 \leqslant S_i \leqslant \max_{j \in v(i)} \{S_j\} + \lambda_i \;\; \forall i \in A \cup P,$$

$$S_i = 0 \;\; \forall i \in E.$$

This problem consists of a non-linear, non-separable objective function and non-linear constraints. Therefore, the analysis is rather difficult for a general network supply chain. Instead of modeling the cost trade-off by service time interaction, an equivalent (see [9]) formulation expressed by the coverage times can be given.

Each coverage time can be characterized by the replenishment lead time L_i and the minimum excess coverage M_i provided by downstream stockpoints:

$$L_i = \lambda_i + \max_{w(l,i)\in W(l,i),\ \forall l\in A} \left\{ \sum_{j\in w(l,i)\setminus\{i\}} (\lambda_j - T_j) \right\},$$

$$M_i = \max_{w(i,m)\in W(l,m),\ \forall m\in E} \left\{ \sum_{j\in w(i,m)\setminus\{i\}} (T_j - \lambda_j) \right\}.$$

The replenishment lead time of i consists of the processing time plus the maximum cumulative uncovered processing time over all supply paths from stockpoints $l \in A$ to i. If a downstream stockpoint's coverage is larger than its processing time, preceeding stockpoints face excess downstream coverage which lowers their own coverage requirements. The minimum excess over all direct successors indicates the time that is already covered for all preceding operations and therefore does not have to be covered by i:

$$T_i = \max\{0; L_i - M_i\}.$$

The remaining coverage requirement equals the difference of lead time and excess coverage (if positive, otherwise the coverage requirement is zero).

2.3. Solution property and combinatorial solution principle

In this subsection some general properties of the optimization problem are discussed and utilized for the development of a solution algorithm. In general, concave minimization optimization procedures can be applied (see [11]) but such an approach does not account for the special structure given by the constraints for this problem. As shown for serial, divergent and convergent supply chains in [9], an optimal safety stock allocation policy is obtained at an extreme point of the feasible region. This implies that coverage is equal to zero or contains the entire replenishment lead time.

Efficient dynamic programming algorithms with a single state variable can be developed for systems with $1:n$ (divergent system) or $n:1$ (convergent system) predecessor relation (see [12]). For spanning tree networks, an evaluation order for the stockpoints can be given and the problem can still be solved by a single-state-variable dynamic programming approach (see [13]). This property does no longer hold in case of a general network. The presence of multiple connections between the stockpoints requires multiple state variables and makes dynamic programming intractable as a general purpose solution method.

By exploiting the cover all or nothing property (see [14]), it is possible to transform the problem into a combinatorial optimization problem. Every candidate safety stock policy is characterized by a binary string $\boldsymbol{x} = (x_i)_{i\in A\cup P}$ that indicates if i holds safety stock ($x_i = 1$) or not ($x_i = 0$). Since final-stage stockpoints at least have to cover the review period, $x_i = 1$ holds for all $i \in E$. For a given representation vector $\boldsymbol{x}$, the corresponding coverage times are determined as follows. First, the impact of downstream excess coverage M_i is neglected and the replenishment lead times L_i are computed starting with the external supply stockpoints:

$$L_i = \begin{cases} \lambda_i & \forall i \in A, \\ \lambda_i + \max_{j\in v(i)}\{L_j(1-x_j)\} & \forall i \in P\cup E. \end{cases} \quad (4)$$

If predecessor j holds safety stock, the corresponding quantity is sufficient to cover its lead time L_j and therefore L_i is not influenced. If j does not hold any safety stock, the material will become available after L_j periods. For given lead times, the coverage and minimum excess times are jointly determined beginning with the final stage:

$$M_i = \begin{cases} 0 & \forall i \in E, \\ \min_{j\in n(i)}\{T_j - \lambda_j + M_j\} & \forall i \in A\cup P. \end{cases} \quad (5)$$

Excess coverage of successor j is given by the coverage time of j plus the minimum excess faced by j less the processing time requirement that can never be covered at upstream levels. After all coverage times are determined, the resulting safety stock quantities can be computed together with the associated costs. Then, the quality of a solution can be compared to other solutions given by different representations.

By exploiting this kind of combinatorial characterization, modern meta heuristics as simulated annealing, tabu search or genetic algorithms can be applied to the problem to find reasonably good solutions as shown in [14].

