Data Streams — A Tutorial

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Data Streams

Situation:

- massive amounts of data
- generated automatically
- continuous, rapid updates

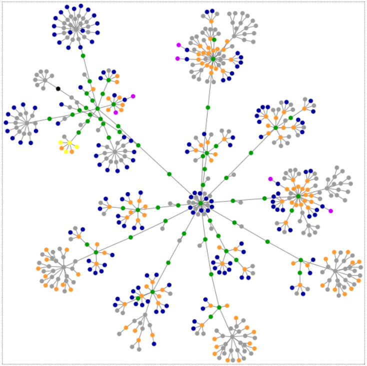
Examples:

- meteorological data (sensor networks)
- astronomical data
- network monitoring
- banking and credit transactions

Challenges:

- cannot wait with processing until "all" the data has arrived
  $\rightsquigarrow$ process data "on-the-fly"
- cannot afford to store all the data $\rightsquigarrow$ store a "sketch"
- data may arrive so rapidly that you cannot even afford to look at each incoming data item $\rightsquigarrow$ "sampling"

Example: Network Monitoring



Let A be a node in the world wide web.
As input, A receives a stream of "packets"

$$p_1, p_2, p_3, p_4, \ldots, p_m.$$

Each packet p_i contains information on

- the sender's IP address,
- the destination's IP address,
- the data that is transmitted

Question: How many distinct IP addresses have sent at least one packet through node A? — I.e., what is the 0-th frequency moment F_0 of the input stream?

Problem: A does not want to store the entire stream $p_1, p_2, p_3, \ldots, p_m$.

Solution:
A suitable randomised algorithm that computes a good approximate answer:

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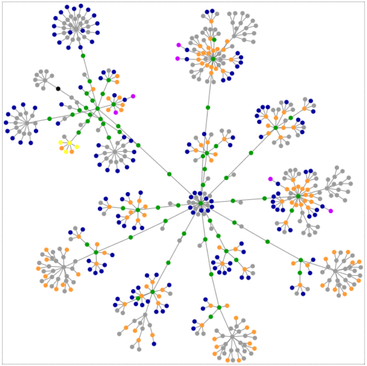

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A suitable randomised algorithm that computes a good approximate answer:

Tight bound for approximating F_0

COMPUTING F_0

Input: A sequence $p_1, p_2, p_3, \ldots, p_m$ of elements in $\{1, \ldots, n\}$.

Task: Compute the number F_0 of *distinct* elements in the input.

Theorem:

(a) *Upper Bound:* *(Flajolet, Martin, FOCS'83)*

For every $c > 2$ there is a randomized one-pass algorithm that uses $O(\log n)$ bits of memory and computes a number Y such that
Prob $\left(\frac{Y}{F_0} \leqslant \frac{1}{c} \text{ or } \frac{Y}{F_0} \geqslant c\right) \leqslant 2/c$.

(b) *Lower Bound:* *(Alon, Matias, Szegedy, STOC'96)*

Any randomized one-pass algorithm computing a number Y such that *Prob* $\left(\frac{Y}{F_0} \leqslant 0.9 \text{ or } \frac{Y}{F_0} \geqslant 1.1\right) \leqslant 0.25$ *uses* $\Omega(\log n)$ *bits of memory.*

Remark: improved bounds: Bar-Yossef, Jayram, Kumar, Sivakumar (RANDOM'00) and Kane, Nelson, Woodruff (PODS'10).

Main issues concerning data streams:

How to design algorithms & how to prove lower bounds

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How to design algorithms & how to prove lower bounds

Overview

One pass over a single stream

Several passes over a single stream

Several passes over several streams in parallel

Read/write streams

Future tasks

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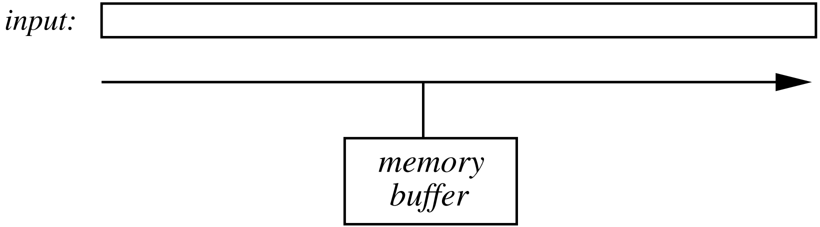
One pass over a single stream

Scenario:



Missing Number Puzzle

MISSING NUMBER

Input: A stream $x_1, x_2, x_3, \ldots, x_{n-1}$ of $n-1$ distinct numbers from $\{1, \ldots, n\}$.

Question: Which number from $\{1, \ldots, n\}$ is missing?

Naive Solution: 2 5 1 3 4 8 6 $\cdots$ n requires n bits of storage

1	2	3	4	5	6	7	8	$\cdots$	n

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Clever Solution: Store running sum $O(\log n)$ bits suffice

$$s := x_1 + x_2 + x_3 + x_4 + \cdots + x_{n-1}$$

$$\text{Missing number} = \frac{n \cdot (n+1)}{2} - s$$

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Lower Bound: at least $\log n$ bits are necessary

Exercise # 1

Find a data stream algorithm that uses at most poly($k \cdot \log n$) bits of memory and solves the following generalization of the "missing numbers puzzle":

k MISSING NUMBERS

Input: Two numbers n, k and a stream $x_1, x_2, x_3, \ldots, x_{n-k}$ of $n-k$ distinct numbers from $\{1, \ldots, n\}$

Task: Find the k missing numbers

The MULTISET-EQUALITY Problem (1/3)

MULTISET-EQUALITY *Total input length:* $N = O(m \cdot \log n)$ *bits*

Input: Two multisets $\{x_1, \ldots, x_m\}$ and $\{y_1, \ldots, y_m\}$ of numbers x_i, y_j in $\{1, \ldots, n\}$.

