Time and Concurrency - Approaches for Intertwining of Time and Petri Nets

Louchka Popova-Zeugmann

Humboldt-Universität zu Berlin
Department of Computer Science

CS&P 2015, Rzeszów

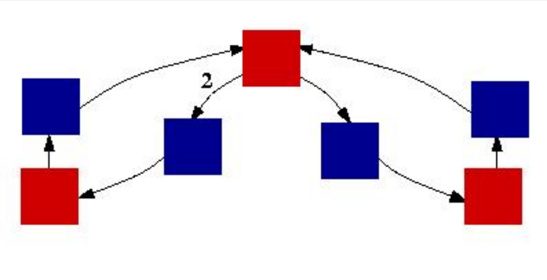
Which of the time-dependent Petri nets is the best?

Clocks were standing or hanging wherever Momo looked
– not only conventional clocks but spherical timepieces
showing what time it was anywhere in the world
...
"Perhaps one needs a watch like yours to recognize them by"
said Momo
Professor Hora smiled and shook his head.
"No, my child, the watch by itself would be no use for anyone.
You have to know how to read it as well."
Michael Ende, Momo

Outline

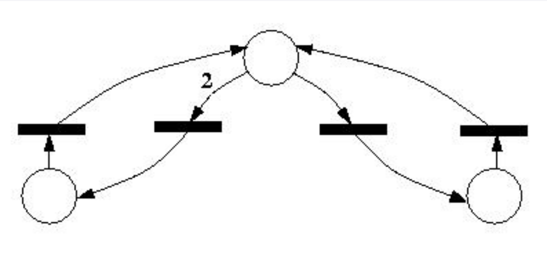
Statics:

non initialized Petri Net



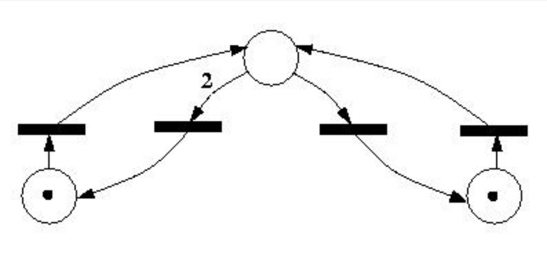
finite two-coloured weighted directed graph

Statics: non initialized Petri Net



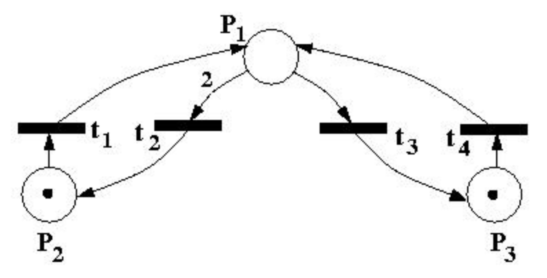
finite two-coloured weighted directed graph

Statics: initialized Petri Net



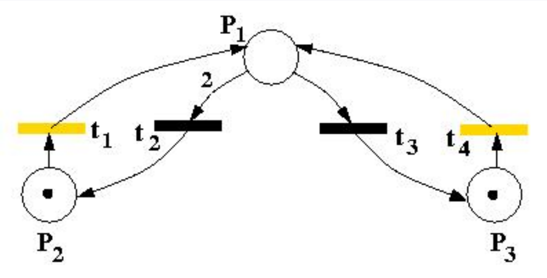
Statics:

initialized Petri Net



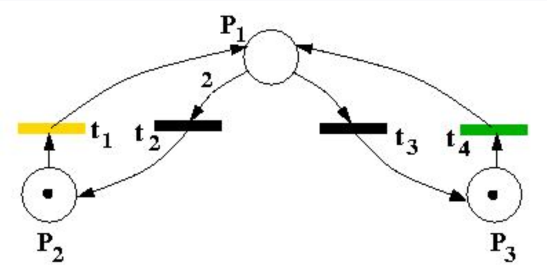
initial marking: $m_0 = (0, 1, 1)$

Dynamics: firing rule



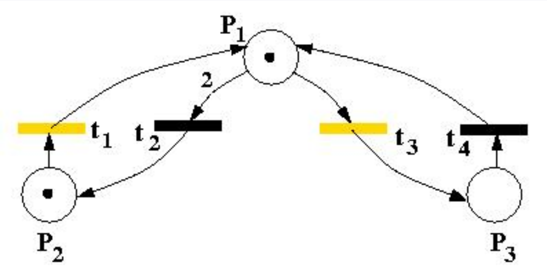
$$m_0 = (0, 1, 1)$$

Dynamics: firing rule



$$m_0 = (0, 1, 1)$$

Dynamics: firing rule



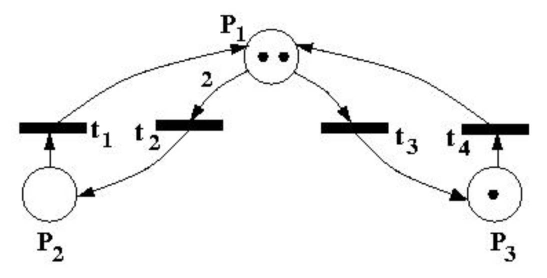
$$m_0 = (0, 1, 1)$$
$$m_1 = (1, 1, 0)$$
$$\vdots$$

Time Assignment

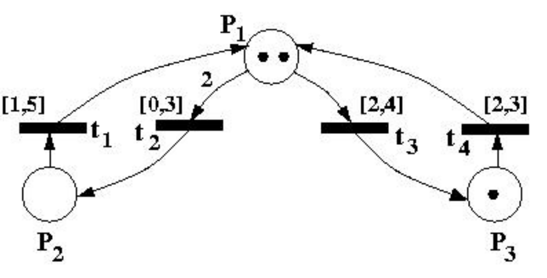
- time dependent Petri Nets with time specification at
  - transitions places
  - arcs
  - tokens
- time dependent Petri Nets with
  - deterministic
  - stochastic

  time assignment.

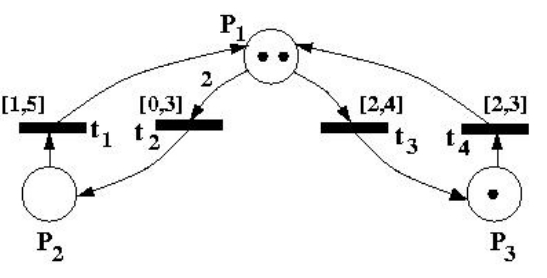
Statics: Petri Net (Skeleton)



Statics: **Time Petri Net**

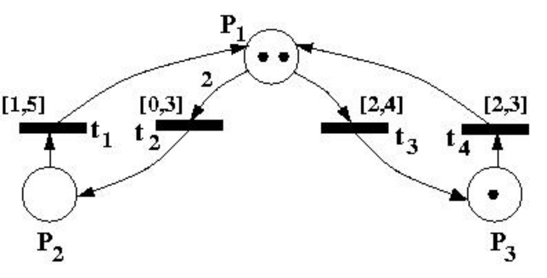


Statics: **Time Petri Net**



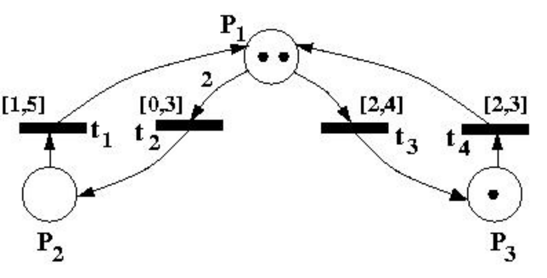
- $m_0 = (2, 0, 1)$

Statics: Time Petri Net



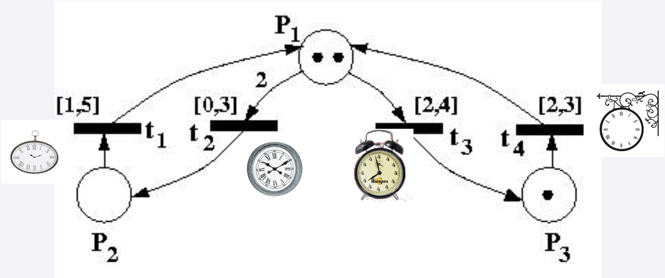
- $m_0 = (2, 0, 1)$ *p*-marking

Statics: Time Petri Net



- $m_0 = (2, 0, 1)$ *p*-marking
- $h_0 = (\sharp, 0, 0, 0)$ *t*-marking

Statics: **Time Petri Net**



- $m_0 = (2, 0, 1)$ *p*-marking
- $h_0 = (\sharp, 0, 0, 0)$ *t*-marking

$h(t)$ is the time shown by the clock of t since the last enabling of t

State

The pair $z = (m, h)$ is called a **state** in a TPN $\mathcal{Z}$, iff:

- m is a p-marking in $\mathcal{Z}$.
- h is a t-marking in $\mathcal{Z}$.

Dynamics: firing rules

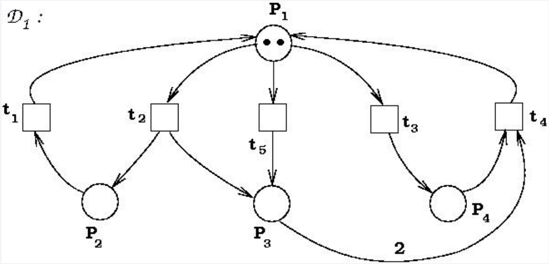
Let $\mathcal{Z}$ be a TPN and let $z = (m, h)$, $z' = (m', h')$ be two states.
$\mathcal{Z}$ changes from state $z = (m, h)$ into the state $z' = (m', h')$ by:

firing a transition / \ **time elapsing**

Notation: $z \xrightarrow{t} z'$ $z \xrightarrow{\tau} z'$

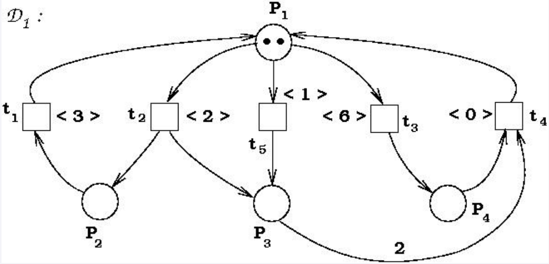
Timed Petri Net: An Informal Introduction

Statics: **Petri Net**



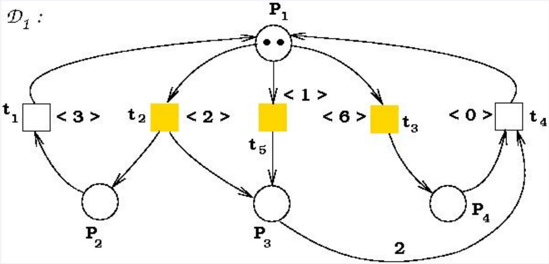
Timed Petri Net: An Informal Introduction