3. Integration of external and internal product returns

The model introduced in Section 2 assumes an acyclic network. In this section, external and internal forward and backward return flows are integrated as shown in Fig. 1.

In the first subsection, the additional assumptions concerning these flows are summarized before, in the second subsection, the extended model is analyzed. The last subsection reports on optimization problem and solution method adjustments.

3.1. Additional assumptions

Products are randomly returned by external customers to stockpoints $i \in F^{E}$. These stockpoints can be either at the final echelon of the supply chain (i.e. merchandise returns) or at an internal stage (i.e. recovery returns). It is assumed that returns R_i are normally distributed with known mean μ_i^{bp} and standard deviation σ_i^{bp}. Correlation of return quantities in time as well as correlation between demands and returns are neglected.

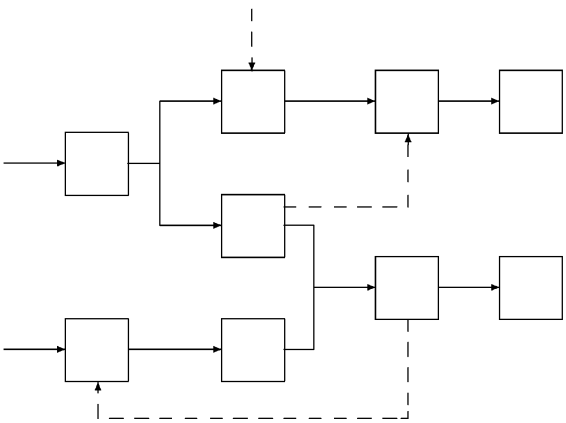

Fig. 1. Supply chain with external return and internal by-product flows.

Internal returns are induced by processes. In extension to the single product outcome assumption, some processes yield by-products that are recovered and then reused as material substitutes in other processes $i \in F^{I}$. Depending on the location of the reuse stockpoint, backward and forward reuse are distinguished. Backward reuse occurs if the by-products of j are reused at a stockpoint $i \in V(j)$, i.e. the recovered material revisits a process. If the recovered by-products are reused in a different product line, this implies a forward reuse operation. A forward reuse arc can never end in a stockpoint $i \in N(j)$.

The by-product quantity that is created by process j and reused as a substitute for product i is assumed to be a deterministic fraction $a_{j,i}^{bp}$ of the desired main product outcome quantity at $j = \text{bp}(i)$. To keep the presentation simple it is further assumed that each by-product of j has a single destination stockpoint and that no allocation decisions concerning the by-products are necessary. Each stockpoint at most reuses material from one (external or internal) return source ($F^{E} \cap F^{I} = \emptyset$). This implies that, together with the regular replenishment source, at most a two supply mode consideration is necessary. Nevertheless, an extension to multiple modes is straightforward.

Returns and by-products are neither disposed of nor kept in inventory. All returns and by-products are immediately recovered with a known time requirement λ_i^{bp} and thereafter kept in stock together with the regularly manufactured products at the same holding cost rate. To account for recovered returns that serve as substitutes for regularly produced items, the replenishment policies of stock points $i \in F^{E} \cup F^{I}$ must be adjusted. It is assumed that each stockpoint operates with a single service time. Return quantities of period t are subtracted from the regular replenishment order in the same time period. In the case of external returns and forward by-product reuse, the returns can exceed the demand driven requirements so that excess returns are subtracted from future regular replenishment orders. Note that, for backward reuse operations, the cyclic dependency that the by-product outcome can only be a fraction of the input, avoids this effect.

3.2. Model analysis

Given the characteristics of customer demand $D_{i,t}$ $\forall i \in E$, external returns $R_{i,t}$ $\forall i \in F^{\mathrm{E}}$, and internal reuse relationships, the internal demand distributions have to be adjusted with respect to the netting of gross requirements $D_{i,t}^{\text{gross}}$ with return quantities. The regular mode replenishment quantity $Q_{i,t}$ is given by the difference of direct gross requirements and direct returns:

$$D_{i,t}^{\text{gross}} = D_{i,t} \quad \forall i \in E,$$

$$D_{i,t}^{\text{gross}} = \sum_{j \in n(i)} a_{i,j} Q_{j,t} \quad \forall i \in A \cup P,$$

$$Q_{i,t} = D_{i,t}^{\text{gross}} - R_{i,t} \quad \forall i \in A \cup P \cup E.$$

The last equation neglects the case where returns exceed gross requirements. From the long run strategic perspective, R will be relatively small in comparison to D so that compensation from netting these excess returns with the next positive regular replenishment is achieved fast. The alternative assumption of immediate netting with pipeline orders, i.e. canceling outstanding orders, even overestimates real safety stock requirements.

