KAPITEL 3 BERUHT AUF EINEM AUSZUG AUS DEM FOLGENDEN VORTRAG:

A tutorial on Database Theory and a talk on database query answering under updates

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Example database and two queries

Movie	
Name	Actor
Alien	Sigourney Weaver
Blade Runner	Harrison Ford
Blade Runner	Sean Young
Brazil	Jonathan Pryce
Brazil	Kim Greist
Casablanca	Humphrey Bogart
Casablanca	Ingrid Bergmann
Gravity	Sandra Bullock
Gravity	George Clooney
Resident Evil	Milla Jovovich
Terminator	Arnold Schwarzenegger
Terminator	Linda Hamilton
Terminator	Michael Biehn
⋮	⋮

Programme		
Cinema	Movietitle	Time
Babylon	Casablanca	17:30
Babylon	Gravity	20:15
Casablanca	Blade Runner	15:30
Casablanca	Alien	18:15
Casablanca	Blade Runner	20:30
Casablanca	Resident Evil	20:30
Kino International	Casablanca	18:00
Kino International	Brazil	20:00
Kino International	Brazil	22:00
Moviemento	Gravity	17:00
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a db	:	$D = (M^D, P^D)$, where
M^D	:	a finite subset of $\mathbf{dom}^2$
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Query evaluation

Consider a query language L
(e.g., SQL, conjunctive queries CQ, first-order logic FO).

Let $\varphi(x_1, \ldots, x_k)$ be a query of signature σ, formulated in L.
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$$\varphi(D) \ := \ [\![\varphi(x_1, \ldots, x_k)]\!](D) \ :=$$

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Special case $k = 0$: Boolean queries:

Evaluate $\varphi()$ on D means Decide if $(\mathrm{adom}(D), D) \models \varphi$

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Typical results obtained in database theory:

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CAVEAT: These notions & results cannot handle updates of the db!

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Overview

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Introduction

Conjunctive Queries on Arbitrary Databases

First-Order Queries on Bounded Degree Databases

Conjunctive queries (CQs)

Conjunctive queries:

$$\varphi(x_1,\ldots,x_\ell) \quad := \quad \exists x_{\ell+1} \cdots \exists x_m \left(R_1(\overline{x}) \wedge \cdots \wedge R_s(\overline{x}) \right)$$

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  counting: [Dalmau, Jonsson 2004], [Chen, Mengel 2015], [Greco, Scarcello 2015]
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Interesting: Sub-linear update time

Scenario

[Berkholz, Keppeler, S., PODS'17]

- **Input:** data complexity
  - Database D arbitrary
  - query $\varphi(x_1, \ldots, x_k)$ CQ
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Main result: This is possible $\iff$ φ is q-hierarchical.

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(i) $\text{atoms}(x) \subseteq \text{atoms}(y)$ or $\text{atoms}(y) \subseteq \text{atoms}(x)$ or $\text{atoms}(x) \cap \text{atoms}(y) = \emptyset$, and

(ii) if $\text{atoms}(x) \subsetneq \text{atoms}(y)$ and $x \in \text{free}(\varphi)$, then $y \in \text{free}(\varphi)$.

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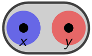

q-hierarchical CQs

Dalvi & Suciu (PODS'07) introduced the hierarchical CQs to characterise the Boolean CQs that can be answered in PTIME on probabilistic dbs.

q-hierarchical CQs are hierarchical CQs where, additionally, the quantifiers respect the query's hierarchical form.

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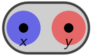

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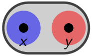

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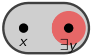

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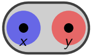

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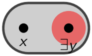
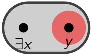


A q-hierarchical query:

$$\theta_{E\text{-}T}(y) \quad := \quad \exists x \; \big(\, E(x,y) \wedge T(y) \, \big)$$



Scenario

[Berkholz, Keppeler, S., PODS'17]

- **Input:** data complexity
  - Database D arbitrary
  - query $\varphi(x_1, \ldots, x_k)$ CQ
- **Preprocessing:**
  Build a suitable data structure that represents D and $\varphi(D)$
- **Output:**
  For Boolean queries:
  - Decide if $D \models \varphi$

  For k-ary queries:
  - Compute the number of tuples in $\varphi(D)$
  - Test for a given tuple $\overline{a}$ whether $\overline{a} \in \varphi(D)$
  - Enumerate the tuples in $\varphi(D)$
- **Dynamic setting:**
  Tuples may be inserted into or deleted from D

After every update we want to update the data structure and report the new query result quickly: in time constant or polylog($\|D\|$).

Main result: This is possible $\iff$ φ is q-hierarchical.

Intractability result for enumerating CQs that are not q-hierarchical

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The OMv-problem: [Henzinger et al., STOC'15]

Input: a Boolean $n \times n$-matrix M and
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Theorem (Enumeration): [Berkholz, Keppeler, S., PODS'17]
Let $\epsilon > 0$ and let $\varphi(\overline{x})$ be a self-join free CQ that is not q-hierarchical.
Then, there is no algorithm with arbitrary preprocessing time and $t_u = O(n^{1-\epsilon})$ update time that enumerates $\varphi(D)$ with $t_d = O(n^{1-\epsilon})$ delay, unless the OMv-conjecture fails.