Statics: **Timed Petri Net**



Timed Petri Net: An Informal Introduction

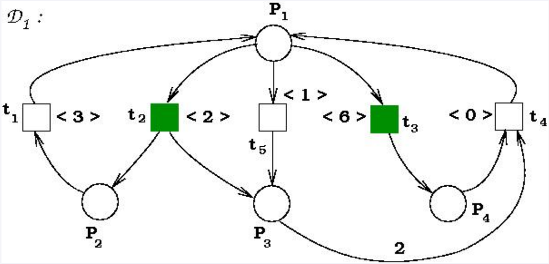
Dynamics:



firing mode: maximal step

Timed Petri Net: An Informal Introduction

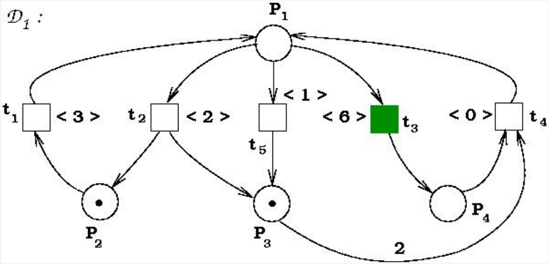
Dynamics:



firing mode: maximal step

Timed Petri Net: An Informal Introduction

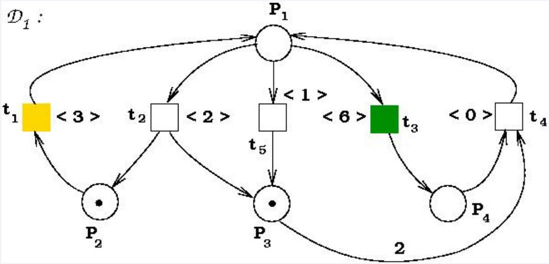
Dynamics:



firing mode: maximal step

Timed Petri Net: An Informal Introduction

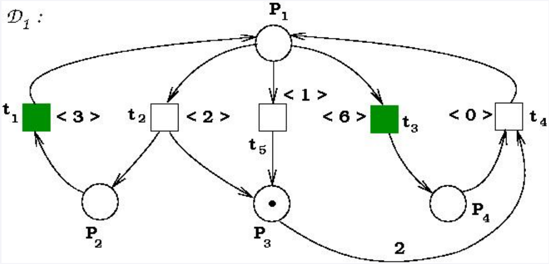
Dynamics:



firing mode: maximal step

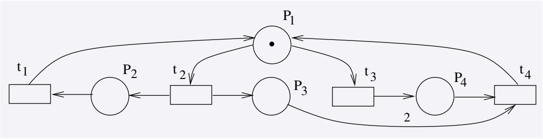
Timed Petri Net: An Informal Introduction

Dynamics:



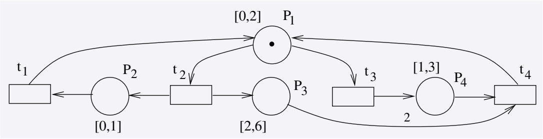
firing mode: maximal step

Petri Nets with Time Windows (tw-PN): An Informal Introduction



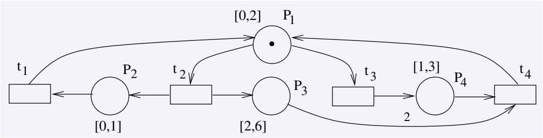
A **Petri Net with Time Windows** $\mathcal{P} = (\mathcal{N}, \mathcal{I})$
is a Petri net $\mathcal{N}$

Petri Nets with Time Windows (tw-PN): An Informal Introduction



A **Petri Net with Time Windows** $\mathcal{P} = (\mathcal{N}, \mathcal{I})$
is a Petri net $\mathcal{N}$
with time intervals (windows) attached to the places.

Initial Time Marking



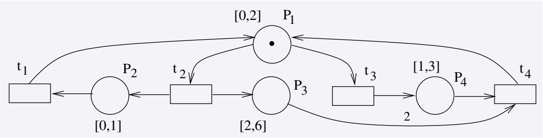
The initial time marking is given by

$$M_0 = (\overbrace{0}^{M(p_1)}; \overbrace{\varepsilon}^{M(p_2)}; \overbrace{\varepsilon}^{M(p_3)}; \overbrace{\varepsilon}^{M(p_4)})$$

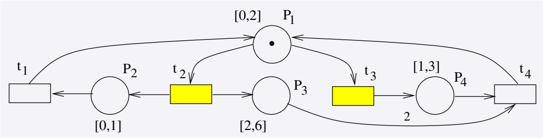
the initial (timeless) marking by

$$m_{M_0} = (1; 0; 0; 0) = m_0$$

Firing a transition t



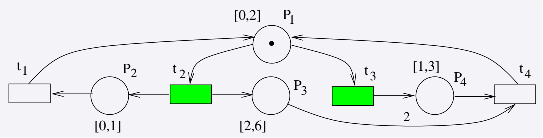
Firing a transition t



"enough" tokens on pre-places of t

$\Rightarrow$ transition t enabled

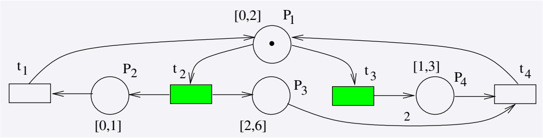
Firing a transition *t*



“enough” tokens on pre-places of *t*
$\Rightarrow$ transition *t* enabled

all needed tokens “old enough”
$\Rightarrow$ transition *t* ready to fire

Firing a transition t



"enough" tokens on pre-places of t

$\Rightarrow$ transition t enabled

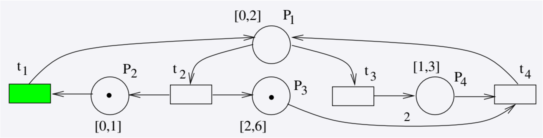
all needed tokens "old enough"

$\Rightarrow$ transition t ready to fire

$M_0 = (0, \varepsilon, \varepsilon, \varepsilon)$

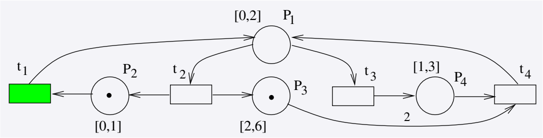
$\Rightarrow$ t_2 and t_3: enabled and ready to fire

Firing a transition t



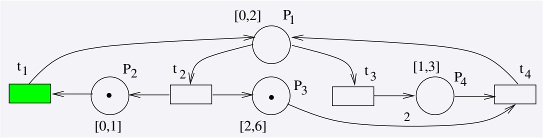
$$M_0 \xrightarrow{t_2} M_1 = (\varepsilon, 0, 0, \varepsilon)$$

Firing a transition t



$$M_0 \xrightarrow{t_2} M_1 = (\varepsilon, 0, 0, \varepsilon)$$
$$M_1 \xrightarrow{1} M_2 = (\varepsilon, 1, 1, \varepsilon)$$

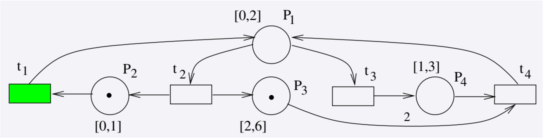
Firing a transition t



$$M_0 \xrightarrow{t_2} M_1 = (\varepsilon, 0, 0, \varepsilon)$$
$$M_1 \xrightarrow{1} M_2 = (\varepsilon, 1, 1, \varepsilon)$$

A transition is not forced to fire!

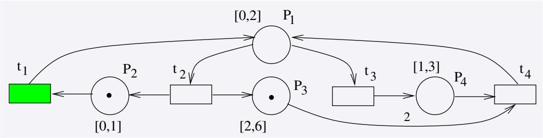
Firing a transition t



$$M_0 \xrightarrow{t_2} M_1 = (\varepsilon, 0, 0, \varepsilon)$$
$$M_1 \xrightarrow{1} M_2 = (\varepsilon, 1, 1, \varepsilon)$$

A transition is not forced to fire!
The age is reset when the retention time is greater than upper time bound.

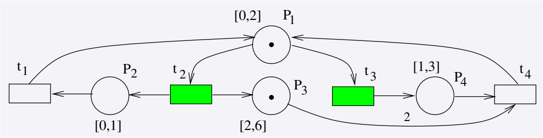
Firing a transition t



$$M_0 \xrightarrow{t_2} M_1 = (\varepsilon, 0, 0, \varepsilon)$$
$$M_1 \xrightarrow{1} M_2 = (\varepsilon, 1, 1, \varepsilon)$$
$$M_2 \xrightarrow{0.5} M_3 = (\varepsilon, 0.5, 1.5, \varepsilon)$$

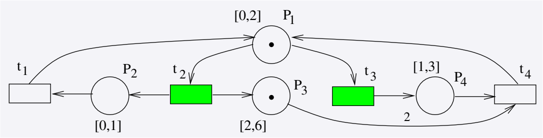
A transition is not forced to fire!
The age is reset when the retention time is greater than upper time bound.

Firing a transition t



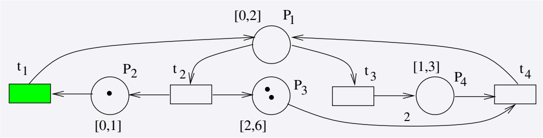
$$M_0 \xrightarrow{t_2} M_1 = (\varepsilon, 0, 0, \varepsilon)$$
$$M_1 \xrightarrow{1} M_2 = (\varepsilon, 1, 1, \varepsilon)$$
$$M_2 \xrightarrow{0.5} M_3 = (\varepsilon, 0.5, 1.5, \varepsilon)$$
$$M_3 \xrightarrow{t_1} M_4 = (0, \varepsilon, 1.5, \varepsilon)$$

Firing a transition t



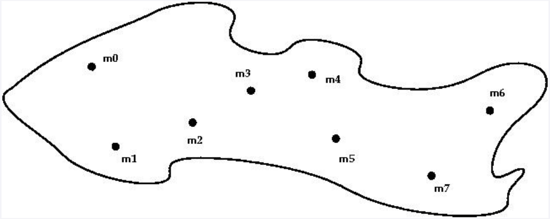
$$M_0 \xrightarrow{t_2} M_1 = (\varepsilon, 0, 0, \varepsilon)$$
$$M_1 \xrightarrow{1} M_2 = (\varepsilon, 1, 1, \varepsilon)$$
$$M_2 \xrightarrow{0.5} M_3 = (\varepsilon, 0.5, 1.5, \varepsilon)$$
$$M_3 \xrightarrow{t_1} M_4 = (0, \varepsilon, 1.5, \varepsilon)$$
$$M_4 \xrightarrow{1} M_5 = (1, \varepsilon, 2.5, \varepsilon)$$

Firing a transition t



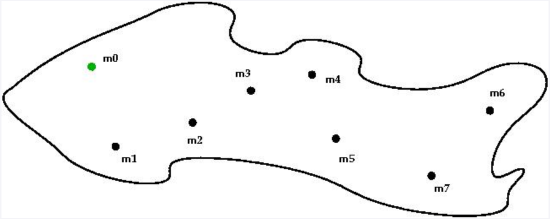
$$M_0 \xrightarrow{t_2} M_1 = (\varepsilon, 0, 0, \varepsilon)$$
$$M_1 \xrightarrow{1} M_2 = (\varepsilon, 1, 1, \varepsilon)$$
$$M_2 \xrightarrow{0.5} M_3 = (\varepsilon, 0.5, 1.5, \varepsilon)$$
$$M_3 \xrightarrow{t_1} M_4 = (0, \varepsilon, 1.5, \varepsilon)$$
$$M_4 \xrightarrow{1} M_5 = (1, \varepsilon, 2.5, \varepsilon)$$
$$M_5 \xrightarrow{t_2} M_6 = (\varepsilon, 0, 2.5\ 0, \varepsilon)$$

State Space of a Classic Petri Net



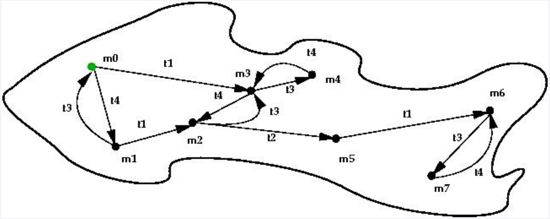
- The state space is the set of all reachable markings starting in m_0.