Question: Is $\{x_1, \ldots, x_m\} = \{y_1, \ldots, y_m\}$?

Observation:

Every deterministic solution requires $\Omega(N)$ bits of storage.

Proof:

- Use fact from Communication Complexity:

Communication Complexity

Yao's 2-Party Communication Model:

- 2 players: Alice & Bob
- both know a function $f : A \times B \to \{0,1\}$
- Alice only sees input $a \in A$, Bob only sees input $b \in B$
- they jointly want to compute $f(a,b)$
- Goal: exchange as few bits of communication as possible

Fact: Deciding if two m-element input sets

$$a = \{x_1, \ldots, x_m\} \subseteq \{1, \ldots, n\} \quad \text{und} \quad b = \{y_1, \ldots, y_m\} \subseteq \{1, \ldots, n\}$$

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  Deciding if two m-element subsets of $\{1, \ldots, n\}$ are equal requires at least $\log\binom{n}{m}$ bits of communication.
- If $n = m^2$, then $\log\binom{n}{m} \geqslant m \cdot \log m$ bits of communication are necessary, and the total length of the corresponding MULTISET-EQUALITY input is $N = \Theta(m \cdot \log m)$.

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The MULTISET-EQUALITY Problem (2/3)

Proof (continued):

- Known: $N = \Theta(m \cdot \log m)$, and $\geqslant m \cdot \log m$ bits of communication are necessary for solving MULTISET-EQUALITY.
- A deterministic data stream algorithm solving MULTISET-EQUALITY with s bits of storage would lead to a communication protocol with s bits of communication.

- Thus: Lower bound on communication complexity $\leadsto$ lower bound on memory size of data stream algorithm

The Multiset-Equality Problem (2/3)

Proof (continued):

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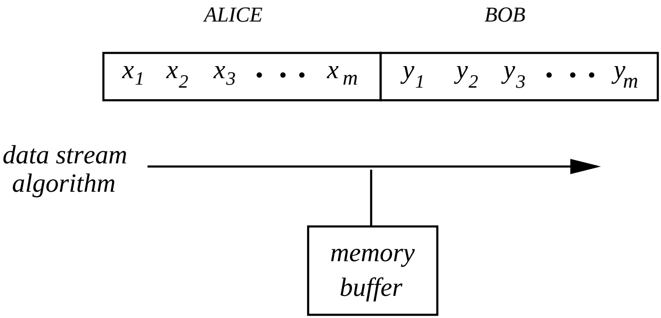

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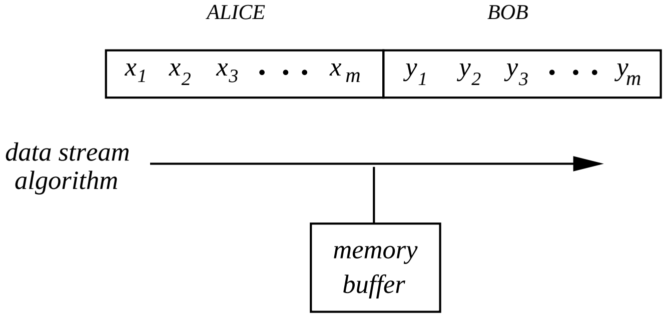

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The MULTISET-EQUALITY Problem (3/3)

Theorem: *(Grohe, Hernich, S., PODS'06)*

The MULTISET-EQUALITY *problem can be solved by a randomised algorithm using $O(\log N)$ bits of storage in the following sense:*

Given m, n, and a stream of numbers $a_1, \ldots, a_m,\ b_1, \ldots, b_m$ from $\{1, \ldots, n\}$, the algorithm

- *accepts with probability 1 if $\{a_1, \ldots, a_m\} = \{b_1, \ldots, b_m\}$*
- *rejects with probability $\geqslant 0.9$ if $\{a_1, \ldots, a_m\} \neq \{b_1, \ldots, b_m\}$.*

Basic idea: Use "Fingerprinting"-techniques:

- represent $\{a_1, \ldots, a_m\}$ by a polynomial $f(x) := \sum_{i=1}^{m} x^{a_i}$
- represent $\{b_1, \ldots, b_m\}$ by a polynomial $g(x) := \sum_{i=1}^{m} x^{b_i}$
- choose a random number r and check if $f(r) = g(r)$
- accept if $f(r) = g(r)$; reject otherwise.

If $\{a_1, \ldots, a_m\} = \{b_1, \ldots, b_m\}$, then $f(x) = g(x)$, and thus the algorithm always accepts. If $\{a_1, \ldots, a_m\} \neq \{b_1, \ldots, b_m\}$, then there are at most *degree*$(f-g)$ many distinct r with $f(r) = g(r)$, and thus the algorithm rejects with high probability.

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Basic idea: Use "Fingerprinting"-techniques:

- represent $\{a_1, \ldots, a_m\}$ by a polynomial $f(x) := \sum_{i=1}^{m} x^{a_i}$
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- choose a random number r and check if $f(r) = g(r)$
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If $\{a_1, \ldots, a_m\} = \{b_1, \ldots, b_m\}$, then $f(x) = g(x)$, and thus the algorithm always accepts. If $\{a_1, \ldots, a_m\} \neq \{b_1, \ldots, b_m\}$, then there are at most $degree(f-g)$ many distinct r with $f(r) = g(r)$, and thus the algorithm rejects with high probability.

