Blatt 6 (Aufg. 1 + Aufg. 2)

Aufg. 1

$$33x_1 + 13x_2 + 18x_3 \to \max$$

$$(P)\begin{cases} 8x_1 + 3x_2 + 4x_3 + u_1 = 32 \\ 12x_1 + 5x_2 + 7x_3 + u_2 = 51 \\ 5x_1 + 2x_2 + 3x_3 + u_3 = 21 \\ x_1 + x_2 + x_3 - u_4 + y_4 = 3 \\ x_i, u_j, y_4 \geq 0 \end{cases}$$

$\hookrightarrow (H)$: $-y_4 \to \max$

$$ZF_{(H)} = -y_4 = -3 + x_1 + x_2 + x_3 - u_4$$ 1 Pkt

		x_1	x_2	x_3	u_4
	-3	-1	-1	-1	1
u_1	32	8	3	4	0
u_2	51	12	5	7	0
u_3	21	5	2	3	0
y_4	3	1	1	1	-1

1 Pkt

		y_4	x_2	x_3	u_4
	0	1	0	0	0
u_1	8	-8	-5	-4	8
u_2	15	-12	-7	-5	12
u_3	6	-5	-3	-2	5
x_1	3	1	1	1	-1

1 Pkt

$$ZF_{(P)} = 33x_1 + 13x_2 + 18x_3 =$$
$$= 33(3 - x_2 - x_3 + u_4) + 13x_2 + 18x_3 =$$
$$= 99 - 33x_2 - 33x_3 + 33u_4 + 13x_2 + 18x_3 =$$
$$= 99 - 20x_2 - 15x_3 + 33u_4$$ 1 Pkt

		x_2	x_3	u_4
	99	20	15	-33
u_1	8	-5	-4	8
u_2	15	-7	-5	12
u_3	6	-3	-2	5
x_1	3	1	1	-1

1 Pkt

		x_2	x_3	u_1
	132	-35/8	-13/2	33/8
u_4	1	-5/8	-1/2	1/8
u_2	3	1/2	1	$-\frac{3}{2}$
u_3	1	1/8	1/2	$-\frac{5}{8}$
x_1	4	3/8	1/2	1/8

1 Pkt

		u_2	x_3	u_1
	$135\frac{3}{4}$	5/4	$-\frac{1}{4}$	9/4
u_4	19/4	5/4	3/4	'
x_2	6	2	2	-3
u_3	1/4	$-\frac{1}{4}$	1/4	-1/4
x_1	7/4	-3	-1/4	'

1 Pkt

		u_2	u_3	u_1
	136	~~3/8~~ 1	1	2
u_4	4	2	-3	-1
x_2	4	4	-8	1
x_3	1	+1	4	-1
x_1	2	+1	+1	1

$\Rightarrow \bar{x} = \begin{pmatrix} 2 \\ 4 \\ 1 \end{pmatrix} \quad ZF(\bar{x}) = 136$ 1 Pkt

Def. 2 $\cong$ Def. 3 Aufg. 2

") Def. 3 $\to$ Def. 2 folgt sofort, in dem es der Def. 3 keine Ungleichungen gibt, d.h. keine A_2, b_2

1 Pkt

$\Rightarrow$) Def. 2 $\to$ Def. 3

Betr. (P_1) : $\max\left\{\langle c, x\rangle \;\middle|\; \begin{array}{l} A_1 x = b_1 \\ A_2 x \le b_2 \\ x \ge 0 \end{array}\right\}$

1 Pkt

z.z.: (D_1): $\min\left\{\left\langle \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}, \begin{pmatrix} v \\ w \end{pmatrix}\right\rangle \;\middle|\; A_1^T v + A_2^T w \ge c,\ w \ge 0\right\}$

dazu: $(P_1) \cong \max\left\{\langle c, x\rangle \;\middle|\; \begin{array}{l} A_1 x = b_1 \\ A_2 x + E u = b_2 \\ x \ge 0,\ u \ge 0 \end{array}\right\}$

1 Pkt

dabei sei $x, c \in \mathbb{R}^n$

$b_1 \in \mathbb{R}^{m_1}$, $b_2 \in \mathbb{R}^{m_2}$

$A_1 \in \mathcal{M}(m_1, n)$ $\quad \Rightarrow u \in \mathbb{R}^{m_2}$

$A_2 \in \mathcal{M}(m_2, n)$

$$\cong \max\left\{\left\langle \begin{pmatrix} c \\ 0 \end{pmatrix}\begin{pmatrix} x \\ u \end{pmatrix}\right\rangle \;\middle|\; \Bigg(\begin{array}{c|c} A_1 & 0_{m_1 \times m_2} \\ \hline \underbrace{A_2}_{n} & \underbrace{E_{m_2}}_{m_2} \end{array}\Bigg) \begin{array}{l} \}m_1 \\ \}m_2 \end{array} \begin{pmatrix} x \\ u \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix},\ \begin{pmatrix} x \\ u \end{pmatrix} \ge 0_{n+m_2} \right\}.$$

2 Pkt

$$\xrightarrow{\text{n. Def. 2}} (D_1) = \min\left\{\left\langle \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}, \begin{pmatrix} v \\ w \end{pmatrix}\right\rangle \;\middle|\; \left(\begin{array}{c|c} A_1 & 0 \\ \hline A_2 & E_{m_2} \end{array}\right)^T \cdot \begin{pmatrix} v \\ w \end{pmatrix} \ge \begin{pmatrix} c \\ 0_{m_2} \end{pmatrix}\right\}$$

2 Pkt

- 2 -

$$= \min\left\{ \left\langle \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}, \begin{pmatrix} v \\ w \end{pmatrix} \right\rangle \;\middle|\; \left(\begin{array}{c|c} A_1^T & A_2^T \\ \hline 0 & E_{m_2} \end{array} \right) \cdot \begin{pmatrix} v \\ w \end{pmatrix} \geq \begin{pmatrix} c \\ 0_{m_2} \end{pmatrix} \right\}$$ 1 Pkt

$$\cong \min\left\{ \left\langle \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}, \begin{pmatrix} v \\ w \end{pmatrix} \right\rangle \;\middle|\; \begin{array}{l} A_1^T \cdot v + A_2^T w \geq c \\ E_{m_2} \cdot w \geq 0_{m_2} \end{array} \right\}$$ 1 Pkt

$$\cong \min\left\{ \left\langle \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}, \begin{pmatrix} v \\ w \end{pmatrix} \right\rangle \;\middle|\; \begin{array}{l} A_1^T v + A_2^T w \geq c \\ w \geq 0 \end{array} \right\}$$ 1 Pkt