In the case of external returns, $R_{i,t}$ denotes an independent random variable whereas $R_{i,t}$ for $i \in F^{\mathrm{I}}$ denotes a random variable that depends on the replenishment quantity $Q_{\text{bp}(i),t}$ of the by-product generating process $j = \text{bp}(i)$ with

$$R_{i,t} = a_{\text{bp}(i),i}^{\text{bp}} Q_{\text{bp}(i),t}.$$

The system of linear equations can be solved for the total requirements coefficients $g_{i,j}$. Then, given the external demand and return vector, internal net demand is

$$Q_{i,t} = \sum_{j \in E \cup F^{\mathrm{E}}} g_{i,j}(D_{j,t} - R_{j,t}). \tag{6}$$

The ordering and replenishment coordination for each stockpoint concerns at most two supply modes, the regular mode and the return reuse mode. Let L_i^{r} denote the lead time of material replenishment from the regular source, L_i^{bp} denote the lead time for recovered materials that are reused at i, and $L_{\text{bp}(i)}^{\mathrm{r}}$ denote the regular replenishment lead time of the by-product generating process $j = \text{bp}(i)$. Using the regular replenishment mode, net requirements $Q_{i,t}$ are replenished with lead time $L_i^{\mathrm{r}} = \max_{j \in v(i)}\{S_j\} + \lambda_i$. The reuse mode provides recovered returns $R_{i,t}$ after $L_i^{\text{bp}} = \lambda_i^{\text{bp}}$ periods for external and after $L_i^{\text{bp}} = L_{\text{bp(i)}}^{\mathrm{r}} + \lambda_i^{\text{bp}}$ periods for internal returns. The main replenishment mode is given by regular ordering. Therefore, the service time S_i of stockpoint i is assumed to be bounded by the regular replenishment lead time L_i^{r} ($S_i \leqslant L_i^{\mathrm{r}}$). The coverage time $T_i = L_i^{\mathrm{r}} - S_i$ has to be adjusted with respect to the two mode replenishment with (in general) offsetting lead times. The total replenishment lead time for the gross requirement of a stockpoint is given by the larger of the two lead times:

$$\tilde{T}_i(S_i, L_i^{\mathrm{r}}, L_i^{\text{bp}}) = \max\{L_i^{\mathrm{r}}; L_i^{\text{bp}}\} - S_i.$$

Therefore, the first period that may be completely influenced is $t + \tilde{T}_i + S_i$. In order to derive a safety stock size to cover against the uncertainty within the $\tilde{T}_i$ periods that accounts for the offsetting lead times, i.e. one of the two modes provides an earlier availability of material, the gross ($D_i^{\text{gross}}[\ ]$) and net ($D_i^{\text{net}}[\ ]$) demand characteristic over the time interval $[t + S_i; t + \tilde{T}_i + S_i]$ is examined. Depending on the values $S_i, L_i^{\mathrm{r}}, L_i^{\text{bp}}$ three cases are distinguished.

Case 1: $S_i \leqslant L_i^{\mathrm{r}} \leqslant L_i^{\text{bp}}$. If the service time is smaller than both replenishment lead times, safety stock coverage is required for both supply modes over gross requirements variations of $L_i^{\text{bp}} - S_i$ time periods ($[t + S_i; t + L_i^{\text{bp}}]$). Within this time interval, net regular replenishment orders over $L_i^{\text{bp}} - L_i^{\mathrm{r}}$ time periods ($[t + L_i^{\mathrm{r}}; t + L_i^{\text{bp}}]$) are received:

$$\begin{aligned} D_i^{\text{net}}[L_i^{\mathrm{r}}, L_i^{\text{bp}}, S_i] &= D_i^{\text{gross}}[t + S_i; t + L_i^{\text{bp}}] \\ &\quad - Q_i[t + L_i^{\mathrm{r}}; t + L_i^{\text{bp}}], \\ &= D_i^{\text{gross}}[t + S_i; t + L_i^{\mathrm{r}}] \\ &\quad + R_i[t + L_i^{\mathrm{r}}; t + L_i^{\text{bp}}]. \end{aligned}$$