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Proof idea for $\varphi_{E\text{-}T}(x) \ := \ \exists y \left(E(x,y) \wedge T(y) \right)$

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Given $n \times n$ matrix M, let

- $E^{D_0} \ := \ \{ \, (i,j) \in [n]^2 \, : \, M(i,j) = 1 \, \}, \qquad T^{D_0} \ := \ \emptyset$

Create data structure for D_0 in time $n^2 \cdot n^{1-\epsilon}$.

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This solves OMv in total time $O(n^{3-\varepsilon})$ $\lightning$

Intractability result for Boolean CQs that are not q-hierarchical

The OuMv-problem: [Henzinger et al., STOC'15]

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The OV-problem: [cf. R. Williams, 2005]

Input: two sets U and V of n Boolean vectors of dimension $d := \lceil \log^2 n \rceil$

Task: decide if there exist $u \in U$ and $v \in V$ with $u^\mathsf{T} v = 0$

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Theorem (Counting): [Berkholz, Keppeler, S., PODS'17]
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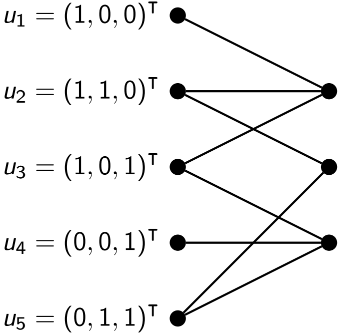
A lower bound for counting via OV

Left: n vertices for the n vectors $u \in U$
Right: $d := \lceil \log^2 n \rceil$ vertices for vector-coordinates



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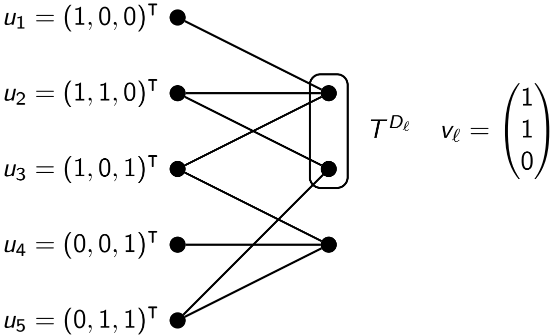

- for each $v_\ell \in V$: update T^{D_ℓ} in time $d \cdot n^{1-\epsilon} \;=\; \lceil \log^2 n \rceil n^{1-\varepsilon}$

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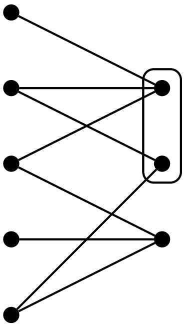
$u_4 = (0,0,1)^\mathsf{T}$

$u_5 = (0,1,1)^\mathsf{T}$



$T^{D_\ell} \quad v_\ell = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$

$\big|\varphi_{E\text{-}T}(D_\ell)\big| = 4 = \big|\{\, u_i \; : \; {u_i}^\mathsf{T} v_\ell \neq 0 \,\}\big|$

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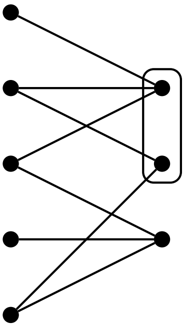

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- there is $u_i \in U$ with ${u_i}^\mathsf{T} v_\ell = 0 \iff |\varphi_{E\text{-}T}(D_\ell)| < n$.
- finished for all $v_\ell \in V$ within time $n \cdot \lceil \log^2 n \rceil n^{1-\varepsilon} = n^{2-\varepsilon'}$ ↯

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[Berkholz, Keppeler, S., PODS'17]

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For every CQ that is q-hierarchical, there is a dynamic data structure that has constant update time and allows to

- answer a Boolean CQ,
- count the number of result tuples,
- enumerate the result relation with constant delay.

q-hierarchical queries

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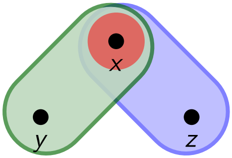

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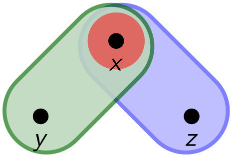

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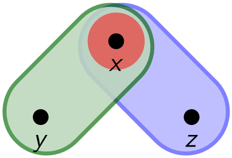

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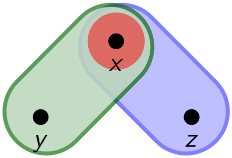

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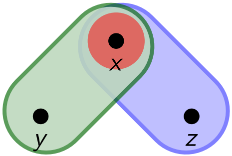

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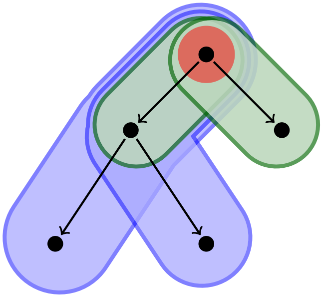

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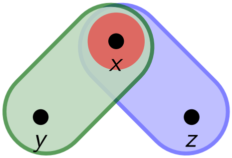

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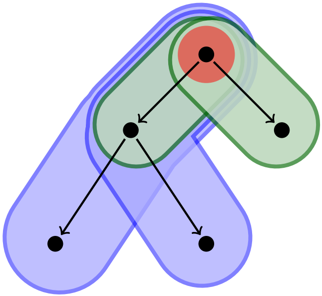