The MULTISET-EQUALITY Problem (3/3)

Theorem: *(Grohe, Hernich, S., PODS'06)*

The MULTISET-EQUALITY *problem can be solved by a randomised algorithm using $O(\log N)$ bits of storage in the following sense:*

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Exercise # 2

Work out the details of the described algorithm and its analysis.

Overview

One pass over a single stream

Several passes over a single stream

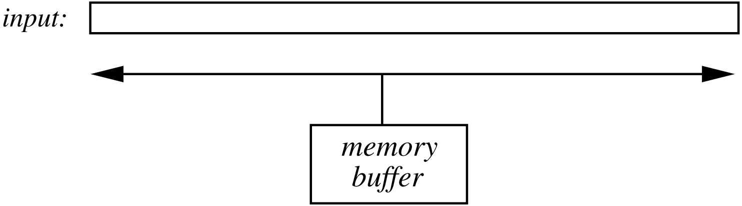
Several passes over several streams in parallel

Read/write streams

Future tasks

Several passes over a single stream

Scenario:



Parameters:

p : number of passes

s : size of memory buffer (number of bits)

We call such computations (p, s)-bounded computations.

If necessary, an output stream can be generated during a computation.

Several passes over a single stream

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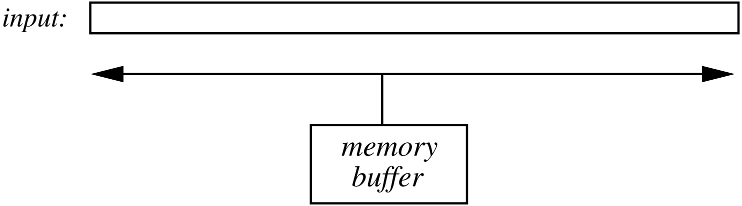

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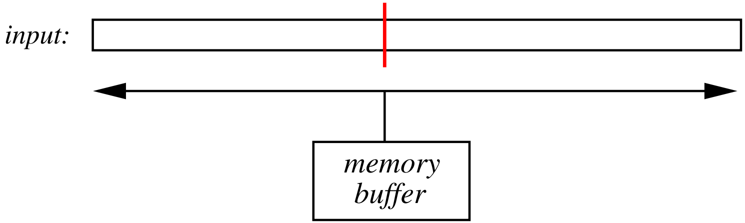

Fact:

During a (p, s)-bounded computation, only $(p \cdot s)$ bits can be communicated between the first and the second half of the input.

Consequence:

Lower bounds on communication complexity lead to lower bounds for (p, s)-bounded computations

... even if backward passes are allowed

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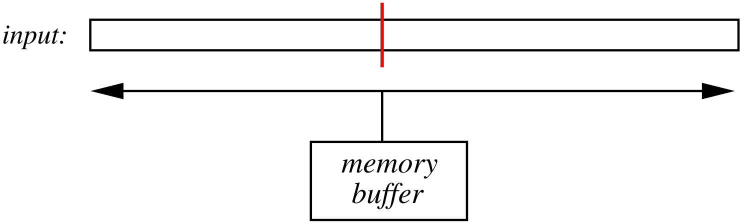

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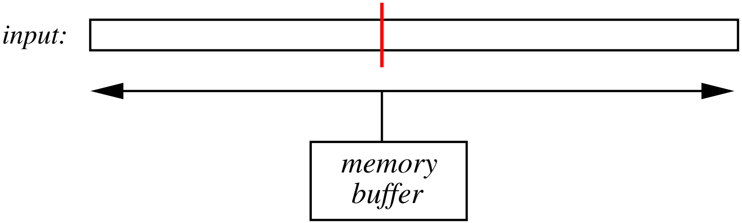

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A lower bound for connectedness of a graph

CONNECTEDNESS *Parameters: m edges on $\leqslant n$ nodes*

Input: A list of edges $e_1, \ldots, e_m$ on node set $V \subseteq \{1, .., n\}$.

Question: Is the input graph connected?

Theorem: *(Henzinger, Raghavan, Rajagopalan, 1998)*

Solving CONNECTEDNESS *with p passes requires $\Omega(n/p)$ bits of memory.*

Proof:

By a reduction using the set disjointness problem.

SET DISJOINTNESS PROBLEM

Input: Two sets $A, B \subseteq \{1, .., n\}$

Question: Is $A \cap B = \emptyset$?

Known communication complexity of the set disjointness problem:
n bits of communication are necessary (and sufficient).

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Exercise # 3

Work out the details of the proof:

(a) prove that n bits of communication are necessary for solving the set disjointness problem in Yao's 2-party communication model, and

(b) use this to show that solving graph connectedness with p passes requires $\Omega(n/p)$ bits of memory.

A lower bound for sorting

SORTING *Input length* $N = O(m \cdot \log n)$ *bits*

Input: A sequence of numbers $x_1, \ldots, x_m \in \{1, \ldots, n\}$ (for arbitrary m, n).

Output: $x_1, \ldots, x_m$ sorted in ascending order.

Theorem: *(Grohe, Koch, S., ICALP'05)*

SORTING *can be solved by a* (p, s)*-bounded computation* $\iff$ $(p \cdot s) \in \Omega(N)$

Proof:

- upper bound: easy.
- lower bound: by a reduction using the set disjointness problem.