Case 2: $S_i \leqslant L_i^{\text{bp}} \leqslant L_i^{\mathrm{r}}$. Safety stock coverage is required for both supply modes. Gross requirements are taken into account over $L_i^{\mathrm{r}} - S_i$ time periods ($[t + S_i; t + L_i^{\mathrm{r}}]$). In this case, reuse return flows over $L_i^{\mathrm{r}} - L_i^{\text{bp}}$ time periods $[(t + L_i^{\text{bp}}; t + L_i^{\mathrm{r}}])$ are received within the coverage time interval:

$$\begin{aligned} D_i^{\text{net}}[L_i^{\mathrm{r}}, L_i^{\text{bp}}, S_i] &= D_i^{\text{gross}}[t + S_i; t + L_i^{\mathrm{r}}] \\ &\quad - R_i[t + L_i^{\text{bp}}; t + L_i^{\mathrm{r}}], \end{aligned}$$

$$= D_i^{\text{gross}}[t + S_i; t + L_i^{\text{bp}}] + Q_i[t + L_i^{\text{bp}}; t + L_i^{\text{r}}].$$

Case 3: $L_i^{\text{bp}} \leqslant S_i \leqslant L_i^{\text{r}}$. Safety stock requirements exist only for the regular replenishment mode. Earlier reuse product availability can be exploited for buffering purposes. Gross requirements over $L_i^{\text{r}} - S_i$ time periods ($[t + S_i; t + L_i^{\text{r}}]$) are netted with received return flows over $L_i^{\text{r}} - L_i^{\text{bp}}$ time periods ($[t + L_i^{\text{bp}}; t + L_i^{\text{r}}]$). This expression can equivalently be expressed by net regular replenishment orders over $L_i^{\text{r}} - S_i$ time periods less available reuse material over $S_i - L_i^{\text{bp}}$ periods:

$$\begin{aligned} D_i^{\text{net}}[L_i^{\text{r}}, L_i^{\text{bp}}, S_i] &= D_i^{\text{gross}}[t + S_i; t + L_i^{\text{r}}] \\ &\quad - R_i[t + L_i^{\text{bp}}; t + L_i^{\text{r}}], \\ &= Q_i[t + S_i; t + L_i^{\text{r}}] \\ &\quad - R_i[t + L_i^{\text{bp}}; t + S_i]. \end{aligned}$$

The safety stock quantity that follows from the net demand over $\tilde{T}_i$ can be expressed by the standard deviation of net demand within this time interval. In the safety stock formula, $\sigma_i\sqrt{T_i}$ has to be replaced by $\sigma_i^{\text{net}}(L_i^{\text{r}}, L_i^{\text{bp}}, S_i) = \sqrt{V(D_i^{\text{net}}(L_i^{\text{r}}, L_i^{\text{bp}}, S_i))}$. When computing these standard deviations, care has to be given to evaluate variances of no time overlapping demand and return expressions.

3.3. Solution principle

By incorporating the adjustments with respect to external and internal returns into the optimization formulation introduced in Section 2, the extensions only change the objective function but not the feasible region. Therefore, the extreme points of the solution set are identical to the ones discussed for the basic model. The additional complexity arises from the separation into three net demand cases. In order to apply the subsequent safety stock formula, the feasible region with respect to a single service time can be divided into three parts, representing the three different net demand situations. Because the objective function is concave with respect to the service time over each of the three regions, a similar extreme point property can be exploited for the development of a solution method. In addition to the extreme points of the feasible region ($S_i \in \{0; \lambda_i + L_i^{\text{r}}\}$), the separation by additional linear constraints provides further extreme points $S_i = L_i^{\text{bp}}$ and $L_i^{\text{r}} = L_i^{\text{bp}}$ which must be taken into account. For $S_i = L_i^{\text{bp}}$, the service time quoted to successors is identical to the reuse lead time. These additional solutions are only possible for stockpoints i with external returns or forward by-product reuse. In case of $L_i^{\text{r}} = L_i^{\text{bp}}$, coverage and service times are chosen in a way that both replenishment lead times become identical and therefore are fully synchronized.

