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Vorlesung Drahtlose Breitbandkommunikationssysteme

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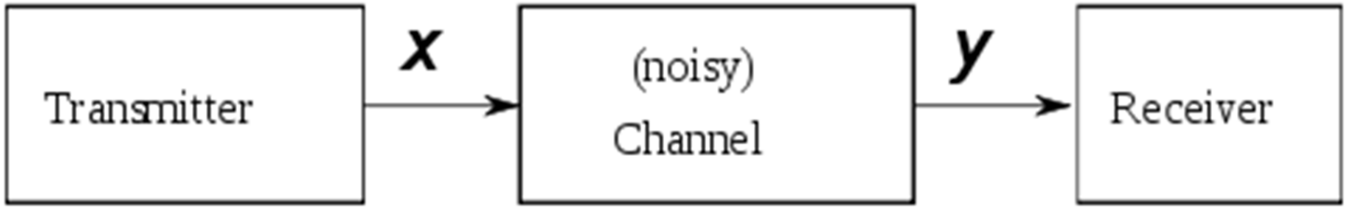
Channel Capacity: Shannon Theorem [Wikipedia]



- The **Shannon limit** or **Shannon capacity** of a communications channel is the theoretical maximum information transfer rate of the channel, for a particular noise level.
- Stated by Claude Shannon in 1948 (with sketchy proof)
- The first rigorous proof by Amiel Feinstein in 1954.
- The Shannon theorem states that given a noisy channel with channel capacity *C* and information transmitted at a rate *R*, then if R<C there exist codes that allow the probability of error at the receiver to be made arbitrarily small.
  - ➢ This means that, theoretically, it is possible to transmit information nearly without error at any rate below a limiting rate, *C*.

Shannon-Hartley Theoreme (for AWGN)

$$C = B \log_2 \left(1 + \frac{S}{N}\right)$$

C: Channel capacity (max. data rate) in bits per second;

B: Bandwidth of the channel in Hertz;

S: Total received signal power over the bandwidth (in case of a modulated signal, often denoted *C*, i.e. modulated carrier), measured in Watt or $(\text{Volt})^2$;

N: Total noise or interference power over the bandwidth, measured in Watt or $(\text{Volt})^2$; and

S/N: Signal-to-noise ratio (SNR) or the carrier-to-noise ratio (CNR) of the communication signal to the Gaussian noise interference expressed as a linear power ratio (not as logarithmic decibels).