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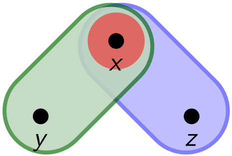

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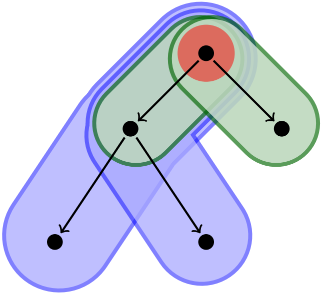

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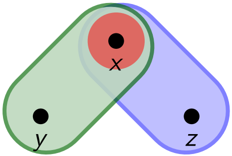

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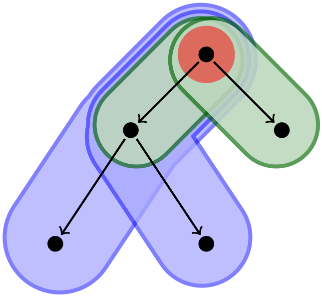

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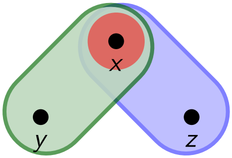

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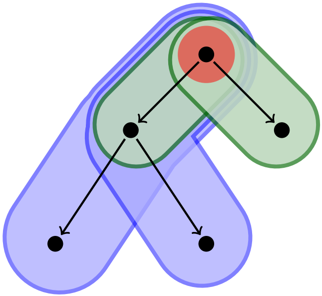
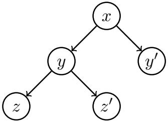


Lemma: A CQ $\varphi(\overline{x})$ is q-hierarchical $\iff$ every connected component of $\varphi(\overline{x})$ has a q-tree.

Data structure for q-hierarchical queries

$$\varphi(x, y, z, y', z') = \big(Rxyz \wedge Rxyz' \wedge Exy \wedge Exy' \wedge Sxyz \big)$$



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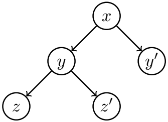

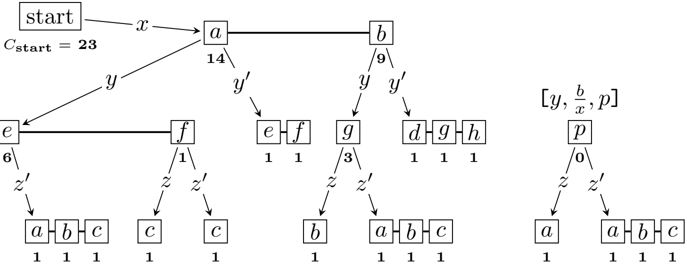

- $S(b,p,a), R(b,p,a), R(b,p,b), R(b,p,c) \in D,\;\; E(b,p) \notin D$

Data structure for q-hierarchical queries

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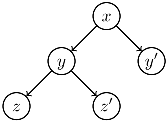

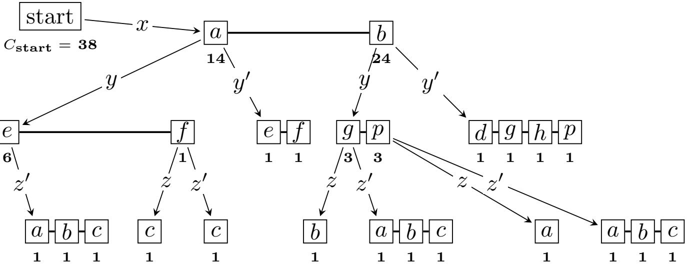

- $S(b,p,a), R(b,p,a), R(b,p,b), R(b,p,c) \in D, \;\; E(b,p) \notin D$
- INSERT $E(b,p)$

Summary

[Berkholz, Keppeler, S., PODS'17]

- **Input:** combined complexity
  - Database D arbitrary $\text{poly}(\varphi) := \|\varphi\|^{O(1)}$
  - query $\varphi(x_1, \ldots, x_k)$ **q-hierarchical** CQ
- **Preprocessing:** in time $\text{poly}(\varphi)\|D\|$
  Build a suitable data structure that represents D and $\varphi(D)$
- **Output:**
  For Boolean queries:
  - Decide if $D \models \varphi$ in time $O(1)$

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  - Compute the number of tuples in $\varphi(D)$ in time $O(1)$
  - Test for a given tuple $\overline{a}$ whether $\overline{a} \in \varphi(D)$ in time $\text{poly}(\varphi)$
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- **Dynamic setting:** update data structure in time $\text{poly}(\varphi)$
  Tuples may be inserted into or deleted from D

After every update we want to update the data structure and report the new query result quickly: in time constant or $\text{polylog}(\|D\|)$.

Main result: This is possible $\iff$ φ is q-hierarchical.

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Ongoing work: Similar results for UCQs & FDs.

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Related work: [Idris, Ugarte, Vansummeren, SIGMOD'17]:
q-hierarchical queries are also efficient in practice!

Overview

Introduction

Conjunctive Queries on Arbitrary Databases

First-Order Queries on Bounded Degree Databases

FO+MOD queries and FOC($\mathbb{P}$) queries

Movie	
Name	Actor
Alien	Sigourney Weaver
Blade Runner	Harrison Ford
Blade Runner	Sean Young
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Brazil	Kim Greist
Casablanca	Humphrey Bogart
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Gravity	Sandra Bullock
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Resident Evil	Milla Jovovich
Terminator	Arnold Schwarzenegger
Terminator	Linda Hamilton
Terminator	Michael Biehn
⋮	⋮

Is the number of movies with Sigourney Weaver even?