A hierarchy on the number of passes

Allowing a single extra scan may be more powerful than significantly increasing the internal memory space:

Theorem: *(Hernich, S., Theor. Comput. Sci. 2008)*

For every logspace-computable function p with $p(N) \in o\left(\frac{N}{\log^2 N}\right)$, there exists a decision problem that

- *can be solved by a $(p+1, s)$-bounded computation, but*
- *that cannot be solved by any (p, S)-bounded computation,*

for $s(N) = O(\log N)$ and $S(N) = o\left(\frac{N}{p(N) \cdot \log N}\right)$.

Remark: An analogous result also holds for randomised computations.

Proof idea:

Use a result by Nisan and Wigderson (1993) on the k-round communication complexity of a particular "pointer jumping" problem.

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A lower bound for finding a longest increasing subsequence

LONGEST-INCREASING-SUBSEQUENCE

Input: a sequence of numbers $x_1, \ldots, x_m \in \{1, \ldots, n\}$ (for arbitrary m, n)

Output: an increasing subsequence $x_{i_1}, \ldots, x_{i_k}$ of maximum length (denoted k)

Theorem: *(Guha, McGregor, ICALP'08)*

Any randomized p-pass algorithm solving LONGEST-INCREASING-SUBSEQUENCE *with p passes (and probability 0.9) requires* $\Omega\left(k^{1+\frac{1}{2^p-1}}\right)$ *bits of memory.*

Proof:

- not by using communication complexity
- introduce a new method of pass elimination (somewhat related to "round elimination" methods in communication complexity, but taylored towards stream processing).

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A matching upper bound was proved by Liben-Nowell, Vee, Zhu, COCOON'05.

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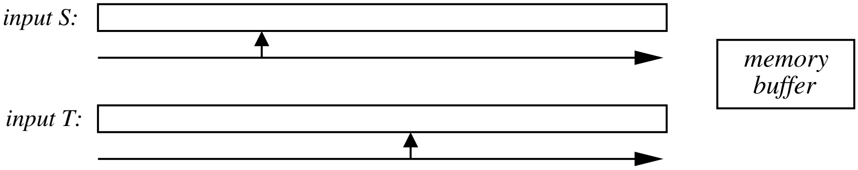

Parameters:

- 2 input streams: $S = s_1, s_2, \ldots, s_n$ and $T = t_1, t_2, \ldots, t_n$.
- one pass over each input; heads may proceed asynchronously
- advancement of heads and new content of memory depends on the current content of memory and the symbols seen at both heads
- for simplicity: advancement of only one head at a time
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- m : number of possible memory configurations, i.e., $\log m = s$

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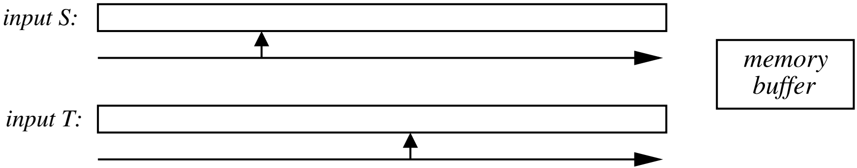

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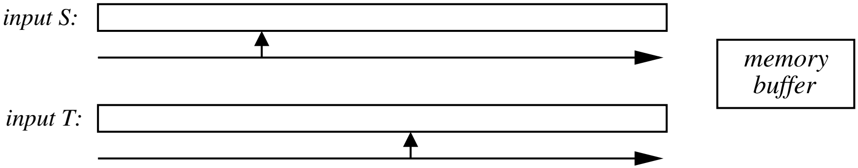

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How to prove lower bounds in this scenario?

Problem:
"Classical" communication complexity results cannot be used so easily here.

Solution: Take a direct look at the "flow of information" during computations.

Consider the following example:

- $n \geqslant 2$
- $\mathbb{D}_n := \{a_1, b_1, c_1 \ldots, a_n, b_n, c_n\}$ — domain of $3n$ input items
- variation of the set disjointness problem:

DISJ$_n$

Input: Two streams $S = s_1, s_2, \ldots, s_n$ and $T = t_1, t_2, \ldots, t_n$ of elements in $\mathbb{D}_n$

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A lower bound proof for $\textsc{Disj}_n$ (1/5)

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$\textsc{Disj}_n$ *cannot be solved by a deterministic algorithm that performs one pass over each stream and that uses less than* $n - \log n - 1$ *bits of memory.*

Proof:

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A lower bound proof for DISJ_n (1/5)

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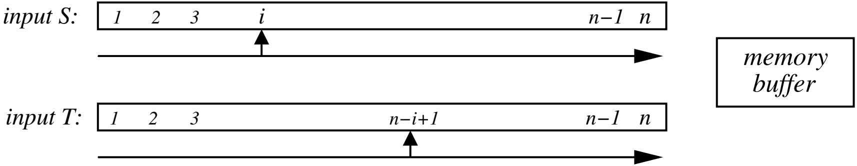
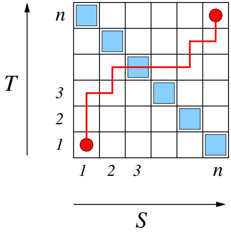
A lower bound proof for $\textsc{Disj}_n$ (2/5)

Situation during a computation:



- potential head positions: (i, j) with $1 \leqslant i, j \leqslant n$
- start: $(1, 1)$
- end: (n, n)



- For each input $D(I, \bar{I})$ there exists exactly one $i \in \{1, \ldots, n\}$ such that the heads visit position $(i, n-i+1)$.