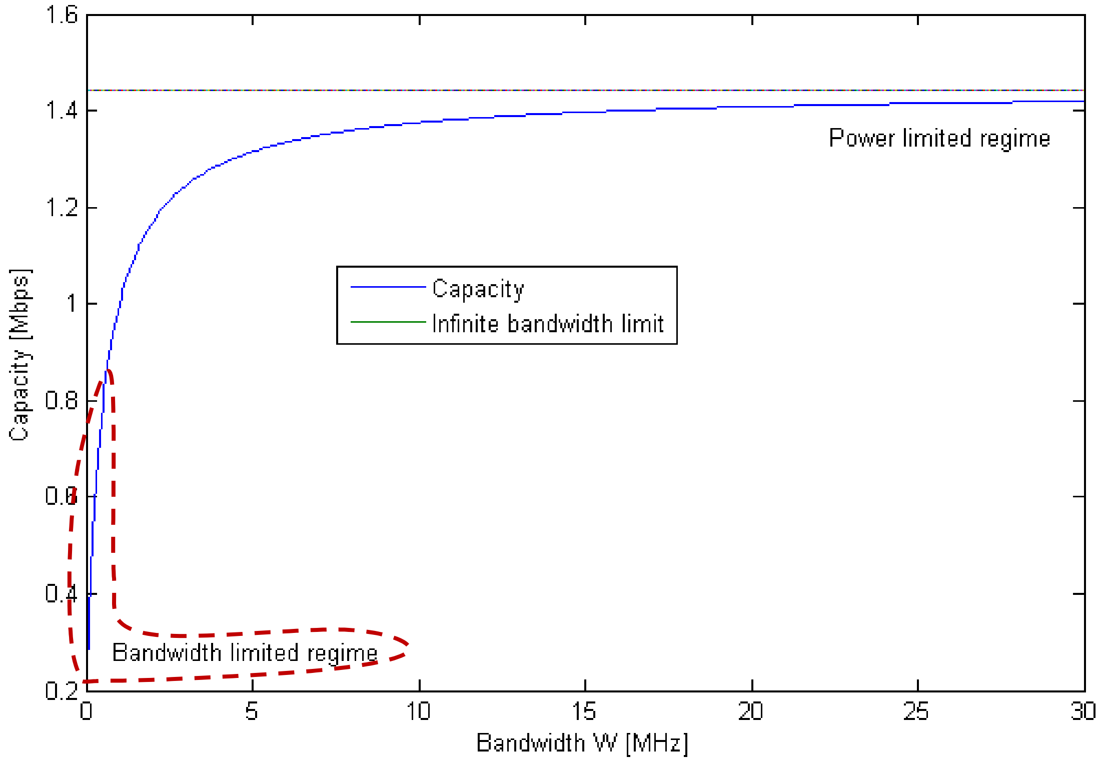
AWGN Channel Capacity

$$C_{awgn} = W \log_2 \left(1 + \frac{\bar{P}}{N_0 W} \right)$$ [bits/s]

- If the average received power is $\bar{P}$ ***[W]*** and the noise power spectral density is N_0 ***[W/Hz]***, the AWGN channel capacity is C_{awgn}
- ***W*** is the used bandwidth (~B)
- $\bar{P}/(N_0 * W)$ is the signal-to-noise-ratio (SNR) at the receiver

Channel Capacity in Bandwidth-Limited Regime

When the SNR is large (SNR >> 0 dB): $C \approx W \log_2 \frac{\bar{P}}{N_0 W}$



$\frac{\bar{P}}{N_o} = 10^6$

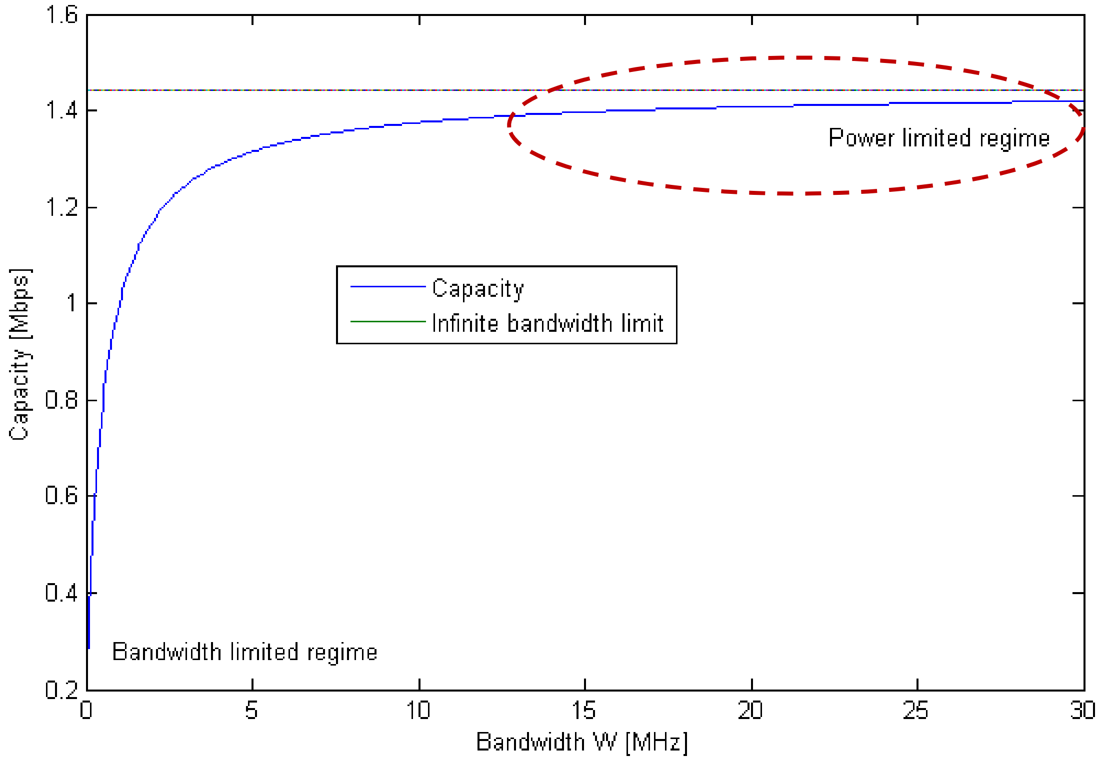
- C is logarithmic in power and approximately linear in bandwidth