State Space of a Classic Petri Net



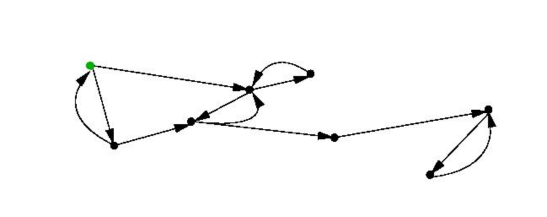
- The state space is the set of all reachable markings starting in m_0.

State Space of a Classic Petri Net

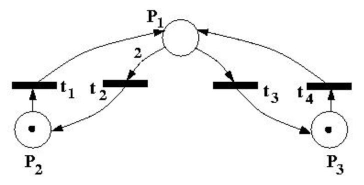


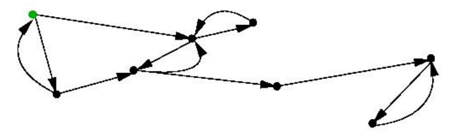
- The state space is the set of all reachable markings starting in m_0.
- All reachable markings + firing relation

State Space of a Classic Petri Net

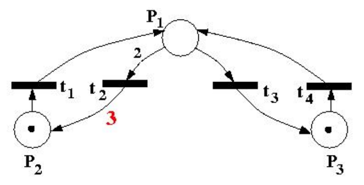


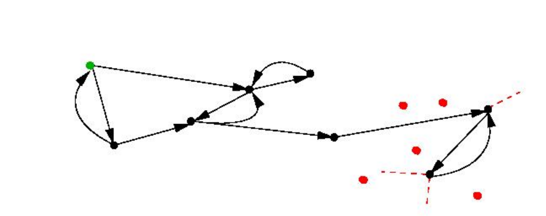
- The state space is the set of all reachable markings starting in m_0.
- All reachable markings + firing relation = reachability graph of the PN





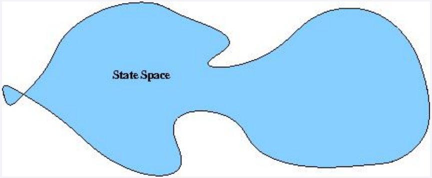
The reachability graph is finite





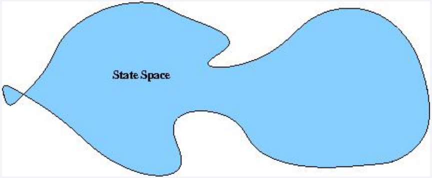
The reachability graph is infinite

The State Space of a Time Petri Net



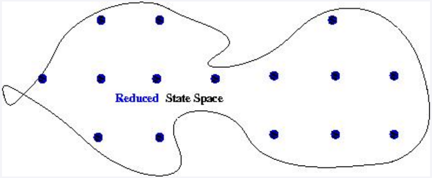
The set of all reachable states is infinite and dense, in general.

The State Space of a Timed Petri Net



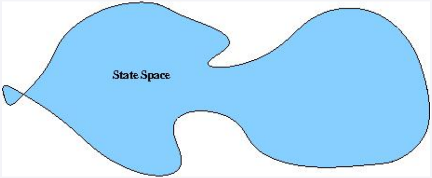
The set of all reachable states is infinite and dense, in general.

The State Space of a Timed Petri Net



The set of all states where a step can fire is a discrete one.

The State Space of a tw- Petri Net



The set of all reachable states is infinite and dense, in general.

Remark 1:

The classic Petri Nets are not Turing-complete.

Remark 2:

Time Petri Nets are Turing-complete.

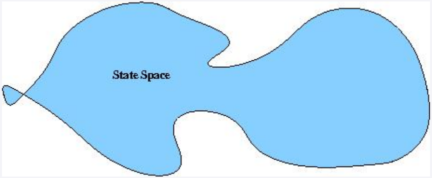
Remark 3:

Timed Petri Nets is are Turing-complete.

Remark 4:

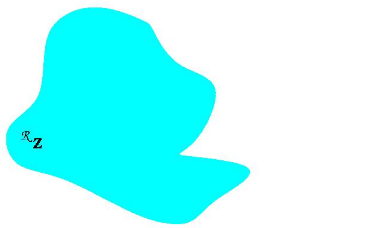
The tw-PNs are not Turing-complete.

Some Problems: The State Space



The set of all reachable states is dense.

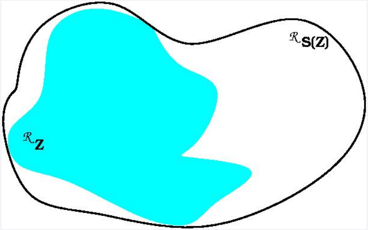
Some Further Problems: Reachability of *p*-markings



$\mathcal{R}_{\mathcal{Z}}$ is the set if all reachable *p*-markings in Z.

$\mathcal{R}_{\mathcal{S}(\mathcal{Z})}$ is the set of all reachable markings in the skeleton of Z (the state space of the skeleton of Z).

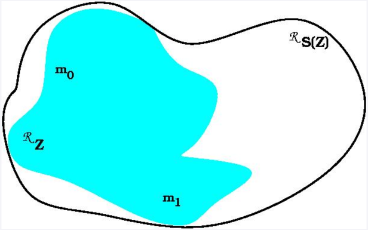
Some Further Problems: Reachability of *p*-markings



$\mathcal{R}_{\mathcal{Z}}$ is the set if all reachable *p*-markings in Z.

$\mathcal{R}_{\mathcal{S}(\mathcal{Z})}$ is the set of all reachable markings in the skeleton of *Z* (the state space of the skeleton of *Z*).

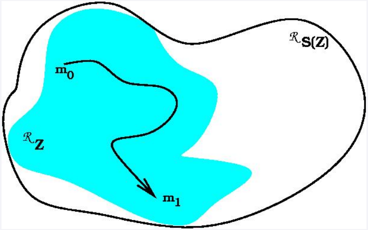
Some Further Problems: Reachability of *p*-markings



$\mathcal{R}_{\mathcal{Z}}$ is the set if all reachable *p*-markings in Z.

$\mathcal{R}_{\mathcal{S}(\mathcal{Z})}$ is the set of all reachable markings in the skeleton of *Z* (the state space of the skeleton of *Z*).

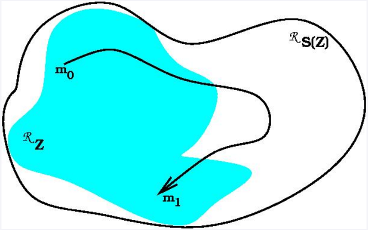
Some Further Problems: Reachability of *p*-markings



$\mathcal{R}_{\mathcal{Z}}$ is the set if all reachable *p*-markings in Z.

$\mathcal{R}_{\mathcal{S}(\mathcal{Z})}$ is the set of all reachable markings in the skeleton of Z (the state space of the skeleton of Z).

Some Further Problems: Reachability of *p*-markings



$\mathcal{R}_{\mathcal{Z}}$ is the set if all reachable *p*-markings in Z.

$\mathcal{R}_{\mathcal{S}(\mathcal{Z})}$ is the set of all reachable markings in the skeleton of *Z* (the state space of the skeleton of *Z*).

Parametric Run, Parametric State

Let $\mathcal{Z} = (P, T, F, V, m_0, I)$ be a TPN and $\sigma = t_1 \cdots t_n$ be a transition sequence in $\mathcal{Z}$.

$(\sigma(x), B_\sigma)$ is a **parametric run** of σ and (z_σ, B_σ) is a **parametric state** in $\mathcal{Z}$ with $z_\sigma = (m_\sigma, h_\sigma)$, if

- $m_0 \xrightarrow{\sigma} m_\sigma$
- $h_\sigma(t)$ is a sum of variables, (h_σ is a parametrical $t-$marking)
- B_σ is a set of conditions (a system of inequalities)

Parametric Run, Parametric State

Let $\mathcal{Z} = (P, T, F, V, m_0, I)$ be a TPN and $\sigma = t_1 \cdots t_n$ be a transition sequence in $\mathcal{Z}$.

$(\sigma(x), B_\sigma)$ is a **parametric run** of σ and (z_σ, B_σ) is a **parametric state** in $\mathcal{Z}$ with $z_\sigma = (m_\sigma, h_\sigma)$, if

- $m_0 \xrightarrow{\sigma} m_\sigma$
- $h_\sigma(t)$ is a sum of variables, (h_σ is a parametrical $t-$marking)
- B_σ is a set of conditions (a system of inequalities)

Obviously

- $z_0, \sigma \rightsquigarrow (z_\sigma, B_\sigma)$,

Parametric Run, Parametric State

Let $\mathcal{Z} = (P, T, F, V, m_0, I)$ be a TPN and $\sigma = t_1 \cdots t_n$ be a transition sequence in $\mathcal{Z}$.

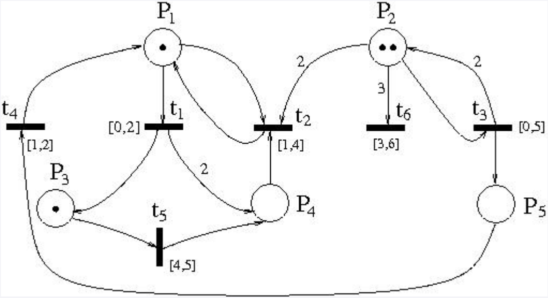
$(\sigma(x), B_\sigma)$ is a **parametric run** of σ and (z_σ, B_σ) is a **parametric state** in $\mathcal{Z}$ with $z_\sigma = (m_\sigma, h_\sigma)$, if

- $m_0 \xrightarrow{\sigma} m_\sigma$
- $h_\sigma(t)$ is a sum of variables, (h_σ is a parametrical $t-$marking)
- B_σ is a set of conditions (a system of inequalities)

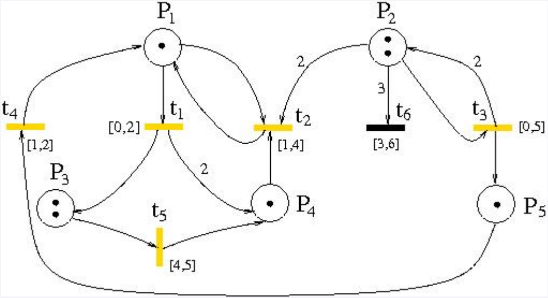
Obviously

- $z_0, \sigma \rightsquigarrow (z_\sigma, B_\sigma)$,
- $StSp(\mathcal{Z}) = \bigcup\limits_{(\sigma(x), B_\sigma)} \underbrace{\{z_{\sigma(x)} | x \text{ solves } B_\sigma\}}_{=:K_\sigma}$.

Runs



Runs



$$\sigma = t_1\ t_3\ t_4\ t_2\ t_3$$

$$\sigma(\tau) := z_0 \xrightarrow{\mathbf{0.7}} \xrightarrow{t_1} \xrightarrow{\mathbf{0.0}} \xrightarrow{t_3} \xrightarrow{\mathbf{0.4}} \xrightarrow{t_4} \xrightarrow{\mathbf{1.2}} \xrightarrow{t_2} \xrightarrow{\mathbf{0.5}} \xrightarrow{t_3} \xrightarrow{\mathbf{1.4}} z$$

$$\tau = 0.7\ 0.0\ 0.4\ 1.2\ 0.5\ 1.4$$

Example - Continuation

The run $\sigma(\tau)$ with

$$z_0 \xrightarrow{\mathbf{0.7}} \xrightarrow{t_1} \xrightarrow{\mathbf{0.0}} \xrightarrow{t_3} \xrightarrow{\mathbf{0.4}} \xrightarrow{t_4} \xrightarrow{\mathbf{1.2}} \xrightarrow{t_2} \xrightarrow{\mathbf{0.5}} \xrightarrow{t_3} \xrightarrow{\mathbf{1.4}} \left(m_\sigma, \begin{pmatrix} 1.9 \\ 1.4 \\ 1.4 \\ 1.4 \\ 4.2 \\ \sharp \end{pmatrix}\right)$$

is feasible.