A lower bound proof for DISJ_n (3/5)

Goal now: "cut-and-paste argument"

Find $I, J \subseteq \{1, .., n\}$ such that computations on $D(I, \bar{I})$ and $D(J, \bar{J})$ can be combined to an accepting computation on $D(I', J')$ for I' and J' with $I' \cap J' \neq \emptyset$.

$\Longrightarrow$ accept a "no"-instance!

(1) Ex. $i \in \{1, .., n\}$ and $X_1 \subseteq \{I : I \subseteq \{1, .., n\}\}$ such that

- for each $I \in X_1$, head position $(i, n{-}i{+}1)$ is visited,
- $|X_1| \geqslant \frac{2^n}{n}$.

(2) Ex. $X_2 \subseteq X_1$ such that

- for all $I, J \in X_2$: $\quad i \in I \iff i \in J$,
- $|X_2| \geqslant \frac{|X_1|}{2} \geqslant \frac{2^n}{2n}$.

(3) Ex. memory configuration c and $X_3 \subseteq X_2$ such that

- for all $I \in X_3$: memory configuration c when at head position $(i, n{-}i{+}1)$,
- $|X_3| \geqslant \frac{|X_2|}{m} \geqslant \frac{2^n}{2nm}$.

Note: $\quad |X_3| > 1 \iff m < \frac{2^n}{2n} \iff s = \log m < n - \log n - 1$.

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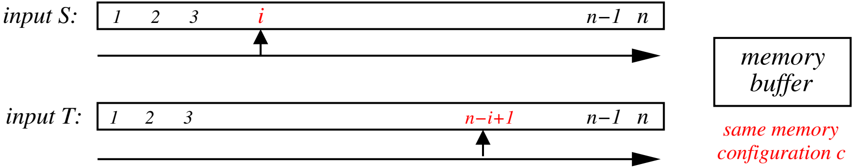
Note: $\quad |X_3| > 1 \quad \Longleftarrow \quad m < \frac{2^n}{2n} \quad \iff \quad s = \log m < n - \log n - 1.$

A lower bound proof for DISJ_n (4/5)

Let $I, J \in X_3$ with $I \neq J$.

Same situation on input $D(I, \bar{I})$ and on input $D(J, \bar{J})$:



- Cut-and-paste argument $\Longrightarrow$ Same situation on inputs $D(I_1, I_2)$ and $D(I_1', I_2')$
- $$\begin{aligned} I_1 &= (I \cap \{1,\ldots,i-1\}) \cup (I \cap \{i\}) \cup (J \cap \{i+1,\ldots,n\}) \\ I_2 &= (\bar{I} \cap \{i+1,\ldots,n\}) \cup (\bar{I} \cap \{i\}) \cup (\bar{J} \cap \{1,\ldots,i-1\}) \end{aligned}$$
- $$\begin{aligned} I_1' &= (J \cap \{1,\ldots,i-1\}) \cup (J \cap \{i\}) \cup (I \cap \{i+1,\ldots,n\}) \\ I_2' &= (\bar{J} \cap \{i+1,\ldots,n\}) \cup (\bar{J} \cap \{i\}) \cup (\bar{I} \cap \{1,\ldots,i-1\}) \end{aligned}$$

Since $I \neq J$, $D(I_1, I_2)$ or $D(I_1', I_2')$ is a "no"-instance. $\square$

A lower bound proof for $\textsc{Disj}_n$ (4/5)

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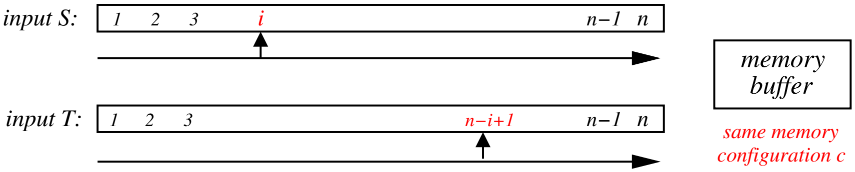

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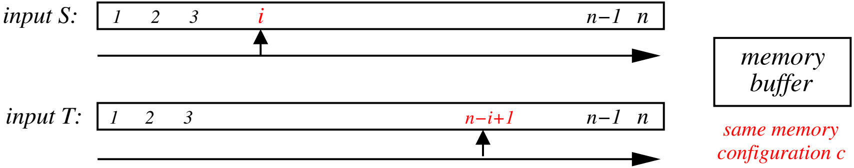

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Since $I \neq J$, $D(I_1, I_2)$ or $D(I_1', I_2')$ is a "no"-instance. $\square$

A lower bound proof for DISJ_n (5/5)

We have proved

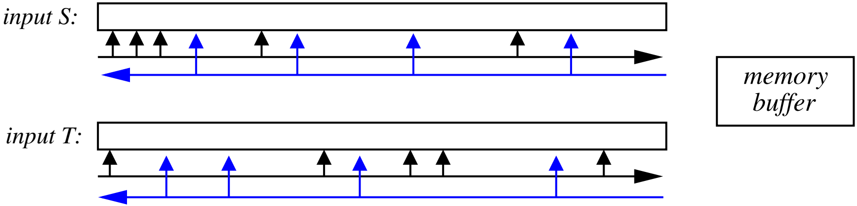
Theorem: *(Bar Yossef, Shalem, ICDE'08)*

DISJ_n *cannot be solved by a deterministic algorithm that performs one pass over each stream and that uses less than* $n - \log n - 1$ *bits of memory.*

The proof given by Bar-Yossef and Shalem (ICDE 2008) is different.
For their proof, they introduce a particular kind of communication model: the token-based mesh communication model.