A first approach to find a (heuristic) solution to the extended optimization problem is to neglect additional synchronization options that add a large amount on computational complexity and to restrict to bang–bang policies, that is to cover nothing or the entire regular replenishment lead time. In this case, the combinatorial solution principle of Section 2 applies except for the safety stock size adjustments with respect to σ_i^{net} in the evaluation of a given safety stock allocation representation $\boldsymbol{x}$. In order to find the optimal solution, a generalized encoding scheme that accounts for the additional synchronization options is necessary. For the safety stock allocation representation vector, let $x_i = 0$ denote zero coverage and $x_i = 1$ denote that the maximum of L_i^{r} and L_i^{bp} is covered. In addition, $x_i = 2$ indicates that stockpoint i holds safety stock in order to guarantee a service time identical to the reuse lead time ($S_i = L_i^{\text{bp}}$), and $x_i = -j$ indicates that the service time of i is chosen in a way that synchronizes both lead times of j, i.e. $L_j^{\text{r}} = L_j^{\text{bp}}$. Because of logical interaction between synchronization and extreme point properties, not all combinations of the above representations are relevant. If $x_i = -j$, no successor l on a path between i and j can cover the entire replenishment lead time ($x_l = 1$) or synchronize for the same destination reuse node $x_l = -j$. With respect to the coverage time determination from the representation vector, the lead time calculations are adjusted:

$$L_i = \lambda_i + \min\left\{L_i^{\text{r}}, \max_{j \in v(i)} \{\tilde{m}_j\}\right\}. \tag{7}$$

The values $\tilde{m}_j$represent the service time outcome of predecessor j depending on x_j, which due to our assumption cannot exceed L_i^{r}. They are defined as follows:

$$x_i = 0 : \tilde{m}_i = L_i^{\mathrm{r}},$$

$$x_i = 1 : \tilde{m}_i = 0,$$

$$x_i = 2 : \tilde{m}_i = \begin{cases} \lambda_i^{\mathrm{bp}} & i \in F^{\mathrm{E}}, \\ L_{\mathrm{bp}(i)}^{\mathrm{r}} + \lambda_i^{\mathrm{bp}} & i \in F^{\mathrm{I}}, \end{cases}$$

$$x_i = -j : \tilde{m}_i = \begin{cases} \left(L_j^{\mathrm{bp}} - \max\limits_{w \in W(i,j)} \left\{ \sum_{m \in w \setminus \{i\}} \lambda_m \right\} \right)^+ & (j \in F^{\mathrm{E}}) \vee (j \in F^{\mathrm{I}} \wedge N(i) \cap \mathrm{bp}(j) = \emptyset, \\ \left(L_j^{\mathrm{r}} - \max\limits_{w \in W(i,j)} \left\{ \sum_{m \in w \setminus \{i\}} \lambda_m \right\} - \lambda_j^{\mathrm{bp}} \right)^+ & j \in F^{\mathrm{I}} \wedge N(i) \cap \mathrm{bp}(j) \neq \emptyset. \end{cases}$$

If $x_i = 0$, no coverage is planned and the entire (regular) replenishment lead time has to be covered by all successors. If $x_i = 1$, all demand uncertainty is covered and no coverage obligations are postponed to succeeding stockpoints. In case of $x_i = 2$, the reuse mode lead time defines the service time and the difference (if positive) between reuse mode and regular mode time is covered. If i offers a service time in order to synchronize both mode lead times of j, the calculation depends on whether (1) j is a stockpoint with external returns or j is an internal reuse stockpoint and i lies on the regular replenishment path of j or (2) i lies on the by-product supply path to j. In case (1), the service time is chosen in a way that service time plus the maximum cumulative time of succeeding processes equals the reuse mode lead time of j. In case (2), the sum of service time of i, maximum cumulative processing time, and recovery time equals the regular mode replenishment time. Note that the required L_i^{r} values are known in the backward reuse case and can be obtained in the forward reuse case due to the restriction that forward reuse is only possible in a different product line. For implementation of this extended solution representation within a local search framework, the neighborhood definition has to undergo some adjustments. Where the representation presented in Section 2 offers only a single transition $(1 - x_i)$ for each stockpoint, the extended version additionally offers transitions to and from synchronization solutions $x_i = 2$ and $x_i = -j$.

4. Numerical example

In this section the impact of incorporating returns into the strategic safety stock placement consideration is illustrated by an example. The original network is a four-stage assembly supply chain with two purchasing processes (1, 2) and a single final product (5) (Fig. 2). The dotted line represents an internal backward reuse arc. Besides the desired manufacturing of product 4, the corresponding process yields a by-product outcome that can be reused at stockpoint 2.

