# Time-Independent Liveness in Time Petri Nets

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## Extended Abstract

Petri nets have been used to describe and study concurrent systems for more than forty-five years. At first glance, time and concurrence do not seem to have much in common. But if one looks closer, the opposite is the case. There are endless examples from different areas showing this. For this reason, a large variety of time dependent Petri nets have been introduced and well studied. One of the first such nets is the Time Petri net (TPN), introduced in [9].

Time Petri nets (TPN) are derived from classical Petri nets. Additionally, each transition $t$ is associated with a time interval $[a_t, b_t]$. Here $a_t$ and $b_t$ are relative to the time, when $t$ was enabled last. When $t$ becomes enabled, it can not fire before $a_t$ time units have elapsed, and it has to fire not later than $b_t$ time units unless $t$ was disabled in between by the firing of another transition. The firing of a transition itself takes no time. The time interval is designed by real numbers, but the interval bounds are nonnegative rational numbers. It is easy to see (cf. [3]) that w.l.o.g. the interval bounds can be considered as integers only. Thus, the interval bounds $a_t$ and $b_t$ of any transition $t$ are natural numbers, including zero and $a_t \leq b_t$ or $b_t = \infty$. $a_t$ is called *earliest firing time* of the transition $t$ (short: $eft(t)$) and $b_t$, the *latest firing time* of $t$ (short: $lft(t)$) .

Every possible situation in a given TPN can be described completely by a state $z = (m, h)$, consisting of a (place-) marking $m$ and a transition marking $h$. The (place-) marking, which is a place vector (i.e. the vector has as many components as places in the considered TPN), is defined as the marking notion in classical Petri nets. The time marking, which is a transition vector (i.e. the vector has as many components as transitions in the considered TPN), describes the time circumstances in the considered situation. In general, each TPN has an infinite number of states. Thus the central problem for analysis of a certain TPN is knowledge about its state space.

In [7]it is shown that the state space can be characterized parametrically and that knowledge about the reachable integer-states, i.e. states whose time markings are (nonnegative) integers, is sufficient to determine the entire behavior of the net at any point in time. In the case that some $lft$s $= \infty$, then a subset of all reachable integer-states, the so-called set of the essential-states, expresses the net behaviour (cf. [5]). A reachability graph $RG(Z)$ for a TPN $Z$ can be defined in such a way that its vertices are the reachable integer-states or the reachable essential-states, respectively. The edges are defined by the triples $(z, t, z')$ and

$(z, \tau, z')$, $\tau \in \mathbb{N}$, where $z \xrightarrow{t} z'$ and $z \xrightarrow{\tau} z'$, respectively. This graph is finite if and only if the set of the reachable markings of the net is finite. The calculation of a single integer-state is very easy.

Actually a reachability graph for TPN was first introduced by Berthomieu and Menasche in [2] res. Berthomieu and Diaz in [1]. They provide a method for analyzing the qualitative behavior of the net based on the computing of certain subsets of reachable states, called state classes. However, the essential-states method is exponentially better in worst case, but in the case that in a TPN the concurrence is rather low, then the state-classes method compute a smaller reachability graph.

A further way to analyze a TPN is the translation into a timed automaton and then to apply the analyzing algorithms used there (cf. [8]).

The liveness definition for TPNs is a consistent expansion of the liveness definition for classical PNs.

**Definition 1.** *Let $\mathcal{Z}$ be a TPN with initial marking $m_0$. Furthermore let $m$ be a reachable (place-) marking and $t$ be a transition in $\mathcal{Z}$. Then*

*(1) $t$ is live in $m$ iff for each (place-) marking $m'$ which is reachable in $\mathcal{Z}$ from $m$ there exists a further (place-) marking $m''$ reachable in $\mathcal{Z}$ from $m'$ and $t$ is enabled in $m''$.*
*(2) $m$ is live in $\mathcal{Z}$ iff all transitions in the TPN are live in $m$.*
*(3) $\mathcal{Z}$ is live iff $m_0$ is live in $\mathcal{Z}$.*

In [3] it is shown that there is no correlation between the livenes behaviour of a TPN and its skeleton (timeless PN), in general. However, a structurally restricted class of TPNs such that each TPN of this class is live iff its skeleton is live can be given. The class is introduced in [4] and a sketch of the proof can be found there. The complete proof is given in [6].

The class contains four restrictions: A TPN $\mathcal{Z}$ belongs to this class iff

(1) the skeleton of $\mathcal{Z}$ is a Free-Choice PN,
(2) the skeleton of $\mathcal{Z}$ is homogeneous one,
(3) for each place $p$ in $\mathcal{Z}$ it holds: $\mathcal{M}in(p) \leq \mathcal{M}ax(p)$, where $\mathcal{M}in(p)$ is the greatest $eft$ of all post-transitions of $p$ and $\mathcal{M}ax(p)$ is the smallest $lft$ of all post-transitions of $p$,
(4) all $lft$s are greater (and not equal) than zero.

The skip of every one of the restriction (2), (3) or (4) leads to the violation of the liveness behaviour equivalence between the TPN and its skeleton. The same is true when (1) is skipped and the skeleton is an arbitrary PN, but not at least Extanded Simple net

In this paper we prove, that the restriction (1) can be replaced by a more weakly restriction (1′), namely:
(1′) the skeleton of $\mathcal{Z}$ is an ES net.

[1] Every Free-Choice PN is an Extended Simple net, as well. The reverse does not hold in general.

Unfortunately, the old proof cannot be generelised for $(1')$ and, thus, the new proof bases on new results. First, all places are classified dependend on their sets of shared post-transitions. It is shown that this classification is unique for ES nets. Afterwards, all transitions are classified using the classification of their pre-places. Furthermore, the relation *conflict* between two transitions defines an equivalence relation in the set of all transition in an ES net. In a natural way a relation *dominant* can be also introduced in each set of the transitions of every TPN. All these make the structure of an ES net more transparent. Eventually the notion *a place is live/dead* is defined. Then, after proving six lemmata the proof is done by contradiction.

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