In FO+MOD:

$$\exists^{0 \bmod 2} y \;\; \mathit{Movie}(y, \text{"Sigourney Weaver"})$$

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FOC($\mathbb{P}$) = extension of first-order logic with formulas of the form $P(t_1, \ldots, t_r)$ for $P \in \mathbb{P}$ of arity r, and where each t_i is a counting term built using integers, $+$, $\cdot$, and basic counting terms $t(\overline{x})$ of the form $\#\overline{y}.\psi(\overline{x}, \overline{y})$

Bounded degree databases

Graph $G = (V, E)$:

degree of a node v	:	the number of neighbours of v in G
degree of G	:	$\max \{\text{degree}(v) : v \in V\}$

Database D:

degree of D	:	degree of the Gaifman graph of D

Gaifman graph of D:

the graph $G = (V, E)$ with $V = \text{adom}(D)$ and an edge between two distinct nodes $a, b \in V$ iff some tuple in some relation of D contains a and b

Known results for the static setting (i.e., without updates)

FO query evaluation on dbs of degree $\leqslant d$

Boolean queries:

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$$f(\varphi, d) \quad = \quad 2^{d^{2^{O(\|\varphi\|)}}} \quad = \quad \text{3-exp}(\|\varphi\| + \lg\lg d)$$

and the 3-fold exponential blow-up is unavoidable assuming FPT $\neq$ AW[$*$].

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Similar results for other classes of databases

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New: Generalisation to the dynamic setting and FO+MOD [Berkholz, Keppeler, S., ICDT'17] and FOC($\mathbb{P}$) [Kuske, S., LICS'17]

Scenario [Berkholz, Keppeler, S., ICDT'17], [Kuske, S., LICS'17]

- **Input:**
  - Database D of degree $\leqslant d$
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- **Input:** combined complexity
  - Database D of degree $\leqslant d$ — $f(\varphi, d) =$
  - query $\varphi(x_1, \ldots, x_k)$ in FO+MOD[σ] — 3-exp($\|\varphi\| + \lg\lg d$)
- **Preprocessing:** in time $f(\varphi, d)\|D\|$

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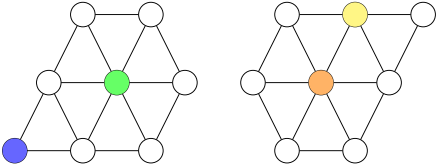
Proof method: use Hanf normal form for FO+MOD

Hanf normal form for FO+MOD

- A type τ with k centres and radius r:



Example type with $k = 4$ centres and radius $r = 1$

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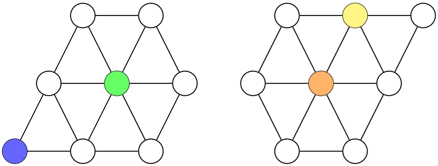


Example type with $k = 4$ centres and radius $r = 1$

- $\mathcal{N}_r^D(\overline{b})$ is the induced substructure of D on

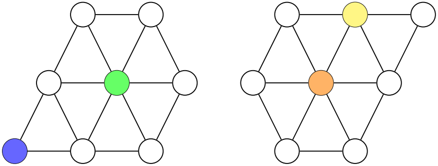
$$N_r^D(\overline{b}) \quad = \quad N_r^D(b_1) \cup \cdots \cup N_r^D(b_k)$$

where

$$N_r^D(b_i) \quad = \quad \{a \in \text{adom}(D) \ : \ \text{dist}^D(b_i, a) \leqslant r\}$$

Hanf normal form for FO+MOD

- A type τ with k centres and radius r:



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- Sphere-formula $\text{sph}_\tau(\overline{x})$:

$$(D, \overline{a}) \models \text{sph}_\tau(\overline{x}) \quad \iff \quad (\mathcal{N}_r^D(\overline{a}), \overline{a}) \cong \tau$$

Hanf normal form for FO+MOD

A Hanf normal form $\psi(\overline{x})$ is a Boolean combination of

- sphere-formulas $\mathsf{sph}_\rho(\overline{x})$ and
- Hanf-sentences $\exists^{\geqslant m} y\ \mathsf{sph}_\tau(y)$ and $\exists^{i \bmod m} y\ \mathsf{sph}_\tau(y)$ where τ is a type with 1 centre and radius r.

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Two queries $\varphi(\overline{x})$ and $\psi(\overline{x})$ are *d-equivalent* iff

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Theorem (Heimberg, Kuske, S., LICS'16)

There is an algorithm which receives as input a degree bound $d \geqslant 2$ and a FO+MOD$[\sigma]$-formula $\varphi(\overline{x})$, and constructs a d-equivalent formula $\psi(\overline{x})$ in Hanf normal form.
The algorithm's runtime is $f(\varphi, d) = 3\text{-}exp(\|\varphi\| + \lg \lg d)$.

Main result for Boolean queries

Theorem

There is a dynamic algorithm that receives as input

- *a degree bound* $d \geqslant 2$,
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Proof Idea: *Step 1:* Transform φ into Hanf normal form ψ.

