Time Petri Nets: Theory, Tools and Applications

Part I

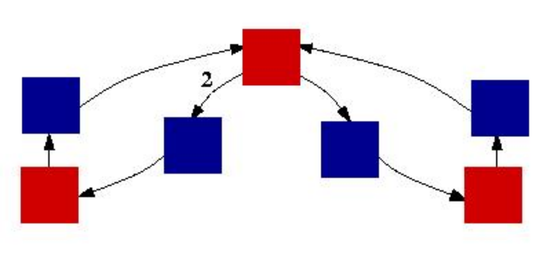
Louchka Popova-Zeugmann

Humboldt-Universität zu Berlin
Department of Computer Science
Unter den Linden 6, 10099 Berlin, Germany

ATPN 2008, Xi'an, China

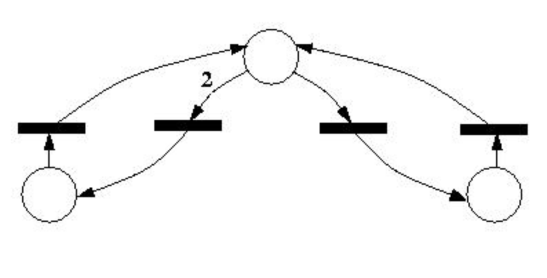
Outline

Statics: non initialized Petri Net



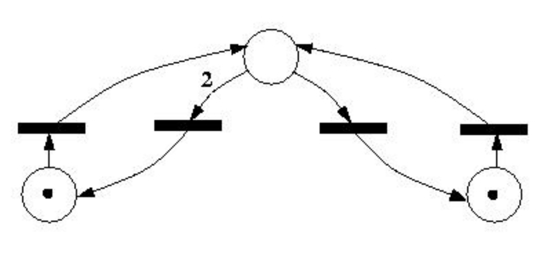
finite two-coloured weighted directed graph

Statics: **non initialized Petri Net**

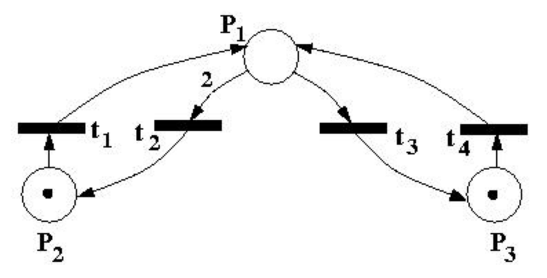


finite two-coloured weighted directed graph

Statics: initialized Petri Net

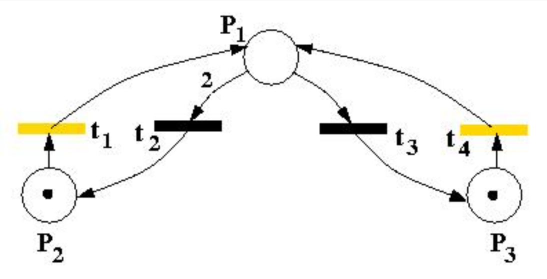


Statics: initialized Petri Net

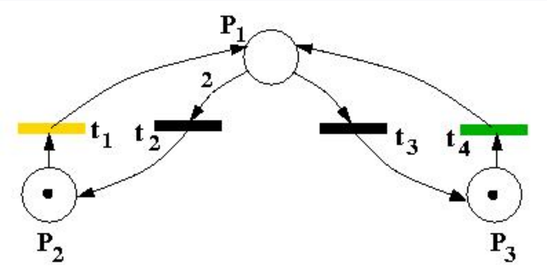


initial marking: $m_0 = (0, 1, 1)$

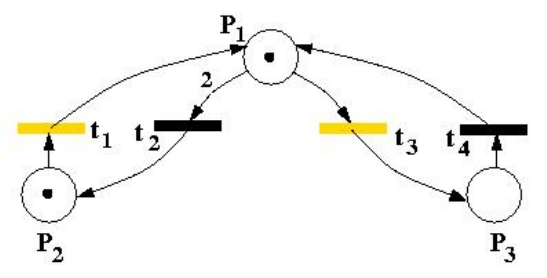
Dynamics: firing rule



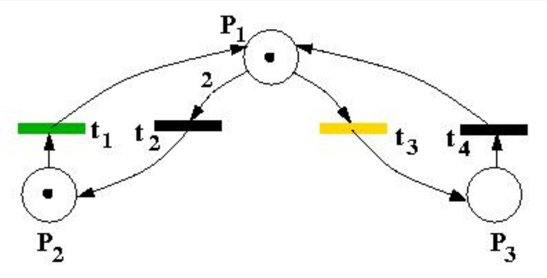
Dynamics: firing rule



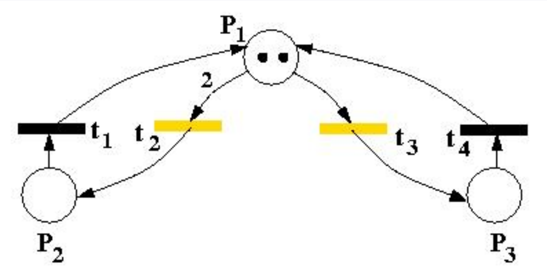
Dynamics: firing rule



Dynamics: firing rule



Dynamics: firing rule



Petri Nets and Turing Machines

Remark:

The power of the classical Petri Nets is less (not equal) to the power of the Turing Machines.

Assuming the opposite easily leads to a contradiction to the halting problem.