-> *bandwidth-limited regime*

Channel Capacity in Power-Limited Regime

When the SNR is small (SNR << 0 dB): $C \approx \frac{\bar{P}}{N_0} \log_2 e$

(W -> ∞)



$$\frac{\bar{P}}{N_o} = 10^6$$

- C is linear in power but insensitive to bandwidth

-> *Power-limited regime*

Approximations

For large or small and constant signal-to-noise ratios, the capacity formula can be approximated:

- If S/N >> 1, then: $C \approx 0.332 \cdot B \cdot \mathrm{SNR\ (in\ dB)}$

  where: $\mathrm{SNR\ (in\ dB)} = 10 \log_{10} \frac{S}{N}.$

- Similarly, if S/N << 1, then: $C \approx 1.44 \cdot B \cdot \frac{S}{N}.$

In this low-SNR approximation, capacity is independent of bandwidth if the noise is white, of spectral density N_0 [W/Hz]. In this case the total noise power is B * N_0 and the channel capacity becomes:

$$C \approx 1.44 \cdot \frac{S}{N_0}$$

Source: http://en.wikipedia.org/wiki/Shannon-Hartley_theorem

Frequency-dependent (colored noise) case

A generalization of the channel capacity equation for the case where the noise is not white (or that the S/N is not constant with frequency over the bandwidth) is obtained by treating the channel as many narrow, independent Gaussian channels in parallel:

$$C = \int_0^B \log_2 \left(1 + \frac{S(f)}{N(f)}\right) df$$

C: channel capacity in bits per second;

B: bandwidth of the channel in Hz;

S(f): signal power spectrum

N(f): noise power spectrum

f: frequency in Hz (within B)

Source: http://en.wikipedia.org/wiki/Shannon-Hartley_theorem

Noise Power Spectral Density N_o

$$C_{awgn} = W \log_2 \left(1 + \frac{\bar{P}}{N_0 W}\right)$$

$N_o = k*T$ [W/Hz]

k: Boltzmann constant [J/K] (= 1.38×10^{-23} J/K)

T: Temperature [K] (~ 300 K @ room temp.)

$N_o = k*T = 1.38 \times 10^{-23}$ J/K * 300 K

$= 4.14 * 10^{-21}$ J

$= 4.14 * 10^{-21}$ Ws (for room temperature)

Example I (Telephone Modem)

- Example I:
  - Consider the operation of a modem on an ordinary telephone line. The SNR is usually about 1000 (30 dB). The bandwidth is 3.4 kHz.

  Therefore:

  $C = 3.4\ \text{kHz} * \log_2(1 + 1000)\ \text{bit}$

  $= 3400\ \text{Hz} * 9.97\ \text{bit}$

  $\underline{= 34\ \text{kbit/s}}$

Source: Rahul Mangharam, Lecture ESE680: Wireless Sensor Networks

- Example II:
  - Consider the operation of a WLAN modem IEEE802.11a. The channel spacing is 20 MHz and the used bandwidth is approximately 16 MHz. The SNR for a good channel and low distance can be about 100 (= 20 dB).

Therefore:

$$C = 16\text{ MHz} * \log_2(1 + 100)\text{ bit}$$
$$\approx 16 * 10^6\text{ Hz} * 6.66\text{ bit}$$
$$\underline{= 106.6\text{ Mbit/s}}$$

Friis free-space path-loss equation

Received Power:

- $P_{rcvd}(d) = P_{tx} + G_t + G_r - PL$ [all in dB]

P_{tx} = Transmit Power
G_t = Gain of Transmit Antenna
G_r = Gain of Receive Antenna
PL = Path Loss

Path loss (PL) in free space:

$PL(a) = (4 * \pi * d * f/c)^2$ (direct form)

$PL(\log) = 10 * \log_{10}(4 * \pi * d * f/c)^2 = 20 * \log_{10}(f) + 20 * \log_{10}(d) + 20 * \log_{10}(4\pi/c)$

(logarithmic form)

d_0

- d: Distance between Sender and Receiver
- f: Carrier Frequency
- c: Speed of light (~ 300 000 km/s)

Friis Equation: Derivation @ board

- Friis Equation
- Link Budget Calculation