Proof idea (by example)

$$\psi \quad = \quad \exists^{0 \bmod 2} y \; \mathsf{sph}_\tau(y) \;\wedge\; \exists^{0 \bmod 2} y \; \mathsf{sph}_\rho(y)$$

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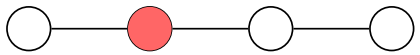
Let τ be the type with 1 center and radius 2:



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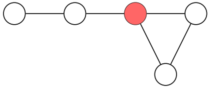

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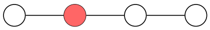

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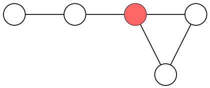

Data structure: `A[`τ`] = 0` , `A[`ρ`] = 0`

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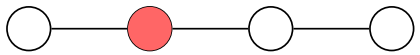

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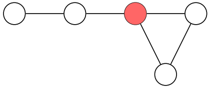

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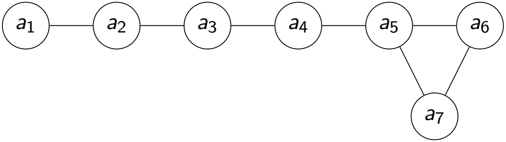

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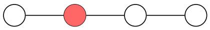

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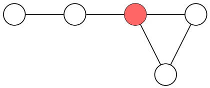

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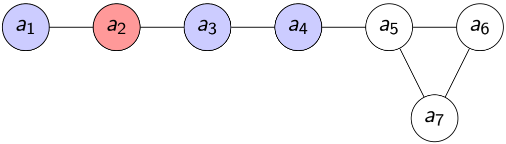

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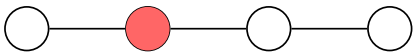

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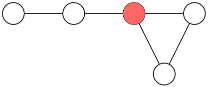

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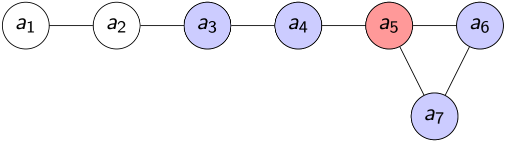

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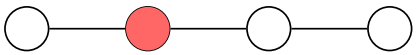

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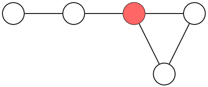

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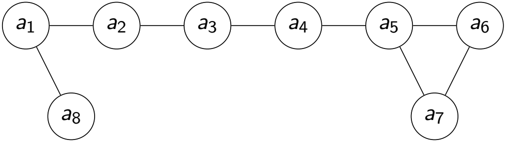

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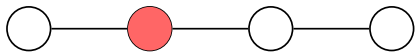

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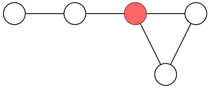

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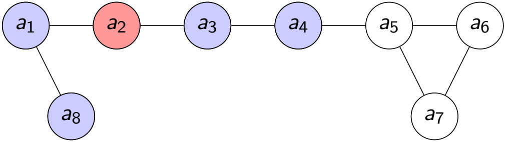

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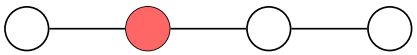

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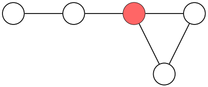

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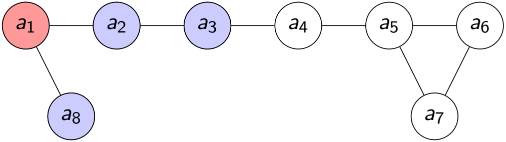

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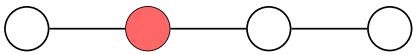

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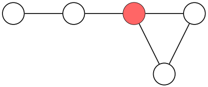

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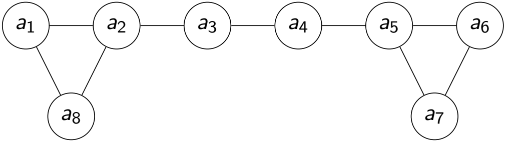

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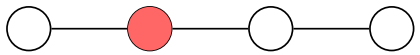

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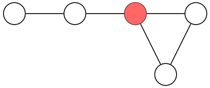

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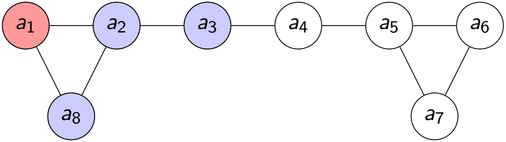

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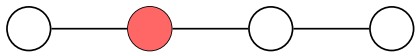

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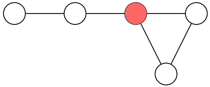

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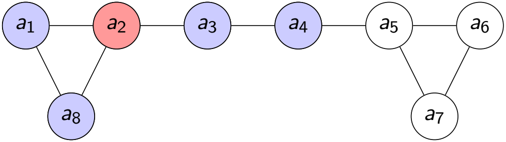

Main result for Boolean queries

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There is a dynamic algorithm that receives as input

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Proof Idea:

Proof idea: Reduction to coloured graphs

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Database D

FO+MOD-query $\varphi(x_1, ..., x_k)$

Same approach as in [Durand, S., Segoufin, PODS'14],
but now we have to take care of updates!