Example - Continuation

$$\underbrace{\left(m_\sigma, \begin{pmatrix} \mathbf{1.0} \\ \mathbf{1.0} \\ \mathbf{1.0} \\ \mathbf{1.0} \\ \mathbf{4.0} \\ \sharp \end{pmatrix}\right)}_{z_0 \xrightarrow{\sigma(?)} \lfloor z \rfloor} \qquad \underbrace{\left(m_\sigma, \begin{pmatrix} \mathbf{1.9} \\ \mathbf{1.4} \\ \mathbf{1.4} \\ \mathbf{1.4} \\ \mathbf{4.2} \\ \sharp \end{pmatrix}\right)}_{z_0 \xrightarrow{\sigma(\tau)} z} \qquad \underbrace{\left(m_\sigma, \begin{pmatrix} \mathbf{2.0} \\ \mathbf{2.0} \\ \mathbf{2.0} \\ \mathbf{2.0} \\ \mathbf{5.0} \\ \sharp \end{pmatrix}\right)}_{z_0 \xrightarrow{\sigma(?)} \lceil z \rceil}$$

Example - Continuation

The runs

$$\sigma(\tau_1^*) := z_0 \xrightarrow{\mathbf{1}} \xrightarrow{t_1} \xrightarrow{\mathbf{0}} \xrightarrow{t_3} \xrightarrow{\mathbf{1}} \xrightarrow{t_4} \xrightarrow{\mathbf{1}} \xrightarrow{t_2} \xrightarrow{\mathbf{0}} \xrightarrow{t_3} \xrightarrow{\mathbf{1}} \lfloor z \rfloor$$

and

$$\sigma(\tau_2^*) := z_0 \xrightarrow{\mathbf{1}} \xrightarrow{t_1} \xrightarrow{\mathbf{0}} \xrightarrow{t_3} \xrightarrow{\mathbf{0}} \xrightarrow{t_4} \xrightarrow{\mathbf{2}} \xrightarrow{t_2} \xrightarrow{\mathbf{0}} \xrightarrow{t_3} \xrightarrow{\mathbf{2}} \lceil z \rceil$$

are also feasible in $\mathcal{Z}$.

Example - Continuation

The runs

$$\sigma(\tau_1^*) := z_0 \xrightarrow{\mathbf{1}} \xrightarrow{t_1} \xrightarrow{\mathbf{0}} \xrightarrow{t_3} \xrightarrow{\mathbf{1}} \xrightarrow{t_4} \xrightarrow{\mathbf{1}} \xrightarrow{t_2} \xrightarrow{\mathbf{0}} \xrightarrow{t_3} \xrightarrow{\mathbf{1}} \lfloor z \rfloor$$

$$\sigma(\tau) = z_0 \xrightarrow{\mathbf{0.7}} \xrightarrow{t_1} \xrightarrow{\mathbf{0.0}} \xrightarrow{t_3} \xrightarrow{\mathbf{0.4}} \xrightarrow{t_4} \xrightarrow{\mathbf{1.2}} \xrightarrow{t_2} \xrightarrow{\mathbf{0.5}} \xrightarrow{t_3} \xrightarrow{\mathbf{1.4}} z$$

$$\sigma(\tau_2^*) := z_0 \xrightarrow{\mathbf{1}} \xrightarrow{t_1} \xrightarrow{\mathbf{0}} \xrightarrow{t_3} \xrightarrow{\mathbf{0}} \xrightarrow{t_4} \xrightarrow{\mathbf{2}} \xrightarrow{t_2} \xrightarrow{\mathbf{0}} \xrightarrow{t_3} \xrightarrow{\mathbf{2}} \lceil z \rceil$$

are also feasible in $\mathcal{Z}$.

Main Property

Theorem 1:

Let $\mathcal{Z}$ be a TPN and $\sigma = t_1 \ \cdots \ t_n)$ be a feasible transition sequence in $\mathcal{Z}$ with a feasable run $\sigma(\tau)$ of σ $\big(\tau = \tau_0 \ldots \tau_n\big)$ i.e.

$$z_0 \xrightarrow{\tau_0}\xrightarrow{t_1} \cdots \xrightarrow{t_n}\xrightarrow{\tau_n} z_n = (m_n, h_n),$$

and all $\tau_i \in \mathbb{R}_0^+$.
Then, there exists a further feasible run $\sigma(\tau^*)$, $\tau^* = \tau_0^* \ldots \tau_n^*$ of σ with

$$z_0 \xrightarrow{\tau_0^*}\xrightarrow{t_1} \cdots \xrightarrow{t_n}\xrightarrow{\tau_n^*} z_n^* = (m_n^*, h_n^*).$$

such that

Main Property

Theorem 1 – Continuation:

$$z_0 \xrightarrow{\tau_0}\xrightarrow{t_1} \cdots \xrightarrow{t_n}\xrightarrow{\tau_n} z_n = (m_n, h_n),\ \tau_i \in \mathbb{R}_0^+.$$
$$z_0 \xrightarrow{\tau_0^*}\xrightarrow{t_1} \cdots \xrightarrow{t_n}\xrightarrow{\tau_n^*} z_n^* = (m_n^*, h_n^*)$$

1. For each $i, 0 \leq i \leq n$ the time τ_i^* is a natural number.
2. For each enabled transition t at marking $m_n (= m_n^*)$ it holds:
   1. $h_n^*(t) = \lfloor h_n(t) \rfloor$.
   2. $\sum_{i=1}^{n} \tau_i^* = \lfloor \sum_{i=1}^{n} \tau_i \rfloor$
3. For each transition $t \in T$ it holds:
   t is ready to fire in z_n iff t is also ready to fire in $\lfloor z_n \rfloor$.

Main Property

Theorem 1 – Continuation:

$$z_0 \xrightarrow{\tau_0} \xrightarrow{t_1} \cdots \xrightarrow{t_n} \xrightarrow{\tau_n} z_n = (m_n, h_n),\ \tau_i \in \mathbb{R}_0^+.$$

$$z_0 \xrightarrow{\tau_0^*} \xrightarrow{t_1} \cdots \xrightarrow{t_n} \xrightarrow{\tau_n^*} z_n^* = (m_n^*, h_n^*)\ ,\ \tau_i^* \in \mathbb{N}.$$

1. For each $i, 0 \leq i \leq n$ the time τ_i^* is a natural number.
2. For each enabled transition t at marking $m_n (= m_n^*)$ it holds:
   1. $h_n^*(t) = \lfloor h_n(t) \rfloor$.
   2. $\sum_{i=1}^{n} \tau_i^* = \lfloor \sum_{i=1}^{n} \tau_i \rfloor$
3. For each transition $t \in T$ it holds:
   t is ready to fire in z_n iff t is also ready to fire in $\lfloor z_n \rfloor$.

Main Property

Theorem 2:

Let $\mathcal{Z}$ be a TPN and $\sigma = t_1 \cdots t_n)$ be a feasible transition sequence in $\mathcal{Z}$, with feasable run $\sigma(\tau)$ of σ $\left(\tau = \tau_0 \ldots \tau_n\right)$ i.e.

$$z_0 \xrightarrow{\tau_0}\xrightarrow{t_1} \cdots \xrightarrow{t_n}\xrightarrow{\tau_n} z_n = (m_n, h_n),$$

and all $\tau_i \in \mathbb{R}_0^+$. Then, there exists a further feasible run $\sigma(\tau^*)$ of σ with

$$z_0 \xrightarrow{\tau_0^*}\xrightarrow{t_1} \cdots \xrightarrow{t_n}\xrightarrow{\tau_n^*} z_n^* = (m_n^*, h_n^*).$$

such that

Main Property

Theorem 2 – Continuation:

1. For each $i, 0 \leq i \leq n$ the time τ_i^* is a natural number.
2. For each enabled transition t at marking $m_n (= m_n^*)$ it holds:
   1. $h_n(t)^* = \lceil h_n(t) \rceil$.
   2. $\sum_{i=1}^{n} \tau_i^* = \lceil \sum_{i=1}^{n} \tau_i \rceil$
3. For each transition $t \in T$ holds:
   t is ready to fire in z_n iff t is also ready to fire in $\lceil z_n \rceil$.

Some Conclusions

- Each feasible transitions sequence σ in $\mathcal{Z}$ can be realized with an **integer** run.
- Each reachable *p*-marking in $\mathcal{Z}$ can be reached using **integer** runs only.
- If z is reachable in $\mathcal{Z}$, then $\lfloor z \rfloor$ and $\lceil z \rceil$ are reachable in $\mathcal{Z}$ as well.
- The length of the shortest and longest time path (if this is finite) between two arbitrary *p*-markings are natural numbers.

A run $\sigma(\tau) = \tau_0 \ t_1 \ \tau_1 \ldots t_n \ \tau_n$ is an **integer** one, if $\tau_i \in \mathbb{N}$ for each $i = 0 \ldots n$.

Integer States

A state $z = (m, h)$ is an **integer** one, if $h(t) \in \mathbb{N}$ for each in m enabled transition t.

Theorem 3:

Let $\mathcal{Z}$ be a finite TPN, i.e. $lft(t) \neq \infty$ for all $t \in T$.
The set of all reachable integer states in $\mathcal{Z}$ is finite
if and only if
the set of all reachable $p-$markings in $\mathcal{Z}$ is finite.

Integer States

A state $z = (m, h)$ is an **integer** one, if $h(t) \in \mathbb{N}$ for each in m enabled transition t.

Theorem 3:

Let $\mathcal{Z}$ be a finite TPN, i.e. $lft(t) \neq \infty$ for all $t \in T$.
The set of all reachable integer states in $\mathcal{Z}$ is finite
if and only if
the set of all reachable $p-$markings in $\mathcal{Z}$ is finite.

Remark:

Theorem 3 can be generalized for all TPNs (applying a further reduction of the state space).

Modified Rule

Let $\mathcal{Z}$ be an arbitrary TPN. The state change **by time elapsing** can be slightly **modified** for each transition t with $lft(t) = \infty$, because to fire such a transition t

- it is important if t is old enough to fire or not, i.e. if t has been enabled last for $eft(t)$ (or more) time units or t is younger.
- Thus, the time $h(t)$ increases **until** $eft(t)$. After that, the clock of t remains in this position (although the time is elapsing), unless t becomes disabled.

Essential States

Theorem 4:

In an arbitrary TPN a *p*-marking is reachable using the non-modified definition iff it is reachable using the modified one.

Essential States

Theorem 4:
In an arbitrary TPN a *p*-marking is reachable using the non-modified definition iff it is reachable using the modified one.

All reachable integer states in an arbitrary TPN, obtained by using the modified definition, are called the **essential states** of this net.

Essential States

Theorem 4:

In an arbitrary TPN a *p*-marking is reachable using the non-modified definition iff it is reachable using the modified one.

All reachable integer states in an arbitrary TPN, obtained by using the modified definition, are called the **essential states** of this net.

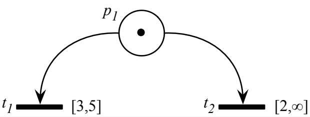
Theorem 5:

An arbitrary TPN is bounded iff the set of its essential states is finite.