Net demand analysis for the second stockpoint with the regular supply mode ($L_2^{\mathrm{r}} = \lambda_2$) and the reuse mode ($L_2^{\mathrm{bp}} = \max\{S_1; S_3\} + \lambda_4 + \lambda_2^{\mathrm{bp}}$) gives the following result:

Case 1: $S_2 \leqslant L_2^{\mathrm{r}} \leqslant L_2^{\mathrm{bp}}$.

$$D_2^{\mathrm{net}}[L_2^{\mathrm{r}}, L_2^{\mathrm{bp}}, S_2] = a_{2,5} D_5[t + S_2; t + L_2^{\mathrm{r}}] + a_{4,2}^{\mathrm{bp}} a_{4,5} D_5[t + L_2^{\mathrm{r}}; t + L_2^{\mathrm{bp}}].$$

Case 2: $S_2 \leqslant L_2^{\mathrm{bp}} \leqslant L_2^{\mathrm{r}}$.

$$D_2^{\mathrm{net}}[L_2^{\mathrm{r}}, L_2^{\mathrm{bp}}, S_2] = a_{2,5} D_5[t + S_2; t + L_2^{\mathrm{bp}}] + (a_{2,5} - a_{4,2}^{\mathrm{bp}}) D_5[t + L_2^{\mathrm{bp}}; t + L_2^{\mathrm{r}}].$$

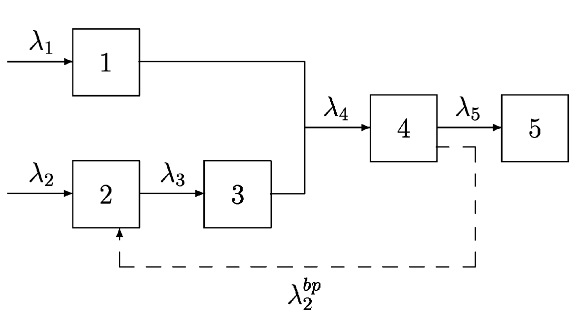

Fig. 2. Example network with one return flow arc.

Case 3: $L_2^{bp} \leqslant S_2 \leqslant L_2^r$.

$$D_2^{net}[L_2^r, L_2^{bp}, S_2] = (a_{2,5} - a_{4,2}^{bp})D_5[t + S_2; t + L_2^r] - a_{4,2}^{bp} D_5[t + L_2^{bp}; t + S_2]$$

with $a_{2,5} = a_{2,3}a_{3,4}a_{4,5}$. In the following, different parameters for the system are analyzed in order to demonstrate three effects. In Example 1, it is shown that the optimal solution can be of the synchronization type. Example 2 presents the natural case where the incorporation of reuse creates additional system inventory and Example 3 shows the opposite – that systems with reuse can require less safety stock.

Example 1. Optimality of a synchronization solution (Table 1).

Final product demand is normally distributed with $\mu_5 = 10$, $\sigma_5 = 3$. All direct material requirements coefficients are $a_{i,j} = 1$ except for $a_{3,4} = 1.5$ and the by-product coefficient $a_{4,2}^{bp} = 0.4$. The reuse processing time λ_2^{bp} is zero. The coefficients for the processes/stockpoints $i = 1, 2, 3, 4, 5$ are processing times λ_i (2, 4, 1, 1, 2), service levels α_i (0.99, 0.99, 0.90, 0.90, 0.95) and value added holding costs h_i (3, 3, 3,5, 8.5, 11.5).

The traditional system without returns yields a safety stock policy with lower costs than the policy under return reuse. As a consequence of the optimal solution of the new system with reuse flow, $L_2^r = L_2^{bp} = 4$ holds and additional inventory at stockpoint 2 is avoided. To illustrate the result in more detail, this solution is compared to the best bang–bang solution for the new system and to the solution with the same allocation pattern as for the traditional solution. The best bang–bang solution is given by $\boldsymbol{x} = (0, 0, 1, 0, 1)$, $\boldsymbol{T} = (0, 0, 3, 0, 5)$ and $\boldsymbol{SST} = (0, 2.79, 9.99, 0, 11.04)$ with $C = 170.26$.

The fact that the two mode lead times of stockpoint 2 ($L_2^r = 4$, $L_2^{bp} = 3$) are not synchronized creates additional inventory at this stockpoint. The same effect shows up when using the optimal allocation pattern of the traditional system. The only difference is that additional inventory of 3.95 units is created at stockpoint 2 ($C = 172.18$).