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$\Downarrow$

$\sigma_k := \{E, C_1, ..., C_k\}$

σ_k-structure $\mathcal{G}$

$\overline{v} \in \psi_k(\mathcal{G})$

$$\psi_k(x_1, \ldots, x_k) \;:=\; \bigwedge_{i=1}^{k} C_i(x_i) \;\wedge\; \bigwedge_{i \neq j} \neg E(x_i, x_j)$$

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$\Uparrow$

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Representing Databases by Coloured Graphs

$$\varphi(x_1, \ldots, x_k) \qquad \equiv_d \bigvee_{i \in \mathcal{I}} \mathsf{sph}_{\tau_i}(x_1, \ldots, x_k) \quad \text{\& sentences}$$

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Representing Databases by Coloured Graphs

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Claim

If D_{new} is obtained from D_{old} by one update step, then $\mathcal{G}_{D_{new}}$ can be obtained from $\mathcal{G}_{D_{old}}$ by $d^{\mathcal{O}(k^2 r + k\|\sigma\|)}$ update steps and additional computing time $2^{\mathcal{O}(\|\sigma\| k^2 d^{2r+2})}$.

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- Updating the colours: Before: $C_j = C_j^{\mathcal{G}_{D_{old}}}$

1: **for** $j = 1$ **to** c **do**
2:     **for** every tuple $\overline{b} \in \bigcup_{\ell=1}^{k} U^\ell$ **do**
3:         **if** $(\mathcal{N}_r^{D_{new}}(\overline{b}), \overline{b}) \cong \tau_j$ **then** $C_j \leftarrow C_j \cup \{v_{\overline{b}}\}$
4:         **else** $C_j \leftarrow C_j \setminus \{v_{\overline{b}}\}$

Afterwards: $C_j = C_j^{\mathcal{G}_{D_{new}}}$

Updating the graph (2)

- $E^{\mathcal{G}_D} := \{(v_{\overline{a}}, v_{\overline{b}}) \in V^2 : \text{dist}^D(\overline{a}, \overline{b}) \leqslant 2r + 1\}$

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```
for every tuple b̄ ∈ ⋃_{ℓ=1}^{k} U^ℓ do
    for every tuple b̄' ∈ ⋃_{ℓ=1}^{k} U^ℓ do
        if condition (1), (2) and (3) holds then
            E ← E ∪ {(v_b̄, v_b̄')}
        else
            E ← E \ {(v_b̄, v_b̄')}
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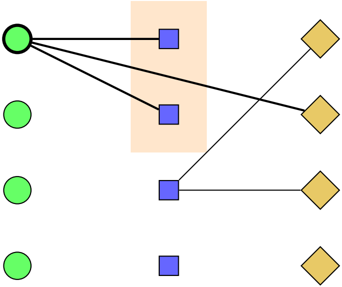
- Conditions:

(1) There is a $j \in \{1, \ldots, c\}$ such that $\overline{b} \in C_j^{\mathcal{G}}$
(2) There is a $j' \in \{1, \ldots, c\}$ such that $\overline{b}' \in C_{j'}^{\mathcal{G}}$
(3) $\text{dist}^{D_{\textit{new}}}(\overline{b}, \overline{b}') \leqslant 2r + 1$

Enumeration with delay $O(k^3 d)$

$$\psi_k(x_1, \ldots, x_k) \quad := \quad \bigwedge_{i=1}^{k} C_i(x_i) \;\wedge\; \bigwedge_{i \neq j} \neg E(x_i, x_j)$$



for all $u_1 \in C_1^{\mathcal{G}}$ **do**
    $\texttt{Enum}(u_1)$.
Output $\texttt{EOE}$.

function $\textsc{Enum}(u_1, \ldots, u_i)$
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        Output $(u_1, \ldots, u_i)$
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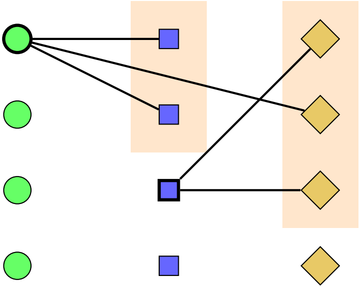

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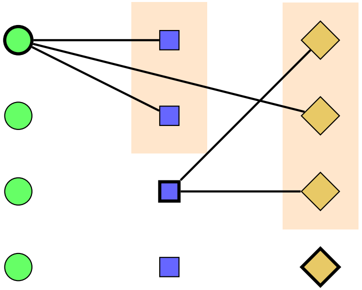

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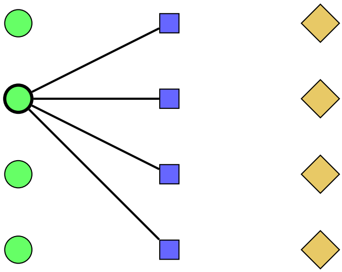

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Problem: Too few blue nodes

Handling small colours

A colour $\ell \in \{1, \ldots, k\}$ is small $\quad :\iff \quad \left|C_\ell^{\mathcal{G}}\right| \leqslant dk$

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$$\overline{s} \in \mathcal{S} \quad \Longleftrightarrow \quad \text{ex. } \overline{a} \text{ such that } (\overline{s}, \overline{a}) \in \varphi(D)$$

The enumeration procedure

for all $(u_1, \ldots, u_s) \in \mathcal{S}$ **do**
    $\texttt{Enum}(u_1, \ldots, u_s)$.
Output the end-of-enumeration message $\texttt{EOE}$.

function $\textsc{Enum}(u_1, \ldots, u_i)$
    if $i = k$ **then**
        output the tuple $(u_1, \ldots, u_i)$
    else
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            if $u_{i+1} \notin \bigcup_{j=1}^{i} N^{\mathcal{G}}(u_j)$ **then**
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where $N^{\mathcal{G}}(u_j) \ := \ \{ v \in V^{\mathcal{G}} \ : \ (u_j, v) \in E^{\mathcal{G}} \}$.