Essential States

Remark:

The sets of all **reachable integer** states and the set of all **essential** states are incomparable in an infinite TPN, in general.



All reachable integer states are:

$\{(1, \begin{pmatrix} 0 \\ 0 \end{pmatrix}), (1, \begin{pmatrix} 1 \\ 1 \end{pmatrix}), (1, \begin{pmatrix} 2 \\ 2 \end{pmatrix}), (1, \begin{pmatrix} 3 \\ 3 \end{pmatrix}), (1, \begin{pmatrix} 4 \\ 4 \end{pmatrix}), (1, \begin{pmatrix} 5 \\ 5 \end{pmatrix}), (0, \begin{pmatrix} \sharp \\ \sharp \end{pmatrix})\}$ and

all essential states are:

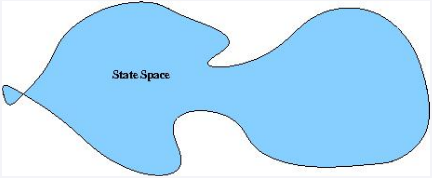
$\{(1, \begin{pmatrix} 0 \\ 0 \end{pmatrix}), (1, \begin{pmatrix} 1 \\ 1 \end{pmatrix}), (1, \begin{pmatrix} 2 \\ 2 \end{pmatrix}), (1, \begin{pmatrix} 3 \\ 2 \end{pmatrix}), (1, \begin{pmatrix} 4 \\ 2 \end{pmatrix}), (1, \begin{pmatrix} 5 \\ 2 \end{pmatrix}), (0, \begin{pmatrix} \sharp \\ \sharp \end{pmatrix})\}$.

Dense Semantics vs. Discrete Semantics

Corollary :

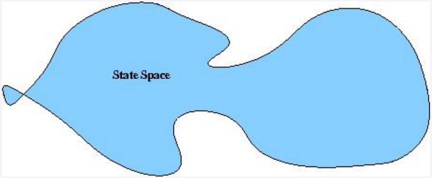
A Time Petri nets with **dense semantics** has the same behavior as the same net with **discrete semantics** w.r.t. boundedness, liveness etc.

Discrete Reduction of the State Space

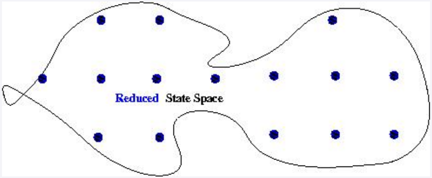


The set of all reachable states

Discrete Reduction of the State Space

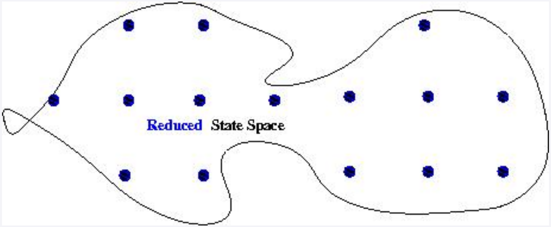


The set of all reachable states

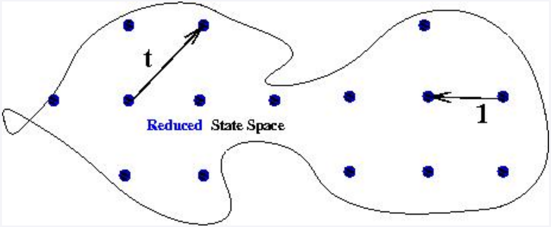


The set of all essential states

(Reduced) Reachability Graph

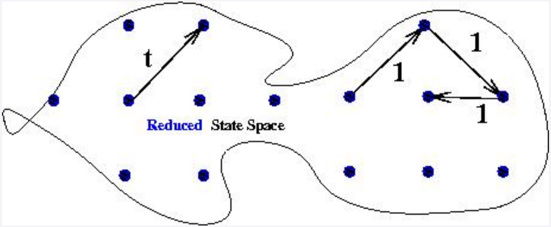


(Reduced) Reachability Graph



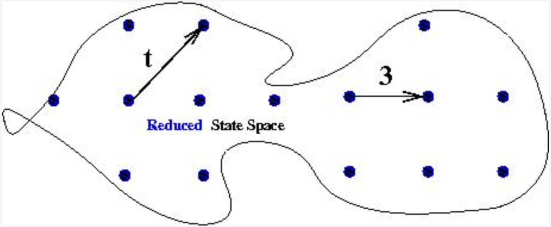
The reachability graph is a weighted directed graph, including the time explicit.

(Reduced) Reachability Graph



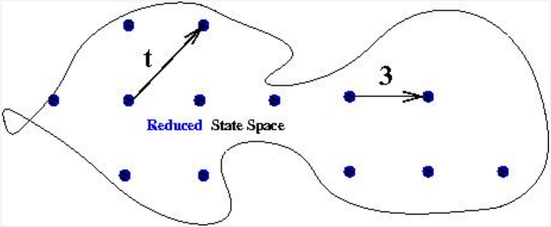
The reachability graph is a weighted directed graph, including the time explicit.

(Reduced) Reachability Graph



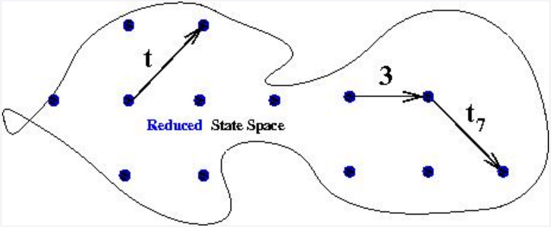
The reachability graph is a weighted directed graph, including the time explicit.

(Reduced) Reachability Graph



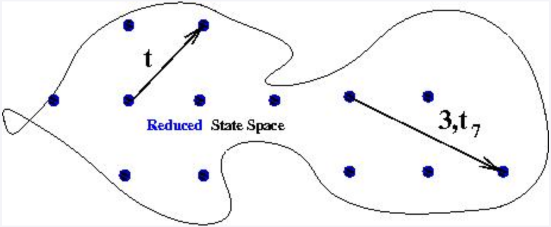
The reachability graph is a weighted directed graph, including the time explicit.

(Reduced) Reachability Graph



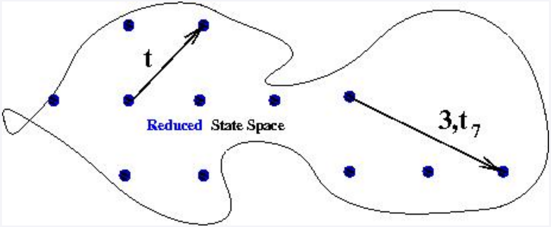
The reachability graph is a weighted directed graph, including the time explicit.

(Reduced) Reachability Graph



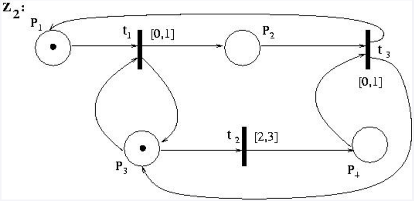
The reachability graph is a weighted directed graph, including the time explicit.

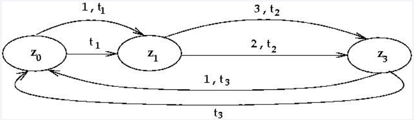
(Reduced) Reachability Graph



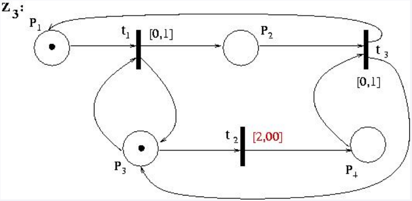
The reachability graph is a weighted directed graph, including the time explicit.

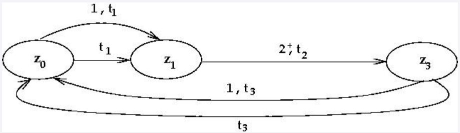
Example: A finite TPN and its reachability graph





Example: A non-finite TPN and its reachability graph





Boundedness: TPN vs. Skeleton

A TPN $\mathcal{Z}$ is bounded if the set of all its reachable *p*-markings is finite.

Theorem 6:

Let $\mathcal{Z}$ be a TPN and $S(\mathcal{Z})$ its skeleton. Than it holds:

- If $S(\mathcal{Z})$ is bounded then $\mathcal{Z}$ is bounded as well.
- If $\mathcal{Z}$ is bounded, then $S(\mathcal{Z})$ can be bounded or unbounded, i.e. the vice versa is not true.

Reachability in finite TPN

Theorem:

Let the skeleton $S(\mathcal{Z})$ of the TPN $\mathcal{Z}$ be bounded. Than it holds:

- The reachability of each p-marking in $\mathcal{Z}$ is decidable.
- The reachability of each rational state $z = (m, h)$ (i.e. $h(t)$ is a rational number for each enabled transition t by m) is decidable.

Reachability: TPN vs. Skeleton

Theorem (speeded nets):

Let $\mathcal{Z}$ be a TPN, $S(\mathcal{Z})$ its skeleton and $eft(t) = 0$ for all transitions t in $\mathcal{Z}$. Than a p-marking m is reachable in $\mathcal{Z}$ iff m is reachable in $S(\mathcal{Z})$.

Theorem (lazy nets):

Let $\mathcal{Z}$ be a TPN, $S(\mathcal{Z})$ its skeleton and $lft(t) = \infty$ for all transitions t in $\mathcal{Z}$. Than a p-marking m is reachable in $\mathcal{Z}$ iff m is reachable in $S(\mathcal{Z})$.

Liveness: Definitions

Let $\mathcal{Z}$ be a TPN, t be a transition in $\mathcal{Z}$ and z, z' two states in $\mathcal{Z}$.

- t is called **live in** $\mathcal{Z}$, if
  $\forall z \, \exists z' \, (\, z_0 \xrightarrow{*} z \xrightarrow{*} z' \xrightarrow{t} \,)$
- t is called **dead in** $\mathcal{Z}$, if
  $\forall z \, (\, z_0 \xrightarrow{*} z \not\xrightarrow{t} \,)$
- $\mathcal{Z}$ is called **live or dead,** resp., if all transitions in $\mathcal{Z}$ are live or dead , resp.

Liveness: Definitions

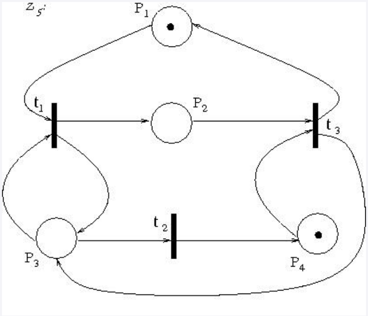
Let $\mathcal{Z}$ be a TPN, t be a transition in $\mathcal{Z}$ and z, z' two states in $\mathcal{Z}$.

- t is called **live in** $\mathcal{Z}$, if
  $\forall z \, \exists z' \, (\, z_0 \xrightarrow{*} z \xrightarrow{*} z' \xrightarrow{t} \,)$
- t is called **dead in** $\mathcal{Z}$, if
  $\forall z \, (\, z_0 \xrightarrow{*} z \overset{t}{\not\longrightarrow} \,)$
- $\mathcal{Z}$ is called **live or dead,** resp., if all transitions in $\mathcal{Z}$ are live or dead , resp.

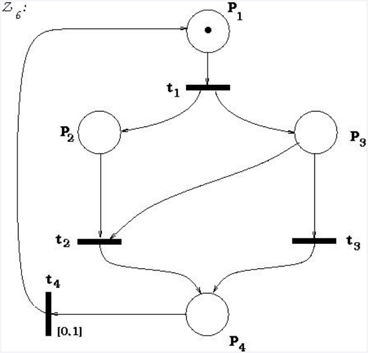
Remark:

There is not a correlation between the liveness behaviors of a TPN and its skeleton.