Example 2. Optimal safety stock solution of the system without reuse arc dominates system with reuse (Table 2).

The probability distribution parameters are again $\mu_5 = 10$, $\sigma_5 = 3$. Direct material requirements are given by $a_{i,j} = 1$, except for $a_{3,4} = 1.5$ and $a_{4,2}^{bp} = 0.5$. Returns processing requires $\lambda_2^{bp} = 1$ period. Processing times for the regular processes λ_i are (3, 1, 1, 1, 2), service levels α_i (0.95, 0.95, 0.95, 0.95, 0.95) and holding cost rates h_i (1, 1, 2, 4, 4.5). Implementing the optimal allocation pattern of the traditional assembly system, additional inventory of 4.94 ($C = 59.34$) is created at the second stockpoint without opening the opportunity to lower safety inventory at the other stockpoints.

Example 3. Optimal safety stock solution of reuse system dominates the traditional system (Table 3).

Table 1
Optimization results for Example 1

	Old system					New system				
	1	2	3	4	5	1	2	3	4	5
μ_i	10	15	15	10	10	10	11	15	10	10
σ_i	3	4.5	4.5	3	3	3	3.3	4.5	3	3
x_i	0	0	0	1	1	0	0	2	0	1
T_i	0	0	0	6	2	0	0	2	0	6
SST_i	0	0	0	9.42	6.98	0	0	8.16	0	12.09
C	160.33					167.58				

Table 2
Optimization results for Example 2

	Old system					New system				
	1	2	3	4	5	1	2	3	4	5
μ_i	10	15	15	10	10	10	10	15	10	10
σ_i	3	4.5	4.5	3	3	3	3	4.5	3	3
x_i	0	0	0	0	1	1	0	0	0	1
T_i	0	0	0	0	6	1	0	0	0	5
SST_i	0	0	0	0	12.24	4.94	4.27	0	0	11.04
C	54.04					58.87				

Table 3
Optimization results for Example 3

	Old system					New system				
	1	2	3	4	5	1	2	3	4	5
μ_i	10	15	15	10	10	10	10	15	10	10
σ_i	3	4.5	4.5	3	3	3	3	4.5	3	3
x_i	0	1	0	0	1	0	1	0	0	1
T_i	0	4	0	0	4	0	4	0	0	4
SST_i	0	14.81	0	0	9.87	0	13.74	0	0	9.87
C	133.26					132.20				

Customers demand, material requirements coefficients, by-product outcome, return processing time and service levels are the same as for Example 2. The processing times λ_i are (1, 4, 1, 1, 2) and the holding cost rates h_i are (2, 1, 4, 8, 12). The effect that the return recovery system is able to operate with less inventory than the corresponding system without return integration is achieved by the coordination of material flows in a way that the return material becomes available ($L_i^{\mathrm{bp}} = 3$) before the regular replenishments ($L_i^{\mathrm{r}} = 4$) and this stock can be used for reducing safety inventory.

5. Conclusion and further research

The integration of external and internal product returns and their recovery for reuse as substitutes for regularly replenished/manufactured items complicates the materials coordination in a general supply chain network. Nevertheless, an extension of a general strategic safety stock placement approach is possible to incorporate these flows as outlined in this paper. Additional inventories arise from the return reuse activities. Depending on the specification of the supply chain characteristics in processing times, service levels, and holding costs, the numerical examples show that in some cases excessive inventory is unavoidable if a perfect synchronization of the multiple supply modes is too expensive. Under different data settings synchronization is possible that enables the additional inventory to be used in order to substitute safety stocks.

In this paper, the pure safety stock cost effect of reverse logistics activities is analyzed. The avoidance of disposal cost together with reduced external material supply quantities are other cost determinants that make reverse logistics attractive. From a strategic point of view, when (re-)designing supply chains with respect to reverse logistics processes, additional safety stock requirements have to be traded off with disposal and purchase cost savings. But even if there is no material cost advantage of product recovery, a synchronized materials coordination that enables a safety stock reduction may exist and make this option profitable.

Further extensions are desirable with respect to the incorporation of multiple reuse sources (i.e. the same by-product appears at different processes), recovery options and allocation (i.e. if a single return or by-product can be reused at several processes), disposal of excess return quantities and stock holding of returns/by-products instead of immediate processing for reuse, and random recovery processing times as well as random yield of remanufacturing and disassembly operations.

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