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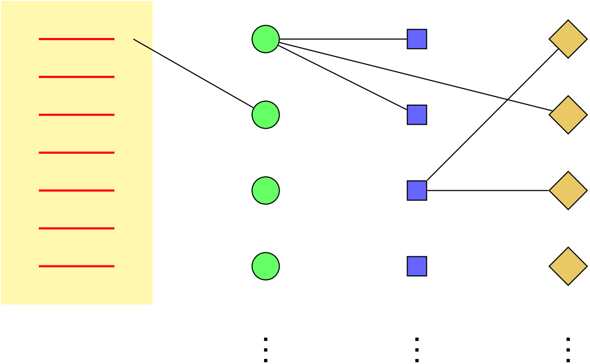

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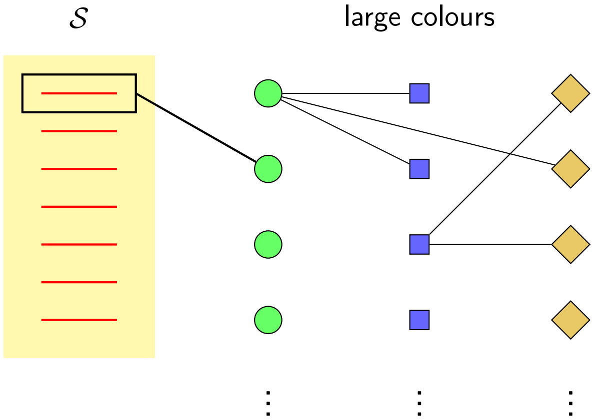

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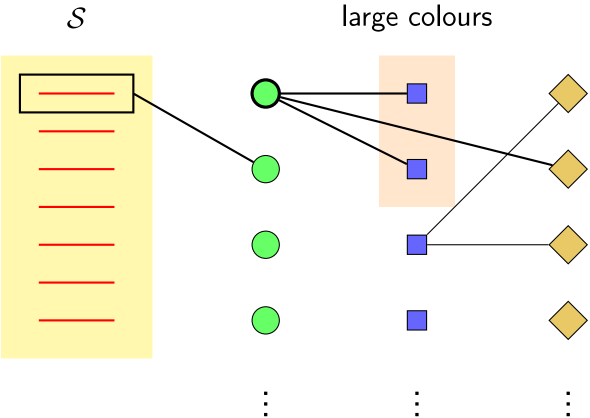

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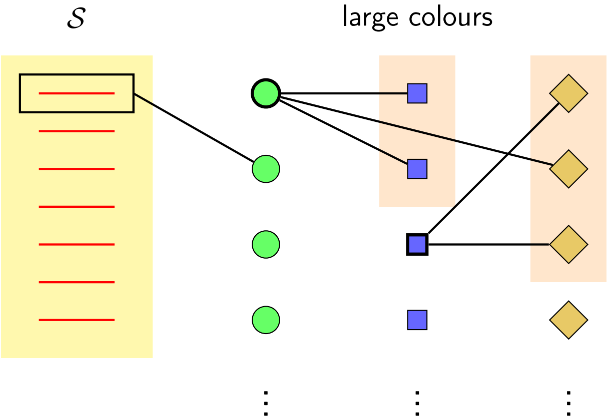

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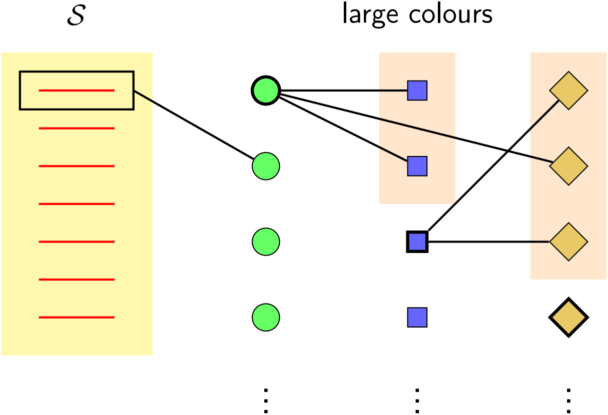

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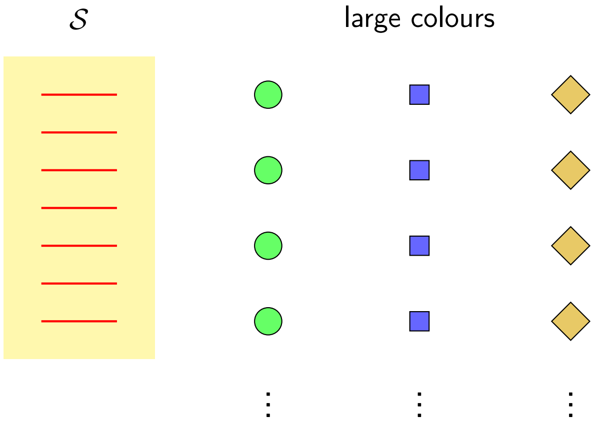