Liveness: TPN vs. Skeleton



$\mathcal{Z}_5$ is live
$S(\mathcal{Z}_5)$ is not live



$\mathcal{Z}_6$ is not live
$S(\mathcal{Z}_6)$ is live

Liveness: TPN vs. Skeleton

Theorem (speeded nets):

Let $\mathcal{Z}$ be a TPN, $S(\mathcal{Z})$ its skeleton and $eft(t) = 0$ for all transitions t in $\mathcal{Z}$. Than $\mathcal{Z}$ is live iff $S(\mathcal{Z})$ is live.

Theorem (lazy nets):

Let $\mathcal{Z}$ be a TPN, $S(\mathcal{Z})$ its skeleton and $lft(t) = \infty$ for all transitions t in $\mathcal{Z}$. Than $\mathcal{Z}$ is live iff $S(\mathcal{Z})$ is live.

Liveness: TPN vs. Skeleton

Theorem:

Let $\mathcal{Z}$ be a TPN , $S(\mathcal{Z})$ its skeleton such that

- $S(\mathcal{Z})$ is a EFC-Net,
- $S(\mathcal{Z})$ is homogeneous,

and it holds:

- $\mathcal{M}in(p) \leq \mathcal{M}ax(p)$ for each place p in $\mathcal{Z}$ and
- $lft(t) > 0$ for each transition t in $\mathcal{Z}$.

Than $\mathcal{Z}$ is live iff $S(\mathcal{Z})$ is live.

Liveness: TPN vs. Skeleton

Theorem:

Let $\mathcal{Z}$ be a TPN , $S(\mathcal{Z})$ its skeleton such that

- $S(\mathcal{Z})$ is a AC-Net,
- $S(\mathcal{Z})$ is homogeneous,

and it holds:

- $\mathcal{M}in(p) \leq \mathcal{M}ax(p)$ for each place p in $\mathcal{Z}$ and
- $lft(t) > 0$ for each transition t in $\mathcal{Z}$.

Than $\mathcal{Z}$ is live iff $S(\mathcal{Z})$ is live.

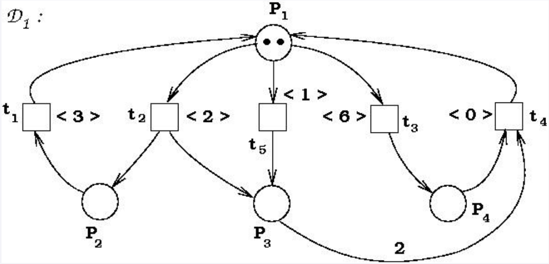
Some Decidable Quantitative Problems

Remark:

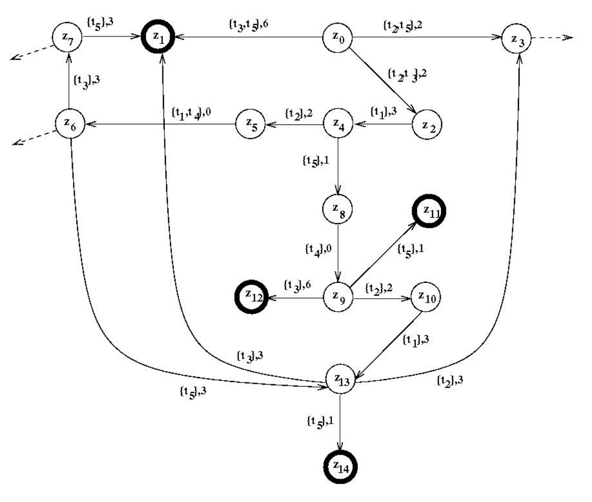
Using parametric states and/or the reachability graph (if it is finite one) a lot of quantitative problems are solvable:

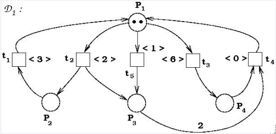
- existence of a run,
- minimal and maximal time length of a firing transition sequence,
- minimal and maximal distance between two essential states and between two *p*-markings, etc.