Several passes over several streams in parallel

General scenario: mp2s-automaton $\mathcal{A}$ with parameters $(\mathbb{D}, m, k_f, k_b)$



Parameters:

- 2 input streams: $S = s_1, s_2, \ldots, s_n$ and $T = t_1, t_2, \ldots, t_n$ of elements in $\mathbb{D}$.
- m : number of possible memory configurations; $s := \log m$ size of the memory buffer (number of bits).
- k_f forward heads on each input stream, k_b backward heads on each input stream
- Depending on (a) the current memory state and (b) the elements in S and T at the current head positions, a deterministic transition function determines (1) the next memory state and (2) which of the heads should be advanced to the next position.

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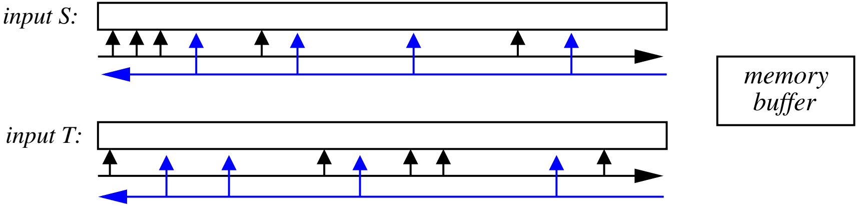

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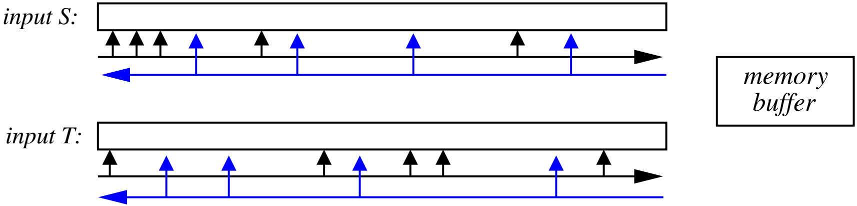

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Solving $\textsc{Disj}_n$ with an mp2s-automaton: upper bound

Proposition:

$\textsc{Disj}_n$ *can be solved by an mp2s-automaton with parameters* $(\mathbb{D}_n, n{+}2, \sqrt{n}, 0)$.
(I.e.: memory buffer of $\log(n{+}2)$ *bits,* $\sqrt{n}$ *forward heads, no backward heads)*

Proof:

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Proof:

Phase 1:

Move heads on S such that they partition S into blocks of length $\sqrt{n}$.
(use $n{+}1 - \sqrt{n}$ states)

Phase 2:

For $j = 1, \ldots, \sqrt{n}$ do

(1) Let j-th head on T pass the entire stream and compare each element of T with the $\sqrt{n}$ elements at head positions in S.

(2) Advance each head on S one step to the right.

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(2) Advance each head on S one step to the right.

(use 2 states)

Solving DISJ_n with an mp2s-automaton: lower bound

Theorem: *(S, STACS'09)*

For all n, m, k_f, k_b such that, for $k = 2k_f + 2k_b$ and $v = (k_f^2 + k_b^2 + 1)\cdot(2k_f k_b + 1)$,

$$k^2 \cdot v \cdot \log(n+1) \; + \; k \cdot v \cdot \log m \; + \; v \cdot (1 + \lg v) \quad \leqslant \quad n,$$

the problem DISJ_n cannot be solved by any mp2s-automaton with parameters $(\mathbb{D}_n, m, k_f, k_b)$.

Proof:

- Similar to the shown proof where only one forward head is available on each stream.
- Divide input streams into blocks and choose a block that is “not checked” by any pair of cursors.

Finite Cursor Machines

Introduced by Grohe, Gurevich, Leinders, S., Tyszkiewicz, Van den Bussche, ICDT'07

- an abstract model for database query processing
- formal model: based on Abstract State Machines

Informal Description of a FCM:

- works on a relational database
  (tables, not sets) (read-only access)
- on each table:
  a fixed number of cursors
- cursors are one-way,
  but can move asynchronously
- internal memory:
  - finite state control
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- manipulation of output row and internal
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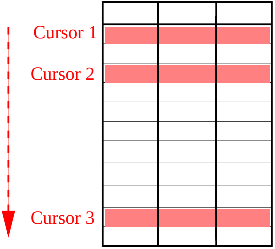

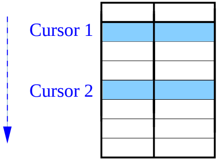

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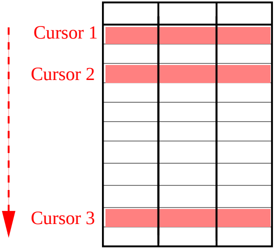

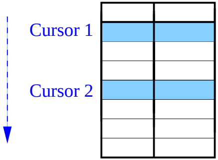

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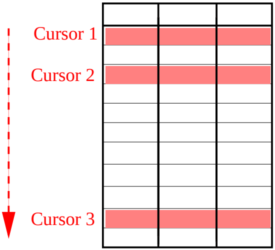

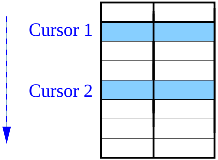

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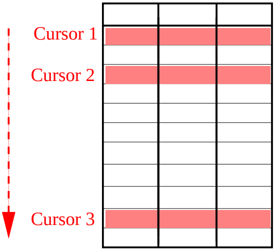

