

$$(P) \max \{-2x_1 + 4x_3 + 3x_4 \mid \left. \begin{array}{l} x_1 + x_2 - x_3 + x_4 \geq 1 \\ 2x_2 - 4x_4 \leq 5 \\ 2x_1 + 7x_3 + x_4 \leq 12 \\ x_i \geq 0, i \in [4] \end{array} \right\}$$

⋮

$$\equiv \max \left\{ \left\langle \underbrace{\begin{pmatrix} -2 \\ 0 \\ 4 \\ 3 \end{pmatrix}}_c, \underbrace{\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}}_x \right\rangle \mid \left. \begin{array}{l} A \cdot x \leq b \\ -x_1 - x_2 + x_3 - x_4 \leq -1 \\ 2x_2 - 4x_4 \leq 5 \\ 2x_1 + 7x_3 + x_4 \leq 12 \\ x_i \geq 0, i \in [4] \end{array} \right\}$$

$$c, x \in \mathbb{R}^4, b \in \mathbb{R}^3 \\ A \in \mathbb{R}^{3 \times 4} \Rightarrow A^T \in \mathbb{R}^{4 \times 3}$$

$$A = \begin{pmatrix} -1 & -1 & 1 & -1 \\ 0 & 2 & 0 & -4 \\ 2 & 0 & 7 & 1 \end{pmatrix}$$

⋮

$$(D) \min \left\{ \left\langle \begin{pmatrix} -1 \\ 5 \\ 12 \end{pmatrix}, \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} \right\rangle \mid \left. \begin{array}{l} A^T y \geq c \\ -y_1 + 2y_3 \geq -2 \\ -y_1 + 2y_2 \geq 0 \\ y_1 + 7y_3 \geq 4 \\ -y_1 - 4y_2 + y_3 \geq 3 \\ y_i \geq 0, i \in [3] \end{array} \right\}$$

$$\equiv \min \left\{ -y_1 + 5y_2 + 12y_3 \mid \left. \begin{array}{l} y_1 - 2y_3 \leq 2 \\ -y_1 + 2y_2 \geq 0 \\ y_1 + 7y_3 \geq 4 \\ -y_1 - 4y_2 + y_3 \geq 3 \\ y_i \geq 0, i \in [3] \end{array} \right\}$$