State Space: Reachability graph

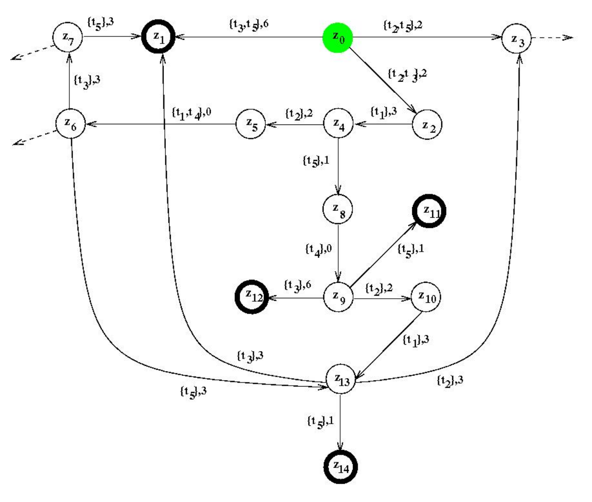


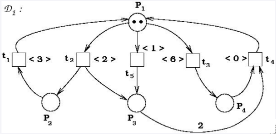
Reachability graph



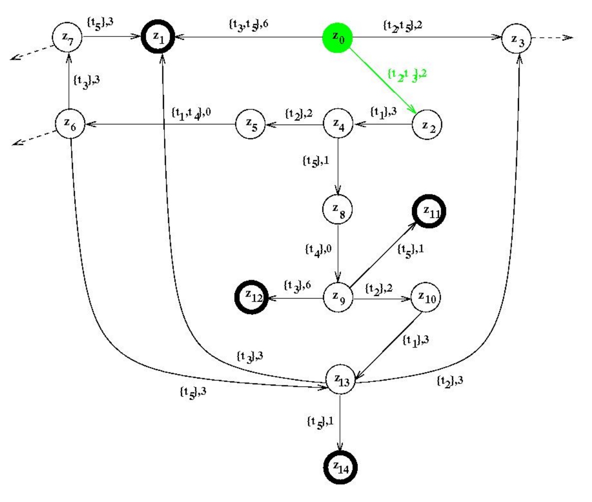


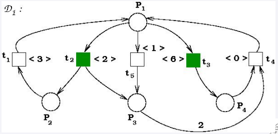
Reachability graph



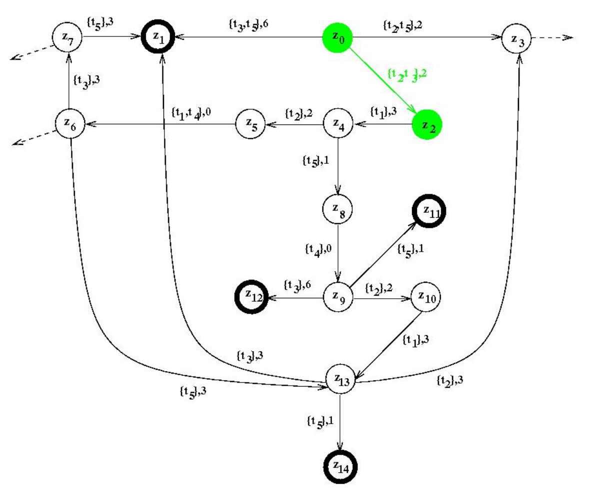


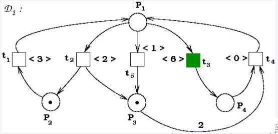
Reachability graph



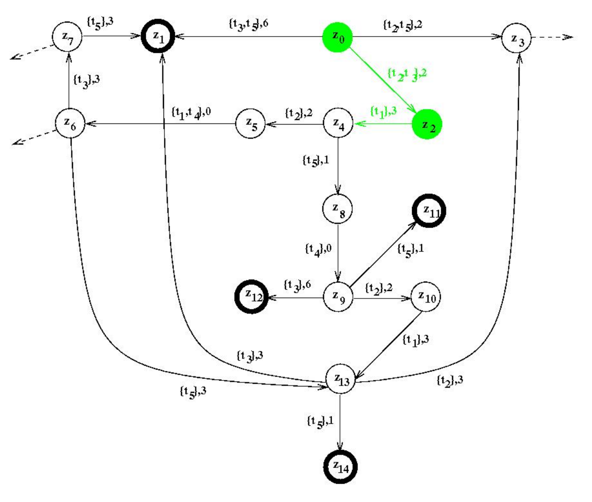


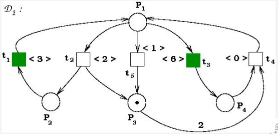
Reachability graph



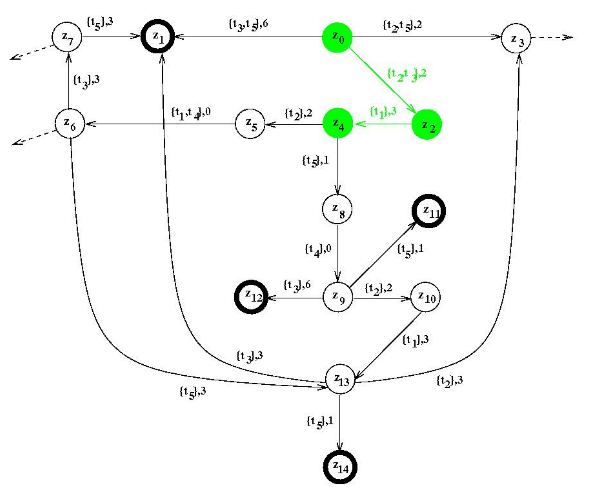


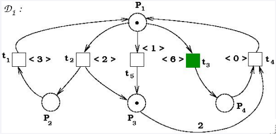
Reachability graph



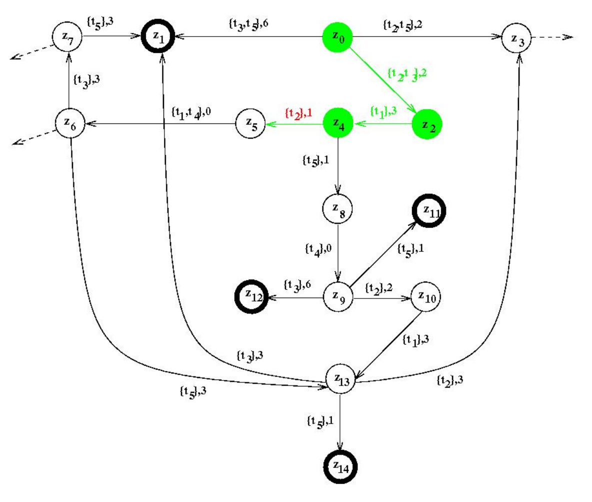


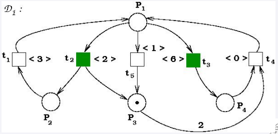
Reachability graph



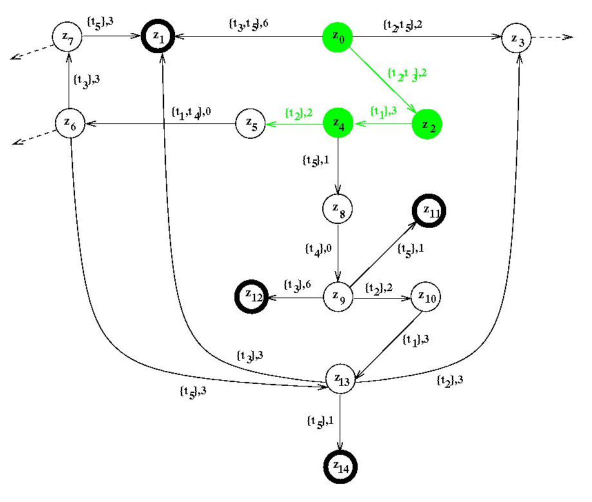


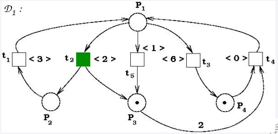
Reachability graph



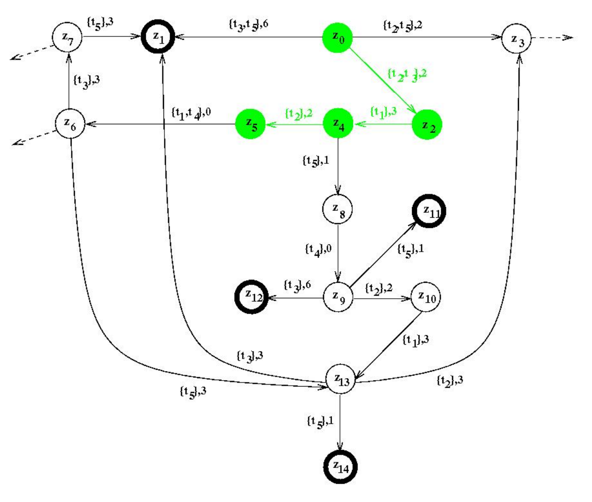


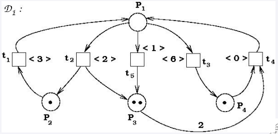
Reachability graph



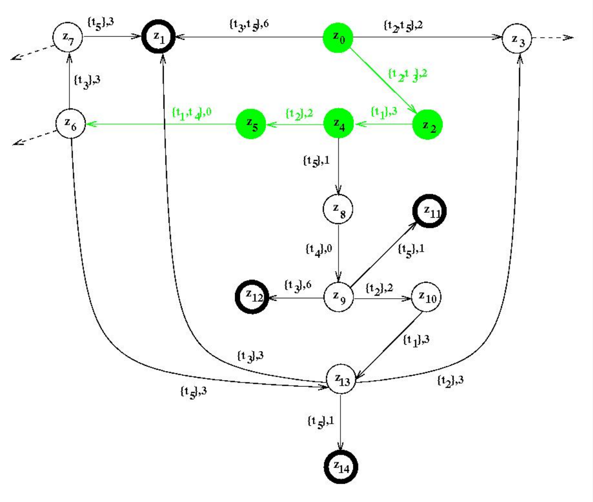


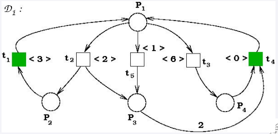
Reachability graph





Reachability graph





State Equation in classic PN

Let $\mathcal{N}$ be a classic PN with

- m_1 and m_2 two markings in $\mathcal{N}$,
- $\sigma = t_1 \dots t_n$ a firing sequence, and
- $m_1 \xrightarrow{\sigma} m_2$.

Then it holds:

$$m_2 = m_1 + C \cdot \pi_\sigma, \text{ (state equation)}$$

where C is the incidence matrix of $\mathcal{N}$ and π_σ is the Parikh vector of σ.

State Equation in classic PN

Let $\mathcal{N}$ be a classic PN with

- m_1 and m_2 two markings in $\mathcal{N}$,
- $\sigma = t_1 \ldots t_n$ a firing sequence, and
- $m_1 \xrightarrow{\sigma} m_2$.

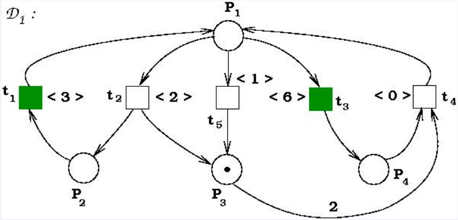
Then it holds:

$$m_2 = m_1 + C \cdot \pi_\sigma, \text{ (state equation)}$$

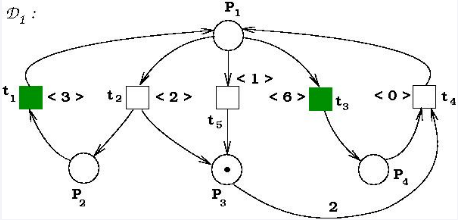
where C is the incidence matrix of $\mathcal{N}$ and π_σ is the Parikh vector of σ.

In each PN $\mathcal{N}$ with initial marking m_0 it holds:
If $m \neq m_0 + C \cdot \pi_\sigma$ for each π_σ then m is not reachable in $\mathcal{N}$.

Extended Form of a Place Marking



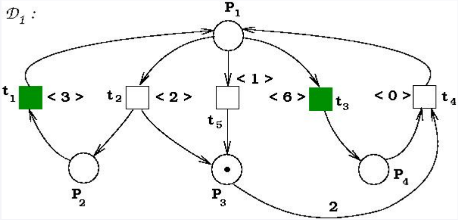
Extended Form of a Place Marking



$$m = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

extended form
of the *p*-markings *m*

Extended Form of a Place Marking



$$m = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{matrix} p_1 \\ p_2 \\ p_3 \\ p_4 \end{matrix}$$

extended form of the p-markings m

after

0 1 2 3 4 5 6

time units

Time Dependent State Equation

Theorem

Let $\mathcal{D}$ be a Timed Petri Net, $z^{(0)}$ be the initial state in extended form and

$$z^{(0)} \xrightarrow{\mathfrak{G}_1} \hat{z}^{(1)} \underset{1}{\longrightarrow} \tilde{z}^{(1)} \xrightarrow{\mathfrak{G}_2} \hat{z}^{(2)} \underset{1}{\longrightarrow} \ldots \xrightarrow{\mathfrak{G}_n} z^{(n)}$$

be a firing sequence ($\mathfrak{G}_i$ is a multiset for each i). Then, it holds:

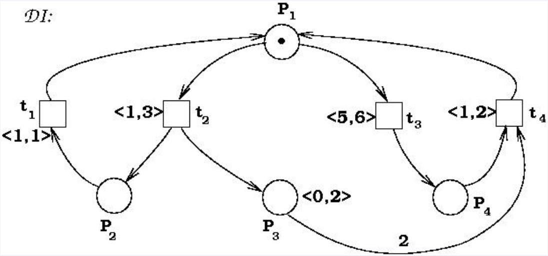
$$m^{(n)} = m^{(0)} \cdot R^{n-1} + C \cdot \Psi_\sigma. \qquad \text{State equation}$$

$$z^{(0)} \xrightarrow{\mathfrak{G}_1} \hat{z}^{(1)} \xrightarrow[1]{} \tilde{z}^{(1)} \xrightarrow{\mathfrak{G}_2} \hat{z}^{(2)} \xrightarrow[1]{} \dots \xrightarrow{\mathfrak{G}_n} z^{(n)}$$

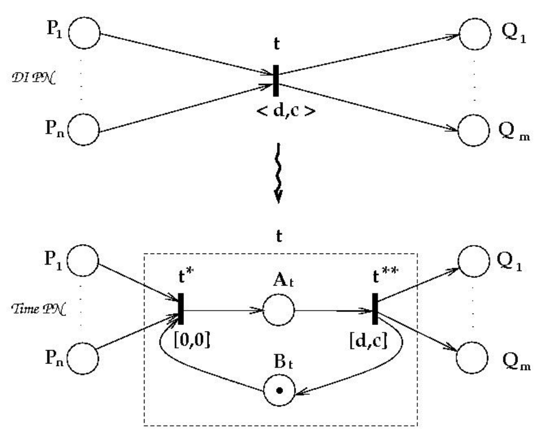
$$m^{(n)} = m^{(0)} \cdot R^{n-1} + C \cdot \Psi_\sigma.$$ State equation

- $m^{(n)}$ and $m^{(0)}$ are place markings in extended form
- R is the progress matrix for $\mathcal{D}$.
- C is the incidence matrix of $\mathcal{D}$ in extended form
- Ψ_σ is the Parikh matix of the sequence $\sigma = \mathfrak{G}_1 \ \mathfrak{G}_2 \ \dots \mathfrak{G}_n$ of multisets of transitions.

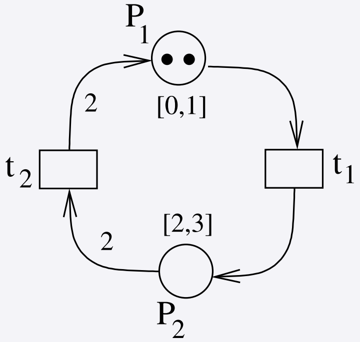
Timed Petri Nets with Uncertain Durations: An Informal Introduction



Transformation Timed PN –> Time PN

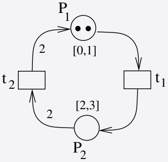


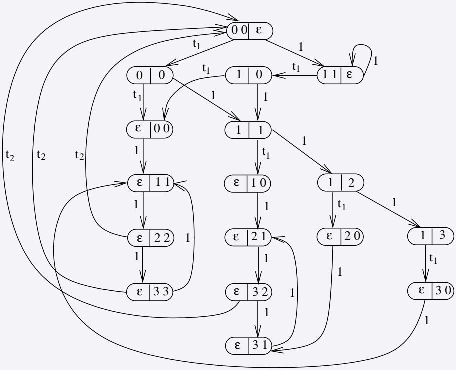
Reachability Graph: Natural Numbers vs. Real Numbers



Reachability Graph: Natural Numbers vs. Real Numbers

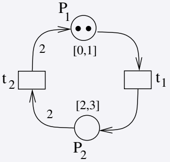
The integer reachability graph

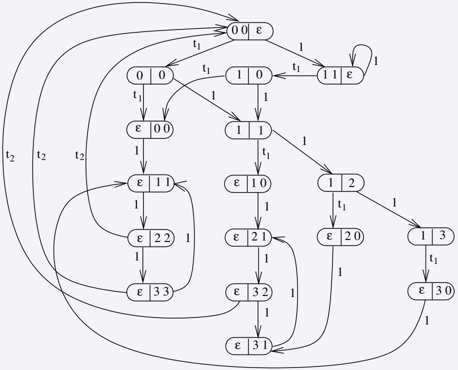




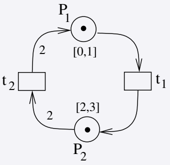
Reachability Graph:
Natural Numbers vs. Real Numbers

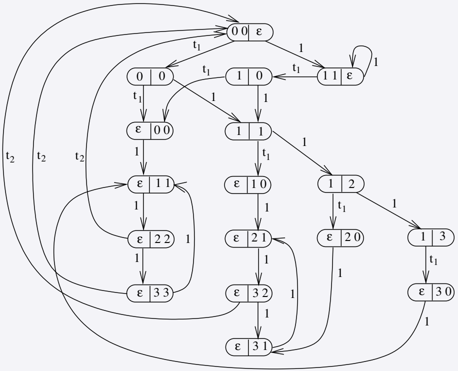
The integer reachability graph





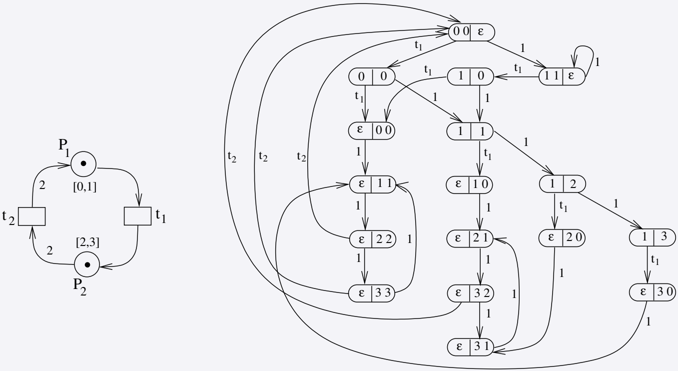
Reachability Graph: Natural Numbers vs. Real Numbers