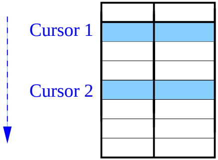

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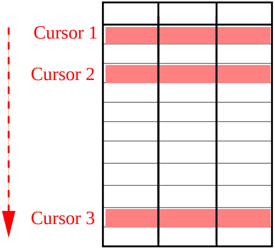

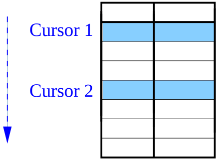

Easy Observations

Consider the operators from Relational Algebra

- Selection $\sigma_{i=j}(R)$ can be implemented by a FCM
- Union $R_1 \cup R_2$ and Projection $\pi_J(R)$ can be implemented by a FCM, provided that input tables are ordered
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Corollary:

Each Semijoin Algebra query can be computed by query plan composed of FCMs and sorting operations. (a.k.a: "classical" 2-pass query processing)

Question: Are intermediate sorting steps really necessary?

Easy Observations

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Answer: Yes! ...

Theorem: *(Grohe, Gurevich, Leinders, S., Tyszkiewicz, Van den Bussche, ICDT'07)*

The query

$$\text{Is} \quad R \ltimes_{x_1=y_1} (S \ltimes_{x_2=y_1} T) \quad \text{nonempty?}$$

where R and T are unary and S in binary, is not computable by an FCM (even if the FCM is allowed to have as input all sorted versions of the input relations).

An Open Question

Is there a Boolean query from Relational Algebra (or, equivalently, a sentence of first-order logic), that cannot be computed by any composition of FCMs and sorting operations?

Conjecture: Yes

... since otherwise FO would have data complexity of time $n \cdot \log n$

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Overview

One pass over a single stream

Several passes over a single stream

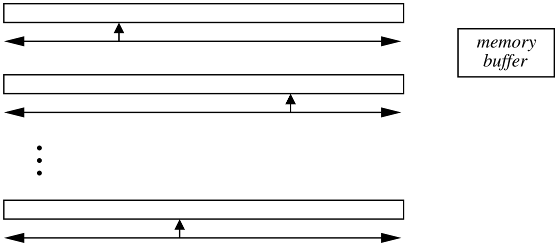
Several passes over several streams in parallel

Read/write streams

Future tasks

Read/write streams

Scenario:



Parameters:

- t read/write streams
- one head on each stream; each head can write onto (and append) the stream
- r : maximum number of head reversals
- s : size of "internal memory" (number of bits)

- input on first read/write stream
- if necessary: output on last read/write stream
- formal model: based on Turing machines.

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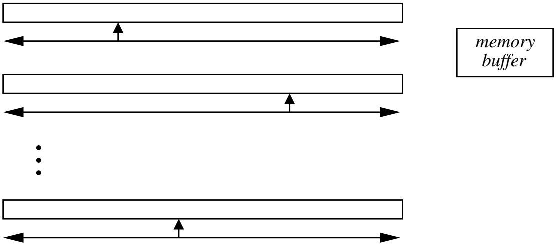

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Complexity classes

$\mathrm{ST}(r, s, t)$:

class of all problems that can be solved by a deterministic algorithm using

- t read/write streams,
- at most r head reversals, and
- a memory buffer of size s.

The sorting problem

SORTING — *Input length* $N = m \cdot (n+1)$

Input: bit-strings $x_1, \ldots, x_m \in \{0,1\}^n$ (for arbitrary m, n)

Output: $x_1, \ldots, x_m$ sorted in ascending order

Already seen in this talk :

Theorem: *(Grohe, Koch, S., ICALP'05)*

SORTING *can be solved by a* (p, s)*-bounded computation* $\iff$ $(p \cdot s) \in \Omega(N)$

Thus: SORTING $\in \mathrm{ST}(r, s, 1) \iff r(N) \cdot s(N) \in \Omega(N)$.

Theorem: *(Chen, Yap, 1991)*

SORTING $\in \mathrm{ST}(O(\log N), O(1), 2)$

Proof method: refinement of Merge-Sort.

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Lower bound for sorting with $\geqslant 2$ r/w streams

Problem:

An additional read/write stream can be used to move around large parts of the input (with just 2 head reversals).

$\rightsquigarrow$ communication complexity does not help to prove lower bounds

Intuition:

Still, the order of the input strings cannot be changed so easily.

Fact:

For sufficiently small $r(N)$, $s(N)$, even with $t \geqslant 2$ read/write streams, sorting by solely comparing and moving around the input strings is impossible.

(For comparison-exchange algorithms, according lower bounds are well-known.)

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Lower bound for sorting with $\geqslant 2$ r/w streams

Problem:

Algorithms for read/write streams are based on Turing machines.
They can perform much more complicated operations than just compare and move around input strings.

Example:

During a first scan of the input, compute the sum of the input numbers modulo a large prime.
(In this way, already a single scan suffices to produce a number that depends in a non-trivial way on the entire input.)

⋮

Do some magic!

— Recall the data stream algorithms for MISSING NUMBER or MULTISET-EQUALITY !

⋮

Write the sorted sequence onto the output read/write stream.

Lower Bound for Sorting

Theorem: *(Grohe, S., PODS'05)*

SORTING $\notin \mathrm{ST}\big(o(\log N), N^{1-\varepsilon}, O(1)\big)$ *(for every $\varepsilon > 0$)*

Proof method:

1. New machine model: List Machines
   - can only compare and move around input strings ($\rightsquigarrow$ weaker than TMs)
   - non-uniform & lots of states and tape symbols ($\rightsquigarrow$ stronger than TMs)
2. Show that list machines can simulate algorithms on read/write streams.
3. Prove that list machines cannot sort (. . . use combinatorics).