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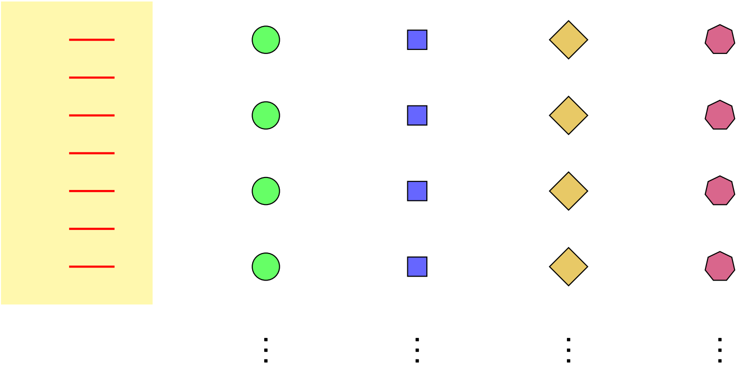

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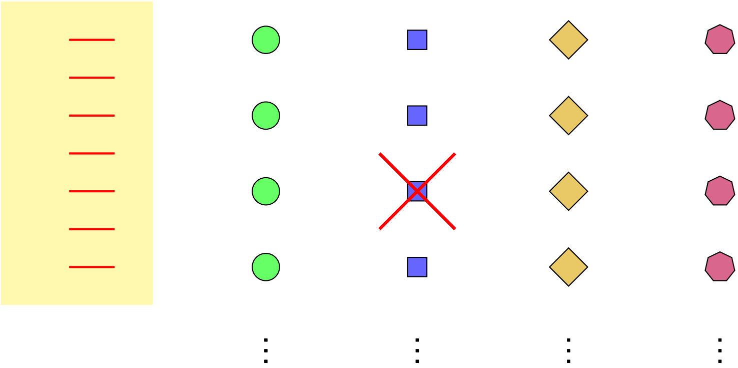

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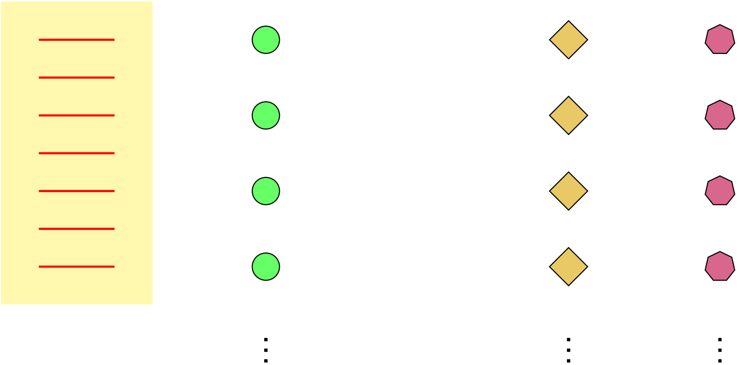

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Main result for enumeration

Theorem

There is a dynamic algorithm that receives as input

- *a degree bound* $d \geqslant 2$,
- *a* k*-ary* FO+MOD[σ]*-query* $\varphi(\overline{x})$, *and*
- *a db* D *of degree* $\leqslant d$,

and computes

- *within* $f(\varphi, d)\|D\|$ *preprocessing time a data structure*
- *that can be updated in time* $f(\varphi, d)$

and allows to enumerate $\varphi(D)$ *with delay* $O(k^3)$.

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Summary [Berkholz, Keppeler, S., ICDT'17]

- **Input:** combined complexity
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  - query $\varphi(x_1, \ldots, x_k)$ in FO+MOD[σ] 3-exp($\|\varphi\| + \lg \lg d$)
- **Preprocessing:** in time $f(\varphi, d)\|D\|$

  Build a suitable data structure that represents D and $\varphi(D)$
- **Output:**

  For Boolean queries:
  - Decide if $D \models \varphi$ in time $O(1)$

  For k-ary queries:
  - Compute the number of tuples in $\varphi(D)$ in time $O(1)$
  - Test for a given tuple $\bar{a}$ whether $\bar{a} \in \varphi(D)$ in time $O(k^2)$
  - Enumerate the tuples in $\varphi(D)$ with delay $O(k^3)$
- **Dynamic setting:** update data structure in time $f(\varphi, d)$

  Tuples may be inserted into or deleted from D

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  - Compute the number of tuples in $\varphi(D)$ in time $O(1)$
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- **Dynamic setting:** update data structure in time $f(\varphi, d)$

  Tuples may be inserted into or deleted from D

Similar results for FO with counting FOC$(\mathbb{P})$ [Kuske, S., LICS'17].

Summary [Berkholz, Keppeler, S., ICDT'17]

- **Input:** combined complexity
  - Database D of degree $\leqslant d$ $f(\varphi, d) =$
  - query $\varphi(x_1, \ldots, x_k)$ in FO+MOD[σ] 3-exp($\|\varphi\| + \lg \lg d$)
- **Preprocessing:** in time $f(\varphi, d)\|D\|$

  Build a suitable data structure that represents D and $\varphi(D)$
- **Output:**

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  - Decide if $D \models \varphi$ in time $O(1)$

  For k-ary queries:
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Future task: Revisit other results on FO model checking in the dynamic setting!

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– Thank you! –