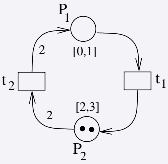
Consider $\sigma(\tau) = t_1$

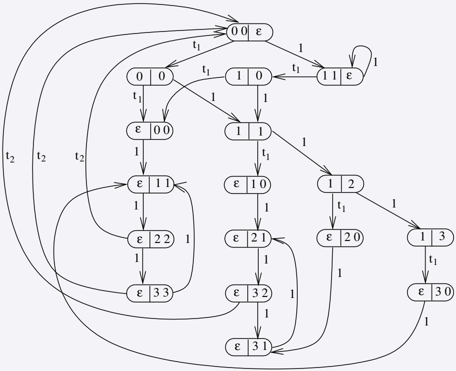
Reachability Graph: Natural Numbers vs. Real Numbers



Consider $\sigma(\tau) = t_1\ 1.5$

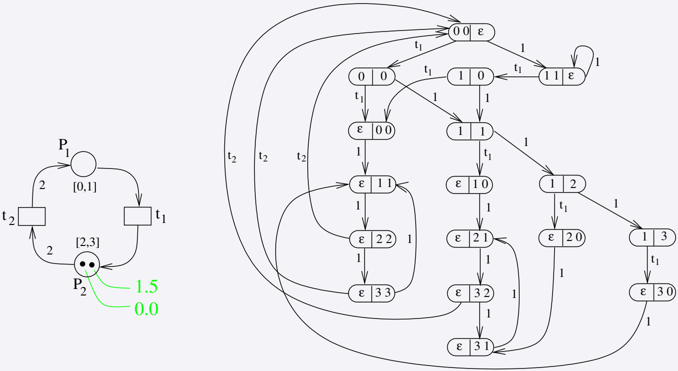
Reachability Graph: Natural Numbers vs. Real Numbers





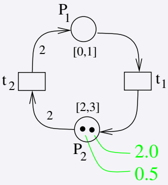
Consider $\sigma(\tau) = t_1 \ 1.5 \ t_1$

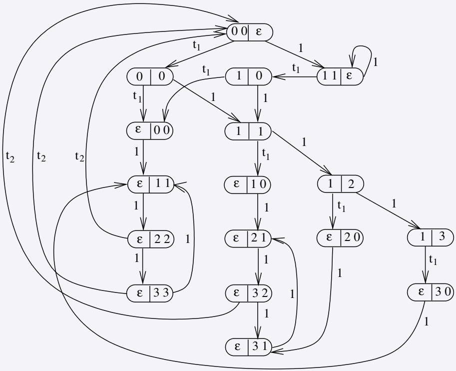
Reachability Graph: Natural Numbers vs. Real Numbers



Consider $\sigma(\tau) = t_1 \; 1.5 \; t_1$

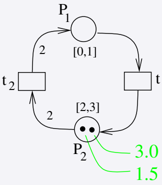
Reachability Graph: Natural Numbers vs. Real Numbers

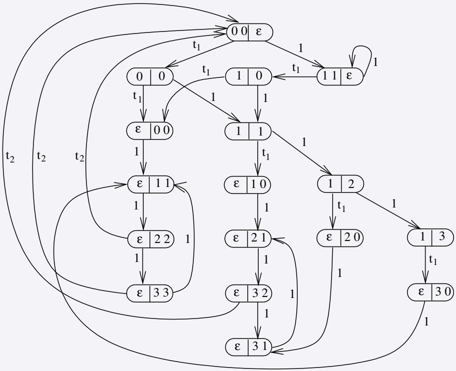




Consider $\sigma(\tau) = t_1\ 1.5\ t_1\ 0.5$

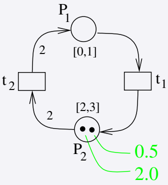
Reachability Graph: Natural Numbers vs. Real Numbers

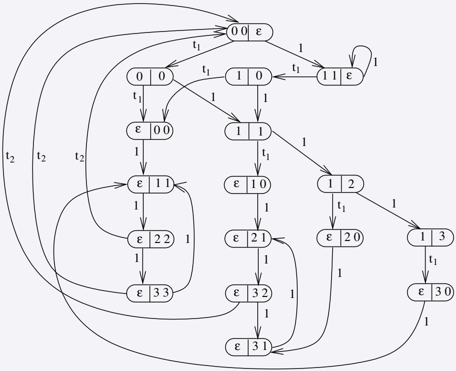




Consider $\sigma(\tau) = t_1\ 1.5\ t_1\ 0.5\ 1.0$

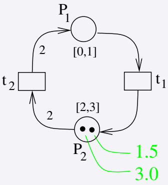
Reachability Graph: Natural Numbers vs. Real Numbers

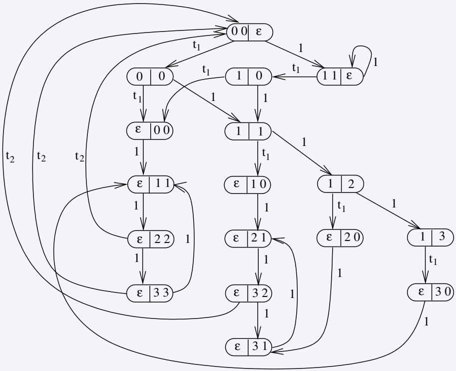




Consider $\sigma(\tau) = t_1\ 1.5\ t_1\ 0.5\ 1.0\ 0.5$

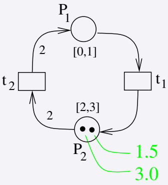
Reachability Graph: Natural Numbers vs. Real Numbers

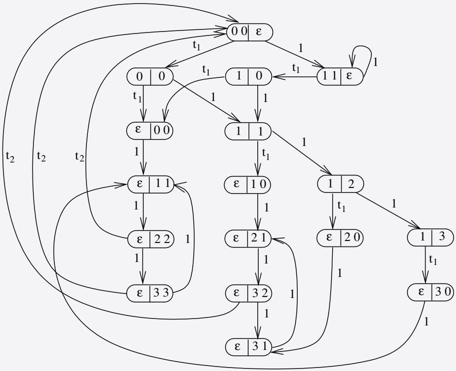




Consider $\sigma(\tau) = t_1\ 1.5\ t_1\ 0.5\ 1.0\ 0.5\ 1.0$

Reachability Graph: Natural Numbers vs. Real Numbers

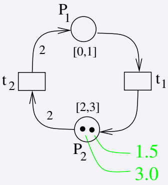


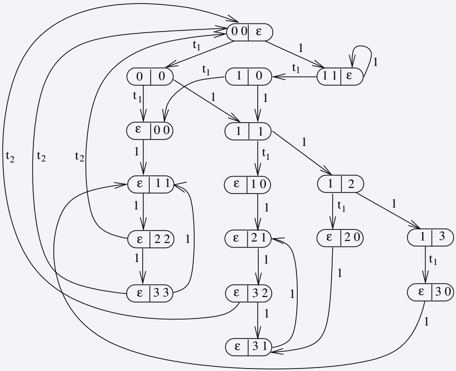


Consider $\sigma(\tau) = t_1\ 1.5\ t_1\ 0.5\ 1.0\ 0.5\ 1.0$
$\Rightarrow t_2$ is in $M = (\varepsilon, 3.0\ 1.5)$ in a t-DL

Reachability Graph: Natural Numbers vs. Real Numbers

There is no “leaf” in the integer reachability graph!





Consider $\sigma(\tau) = t_1\ 1.5\ t_1\ 0.5\ 1.0\ 0.5\ 1.0$
$\Rightarrow t_2$ is in $M = (\varepsilon, 3.0\ 1.5)$ in a t-DL

Theorem:

Let $\mathcal{P}$ be a PN with Time Windows and T be the set of its transitions. Than the transition sequence

$$\sigma = t_1 \cdots t_n$$

is a firing sequence in its skeleton $S(\mathcal{P})$ **iff** there exists a feasible run

$$\sigma(\tau) = \tau_0 t_1 \tau_1 t_2 \tau_2 \ldots \tau_{n-1} t_n$$

in $\mathcal{P}$ with $\tau_i \in \mathbb{R}_0^+$, for all i, $0 \leq i \leq n-1$.

Properties

Property "Reachability"

A marking M is reachable in a tP-PN $\mathcal{P}$ iff m_M is reachable in $S(\mathcal{P})$.

Properties

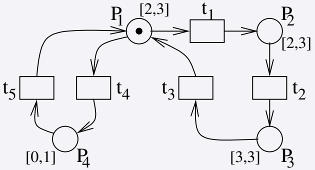
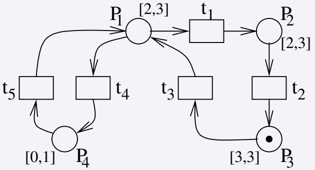
Property "Reachability"

A marking M is reachable in a tP-PN $\mathcal{P}$ iff m_M is reachable in $S(\mathcal{P})$.

Property "Liveness"

There is not a correlation between the liveness behaviors of a tP-PN and its skeleton.

Time Gaps

	
$\sigma(\tau_\alpha) = 3\ t_1\ 3\ t_2\ 3\ t_3$ $\Rightarrow \alpha = 9$	$\sigma(\tau_\beta) = 5\ t_1\ 2\ t_2\ 3\ t_3$ $\Rightarrow \beta = 10$

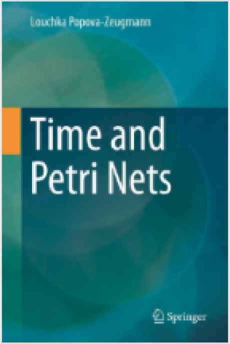
~~$\gamma = 9.5?$~~

- Given: Time dependent Petri Net
- Aim: Analysis of the time dependent Petri Net
- Problem: Infinite (dense) state space, TM-Completeness
- Solution:
  - Parametrisation and discretisation of the state space.
  - Definition of a reachability graph.
  - Structurally restricted classes of time dependent Petri Nets.
  - Time dependent state equation.

Softwate tools

- INA: http://www2.informatik.hu-berlin.de/ starke/ina.html
- tina: http://projects.laas.fr/tina//papers.php
- charlie:
  http://www-dssz.informatik.tu-cottbus.de/DSSZ/Software/Charlie

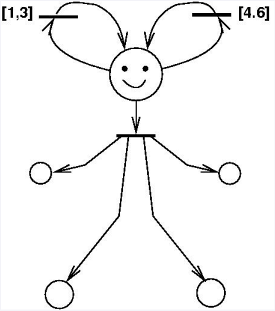
More about Time and Petri nets in



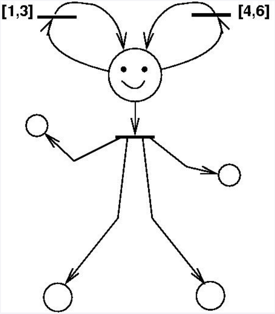
TIME AND PETRI NETS – 2013

AUTHORS	Louchka Popova-Zeugmann
ISBN	9783642411151 · 9783642411144
DOI	10.1007/978-3-642-41115-1
DISCIPLINES	Computer Science
SUBDISCIPLINES	SWE · Theoretical Computer Science · Bioinformatics

Thank you!



Thank you!



Thank you!