Randomised ST-Classes: RST and co-RST

Definition of RST: analogous to the class RP (randomised polynomial time):

An RST-algorithm produces

- no "false positives", i.e., it rejects "no"-instances with prob. 1
- "false negatives" with prob. < 0.1, i.e. it accepts "yes"-inst. with prob. > 0.9

A co-RST-algorithm has complementary probabilities for accepting resp. rejecting:

- no "false negatives", i.e. it accepts "yes"-instances with prob. 1
- "false positives" with prob. < 0.1, i.e. it rejects "no"-inst. with prob. > 0.9

Theorem: *(Grohe, Hernich, S., PODS'06)*

MULTISET-EQUALITY
$$\begin{cases} \notin \text{RST}(o(\log N), N^{1-\varepsilon}, O(1)) & \textit{(for every } \varepsilon > 0) \\ \in \text{co-RST}(2, O(\log N), 1) & \\ \in \text{ST}(O(\log N), O(1), 2) & \end{cases}$$

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Consequences

Separation of deterministic, randomised, and nondeterministic $\mathrm{ST}(\cdots)$-classes:

$$\begin{array}{ll} \mathrm{NST}(R, S, O(1)) & \\ \quad\quad | & \leftarrow \textsc{Multiset-Equality} \in \mathrm{NST}(3, O(\log N), 2) \\ \mathrm{RST}(R, S, O(1)) & \\ \quad\quad | & \leftarrow \textsc{Multiset-Equality} \in \text{co-}\mathrm{RST}(2, O(\log N), 1) \\ \mathrm{ST}(R, S, O(1)) & \end{array}$$

for all $R \subseteq o(\log n)$ and $O(\log n) \subseteq S \subseteq O(N^{1-\varepsilon})$

ST-Classes with 2-Sided Bounded Error

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Theorem: *(Beame, Jayram, Rudra, STOC'07)*

SET-DISJOINTNESS $\notin$ BPST$\left(o\left(\frac{\log N}{\log\log N}\right), N^{1-\varepsilon}, O(1)\right)$ *(for every $\varepsilon > 0$)*

Theorem: *(Beame, Huynh-Ngoc, FOCS'08)*

Approximating the frequency moments F_k with a randomised read/write stream algorithm with $o(\log N)$ head reversals requires (almost) as much internal memory as a "conventional" one-pass data stream algorithm.

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A few directions for future research

- Consider randomized versions of mp2s-automata:

  Design efficient randomized approximation algorithms for particular problems and develop techniques for proving lower bounds in the randomized model.

- Study the extension of the read/write stream model in which intermediate sorting steps are available.

  This is the StrSort model by Aggarwal, Datar, Rajagopalan, Ruhl, FOCS'04.

- An open question concerning finite cursor machines:

  Is there a sentence from first-order logic that cannot be evaluated by a composition of finite cursor machines and sorting operations?
  (Conjecture: yes!)

- An open question from complexity theory:

  Can the sorting problem be solved by a linear time multi-tape Turing machine?

Data stream talks during DEIS'10

- Data stream management systems and query languages (Tuesday, 8:45–9:45) — Sandra Geisler
- Basic algorithmic techniques for processing data streams (Tuesday, 9:45–10:45) — Mariano Zelke
- Querying and mining data streams (Wednesday, 11:15–12:15) — Elena Ikonomovska
- Stream-based processing of XML documents (Thursday, 11:15–12:15) — Cristian Riveros
- Distributed processing of data streams and large data sets (Thursday 1:45–2:45) — Marwan Hassani

Exercise # 4

Let s be a number with $0 < s < 1$.
The goal is to find a data stream algorithm that processes an input stream

$$x_1,\ x_2,\ x_3,\ \ldots,\ x_n$$

of elements from $\{1, \ldots, m\}$ and outputs a set M of input elements such that M contains (at least) all those elements that occur for $\geqslant s \cdot n$ times in the input stream.

Note:

- The output has to be a set — i.e., it is not allowed to output elements more than once. (In particular, this means that you cannot simply output the entire input stream.)
- The problem can be solved by a deterministic data stream algorithm using $O(\frac{1}{s} \cdot \log m \cdot \log n)$ memory bits.

References

References to the literature can be found in the following surveys:

- N. Schweikardt. Machine models and lower bounds for query processing. In Proc. PODS'07, pp. 41–52.
- N. Schweikardt. Machine models for query processing. SIGMOD Record 38(2), pp. 18–28, 2009.

Solutions to the exercises can be found in the following articles:

#1: S. Ganguly, A. Majumder: Deterministic K-set structure. Information Processing Letters 109(1), pp. 27–31, 2008.

#2: M. Grohe, A. Hernich, N. Schweikardt: Lower bounds for processing data with few random accesses to external memory. Journal of the ACM 56(3), 2009. — See Theorem 3.5.

#3: M. Henzinger, P. Raghavan, S. Rajagopalan: Computing on data streams. In *External Memory Algorithms*, J.M. Abello and J.S. Vitter (eds.). DIMACS Series in Discrete Mathematics and Theoretical Computer Science, vol. 50. AMS, New York, pp. 107–118, 1999. — See Theorem 6.

#4: G. Schnitger: Lecture notes on "Internet Algorithmen" (in German). Goethe-Universität Frankfurt am Main, 2009. `http://www.thi.informatik.uni-frankfurt.de/Internet0809/skript.pdf` — See Algorithm 4.20 on page 72.

Thank You!