Für alle $\Gamma, \Gamma' \subseteq_e \mathsf{FO}[\sigma]$, alle $\varphi, \psi, \chi \in \mathsf{FO}[\sigma]$, alle $t, u \in \mathsf{T}_\sigma$ und alle $x, y \in \mathsf{VAR}$ betrachten wir die folgenden Sequenzenregeln:

- *Voraussetzungsregel* (V), *Erweiterungsregel* (E):

$$\frac{}{\Gamma, \varphi \vdash \varphi} \qquad \frac{\Gamma \vdash \varphi}{\Gamma' \vdash \varphi} \quad \text{falls } \Gamma \subseteq \Gamma'$$

- *Fallunterscheidungsregel* (FU), *Widerspruchsregel* (W):

$$\frac{\begin{array}{l}\Gamma, \psi \vdash \varphi \\ \Gamma, \neg\psi \vdash \varphi\end{array}}{\Gamma \vdash \varphi} \qquad \frac{\begin{array}{l}\Gamma \vdash \psi \\ \Gamma \vdash \neg\psi\end{array}}{\Gamma \vdash \varphi} \quad (\text{für alle } \varphi \in \mathsf{FO}[\sigma])$$

- $\wedge$-*Einführung* ($\wedge$S), ($\wedge$A$_1$), ($\wedge$A$_2$) *und* $\vee$-*Einführung* ($\vee$S$_1$), ($\vee$S$_2$), ($\vee$A):

$$\frac{\begin{array}{l}\Gamma \vdash \varphi \\ \Gamma \vdash \psi\end{array}}{\Gamma \vdash (\varphi \wedge \psi)} \qquad \frac{\Gamma, \varphi \vdash \chi}{\Gamma, (\varphi \wedge \psi) \vdash \chi} \quad \frac{\Gamma, \psi \vdash \chi}{\Gamma, (\varphi \wedge \psi) \vdash \chi} \qquad \frac{\Gamma \vdash \varphi}{\Gamma \vdash (\varphi \vee \psi)} \quad \frac{\Gamma \vdash \psi}{\Gamma \vdash (\varphi \vee \psi)} \qquad \frac{\begin{array}{l}\Gamma, \varphi \vdash \chi \\ \Gamma, \psi \vdash \chi\end{array}}{\Gamma, (\varphi \vee \psi) \vdash \chi}$$

- $\forall$-*Einführung und* $\exists$-*Einführung* ($\forall$A), ($\exists$S), ($\forall$S), ($\exists$A) ($y \notin \text{frei}(\Gamma, \forall x\varphi)$):

$$\frac{\Gamma, \varphi\frac{t}{x} \vdash \psi}{\Gamma, \forall x\varphi \vdash \psi} \qquad \frac{\Gamma \vdash \varphi\frac{t}{x}}{\Gamma \vdash \exists x\varphi} \qquad \frac{\Gamma \vdash \varphi\frac{y}{x}}{\Gamma \vdash \forall x\varphi} \qquad \frac{\Gamma, \varphi\frac{y}{x} \vdash \psi}{\Gamma, \exists x\varphi \vdash \psi}$$

- *Reflexivität der Gleichheit* (G) *und Substitutionsregel* (S):

$$\frac{}{\Gamma \vdash t{=}t} \qquad \frac{\Gamma \vdash \varphi\frac{t}{x}}{\Gamma, t{=}u \vdash \varphi\frac{u}{x}}$$

Ableitbare Regeln:

- *Kettenschluss* (KS), *Disjunktiver Syllogismus* (DS) *und Modus Ponens* (MP)

$$\frac{\begin{array}{l}\Gamma \vdash \varphi \\ \Gamma, \varphi \vdash \psi\end{array}}{\Gamma \vdash \psi} \qquad \frac{\begin{array}{l}\Gamma \vdash \neg\varphi \\ \Gamma \vdash (\varphi \vee \psi)\end{array}}{\Gamma \vdash \psi} \qquad \frac{\begin{array}{l}\Gamma \vdash \varphi \\ \Gamma \vdash (\varphi \to \psi)\end{array}}{\Gamma \vdash \psi}$$

- *Quantorenaustauschregeln* (QA) *und Kontrapositionen* (KP)

$$\frac{\Gamma, \varphi \vdash \psi}{\Gamma, \neg\psi \vdash \neg\varphi} \qquad \frac{\Gamma, \varphi \vdash \neg\psi}{\Gamma, \psi \vdash \neg\varphi} \qquad \frac{\Gamma \vdash \neg\forall x\varphi}{\Gamma \vdash \exists x\neg\varphi} \qquad \frac{\Gamma \vdash \forall x\neg\varphi}{\Gamma \vdash \neg\exists x\varphi}$$

$$\frac{\Gamma, \neg\varphi \vdash \neg\psi}{\Gamma, \psi \vdash \varphi} \qquad \frac{\Gamma, \neg\varphi \vdash \psi}{\Gamma, \neg\psi \vdash \varphi} \qquad \frac{\Gamma \vdash \neg\exists x\varphi}{\Gamma \vdash \forall x\neg\varphi} \qquad \frac{\Gamma \vdash \exists x\neg\varphi}{\Gamma \vdash \neg\forall x\varphi}$$

- *Symmetrie und Transitivität der Gleichheit* (SG), (TG) *und Verträglichkeiten* (VR), (VF)

$$\frac{\Gamma \vdash t{=}u}{\Gamma \vdash u{=}t} \qquad \frac{\begin{array}{l}\Gamma \vdash t_1{=}t_2 \\ \Gamma \vdash t_2{=}t_3\end{array}}{\Gamma \vdash t_1{=}t_3} \qquad \frac{\begin{array}{l}\Gamma \vdash R(t_1, \dots, t_r) \\ \Gamma \vdash t_1{=}u_1 \\ \vdots \\ \Gamma \vdash t_r{=}u_r\end{array}}{\Gamma \vdash R(u_1, \dots, u_r)} \qquad \frac{\begin{array}{l}\Gamma \vdash t_1{=}u_1 \\ \vdots \\ \Gamma \vdash t_r{=}u_r\end{array}}{\Gamma \vdash f(t_1, \dots, t_r){=}f(u_1, \dots, u_r)}$$