

Einführung in die Datenbanktheorie

Wintersemester 2020/21

Übungsblatt 4

Zu Bearbeiten bis zur Übungsstunde am *9. Dezember 2020*

Aufgabe 1:

Betrachten Sie die beiden folgenden Tableaunanfragen $Q_1 := (\mathbf{T}', u')$ und $Q_2 := (\mathbf{T}'', u'')$, wobei a und b Konstanten sind, $u' = u'' = ()$, sowie

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| $\mathbf{T}'(R)$ <table style="border-collapse: collapse; margin-left: 10px;"> <tr><td style="border-right: 1px solid black; border-bottom: 1px solid black; width: 30px;"></td><td style="border-bottom: 1px solid black; width: 30px;"></td><td style="border-bottom: 1px solid black; width: 30px;"></td><td style="border-bottom: 1px solid black; width: 30px;"></td></tr> <tr><td style="border-right: 1px solid black;">x₁</td><td>x₂</td><td>x₃</td><td></td></tr> <tr><td style="border-right: 1px solid black;">x₂</td><td>x₂</td><td>x₃</td><td></td></tr> <tr><td style="border-right: 1px solid black;">a</td><td>x₂</td><td>x₄</td><td></td></tr> <tr><td style="border-right: 1px solid black;">x₂</td><td>x₆</td><td>x₃</td><td></td></tr> </table> | | | | | x ₁ | x ₂ | x ₃ | | x ₂ | x ₂ | x ₃ | | a | x ₂ | x ₄ | | x ₂ | x ₆ | x ₃ | | $\mathbf{T}''(R)$ <table style="border-collapse: collapse; margin-left: 10px;"> <tr><td style="border-right: 1px solid black; border-bottom: 1px solid black; width: 30px;"></td><td style="border-bottom: 1px solid black; width: 30px;"></td><td style="border-bottom: 1px solid black; width: 30px;"></td><td style="border-bottom: 1px solid black; width: 30px;"></td></tr> <tr><td style="border-right: 1px solid black;">x₂</td><td>x₂</td><td>x₃</td><td></td></tr> <tr><td style="border-right: 1px solid black;">a</td><td>x₂</td><td>x₄</td><td></td></tr> </table> | | | | | x ₂ | x ₂ | x ₃ | | a | x ₂ | x ₄ | |
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| x ₂ | x ₂ | x ₃ | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| a | x ₂ | x ₄ | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
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| x ₄ | x ₂ | x ₂ | x ₃ | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
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| x ₄ | b | x ₁ | x ₅ | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |

Ziel der Aufgabe ist es zu entscheiden, ob $Q_1 \sqsubseteq Q_2$ bzw. $Q_2 \sqsubseteq Q_1$ gilt.

- (a) Geben Sie die kanonischen Tupel $u_{Q_2}^{Q_1}$ und $u_{Q_1}^{Q_2}$, sowie die kanonischen Datenbanken $\mathbf{I}_{Q_2}^{Q_1}$ und $\mathbf{I}_{Q_1}^{Q_2}$ an.
- (b) Entscheiden Sie, ob $Q_1 \sqsubseteq Q_2$ gilt und ob $Q_2 \sqsubseteq Q_1$ gilt.
- (c) Gibt es einen Homomorphismus von Q_1 auf Q_2 ? Gibt es einen Homomorphismus von Q_2 auf Q_1 ? Geben Sie jeweils einen Homomorphismus an oder begründen Sie, warum er nicht existiert.

Aufgabe 2:

Das Datenbankschema bestehe aus zwei Relationsnamen R und S der Stelligkeiten 4 und 3. Wenden Sie den Algorithmus aus dem Beweis von Theorem 3.39 (a) an, um die folgende Tableaunanfrage $Q = (\mathbf{T}, ())$ (wobei a eine Konstante ist und y_1, \dots, y_5 verschiedene Variablen sind) zu minimieren.

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| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| y ₁ | a | y ₄ | a | | | | | | | | | | | | | | | | | | | | | | | | | | |
| y ₄ | y ₁ | y ₄ | y ₁ | | | | | | | | | | | | | | | | | | | | | | | | | | |
| y ₁ | y ₃ | y ₂ | a | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| y ₃ | y ₂ | a | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| a | y ₄ | a | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| a | y ₂ | y ₅ | | | | | | | | | | | | | | | | | | | | | | | | | | | |

Aufgabe 3:

Betrachten Sie die beiden folgenden regelbasierten konjunktiven Anfragen Q_1 und Q_2 (wobei a , b und c Konstanten sind):

$$\text{Ans}() \leftarrow R(a, x_3, x_5, x_2), R(x_1, a, x_2, x_4), S(x_3, x_4, x_1), S(x_3, x_2, x_1)$$

$$\text{Ans}() \leftarrow R(y_1, a, y_4, y_4), R(a, a, b, y_4), R(y_1, y_1, b, y_4), S(a, y_4, a), S(a, y_4, y_1)$$

- (a) Stellen Sie Q_1 und Q_2 als Tableau-Anfragen Q'_1 und Q'_2 dar.
- (b) Geben Sie die kanonischen Tupel $u_{Q'_2}^{Q'_1}$ und $u_{Q'_1}^{Q'_2}$, sowie die kanonischen Datenbanken $\mathbf{I}_{Q'_2}^{Q'_1}$ und $\mathbf{I}_{Q'_1}^{Q'_2}$ an.
- (c) Gibt es einen Homomorphismus von Q'_1 auf Q'_2 bzw. einen Homomorphismus von Q'_2 auf Q'_1 ? Geben Sie jeweils einen Homomorphismus an oder begründen Sie, warum er nicht existiert.
- (d) Entscheiden Sie, ob $Q_1 \sqsubseteq Q_2$ bzw. $Q_2 \sqsubseteq Q_1$ gilt.

Aufgabe 4:

Zeigen Sie für Theorem 3.30 die Richtung $(d) \rightsquigarrow (a)$, d.h. zeigen Sie, dass zu jeder erfüllbaren Anfrage der SPC-Algebra eine regelbasierte konjunktive Anfrage existiert, welche die gleiche Anfragefunktion ausdrückt, und dass ein Algorithmus existiert, der selbige in polynomieller Zeit berechnet.