Bsp. 2.10.2. Lösen Sie das AWP

(1) $xy' + 3y = x^3 + 4x, \quad x \in \mathbb{R}^+, \; y(1) = \frac{1}{6}$

Lsg: $(1) \underset{x \in \mathbb{R}^+}{\Leftrightarrow} y' + \underbrace{\frac{3}{x}}_{a(x)} y = \underbrace{x^2 + 4}_{f(x)}$

$$A(x) = \int \frac{3}{x}\,dx = 3 \cdot \ln x = \ln x^3$$

$$\underset{S.2.8.}{\Rightarrow} \quad y(x) = \left(\int f(x) \cdot e^{A(x)}\,dx + c\right) e^{-A(x)}$$

$$= \left(\int (x^2+4) \cdot e^{\ln x^3}\,dx + c\right) \cdot e^{-\ln x^3}$$

$$= \left(\int (x^2+4) \cdot x^3\,dx + c\right) \cdot e^{+\ln (x^3)^{-1}}$$

$$= \left(\int (x^5 + 4x^3)\,dx + c\right) \cdot \frac{1}{x^3}$$

$$= \left(\frac{x^6}{6} + x^4 + c\right) \cdot \frac{1}{x^3} \quad \underline{\underline{\text{allg. Lsg.}}}$$

$$= \frac{x^3}{6} + x + \frac{c}{x^3}$$

AWP: $\frac{1}{6} = y(1) = \frac{1}{6} + 1 + c \Leftrightarrow c = -1$

$$\Rightarrow y(x) = \frac{x^3}{6} + x - \frac{1}{x^3}$$

part. Lsg des AWP

□