Time Assignment

- time dependent Petri Nets with time specification at
  - transitions
  - places
  - arcs
  - tokens

Time Assignment

- time dependent Petri Nets with time specification at
  - transitions
  - places
  - arcs
  - tokens

- time dependent Petri Nets with
  - deterministic
  - stochastic

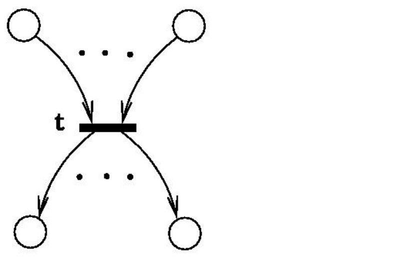
  time assignment.

Time Assignment

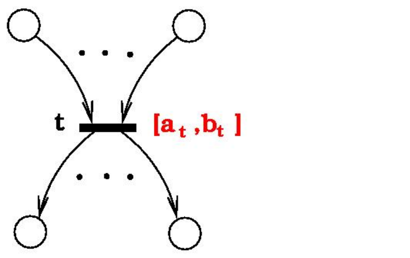
- time dependent Petri Nets with time specification at
  - transitions
  - places
  - arcs
  - tokens
- time dependent Petri Nets with
  - deterministic
  - stochastic

  time assignment.

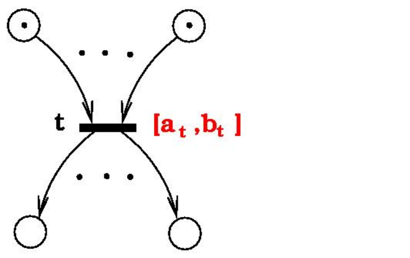
Conception



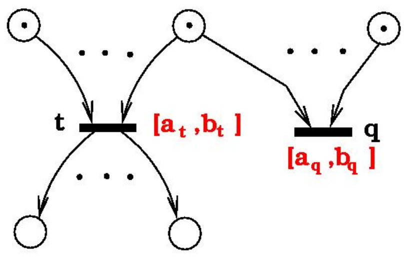
Conception



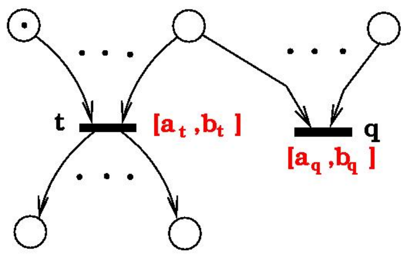
Conception



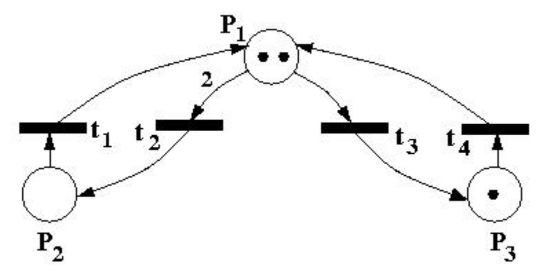
Conception



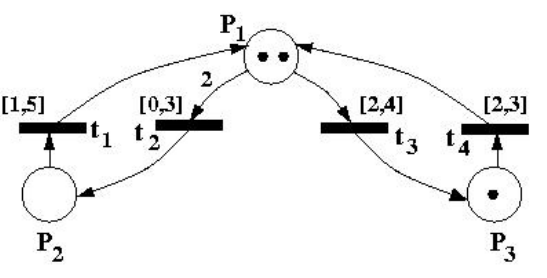
Conception



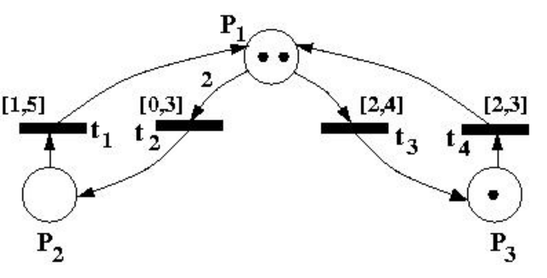
Statics: Petri Net (Skeleton)



Statics: **Time Petri Net**

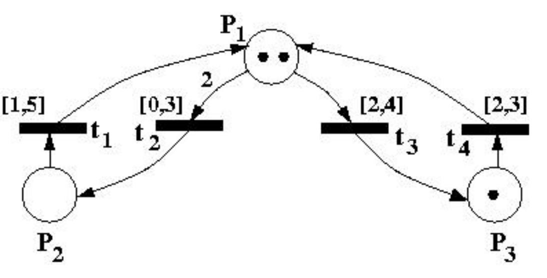


Statics: **Time Petri Net**



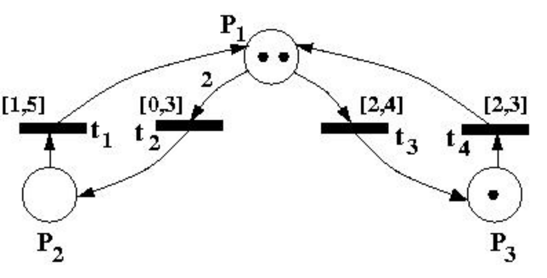
- $m_0 = (2, 0, 1)$

Statics: **Time Petri Net**



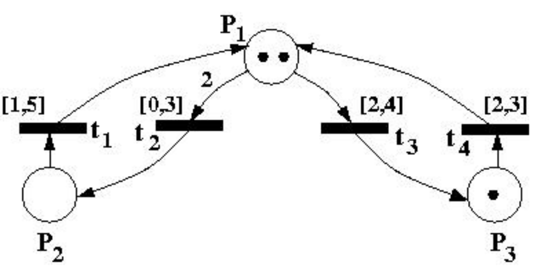
- $m_0 = (2, 0, 1)$ *p*-marking

Statics: **Time Petri Net**



- $m_0 = (2, 0, 1)$ *p*-marking
- $h_0 = (\sharp, 0, 0, 0)$ *t*-marking

Statics: **Time Petri Net**



- $m_0 = (2, 0, 1)$ *p*-marking
- $h_0 = (\sharp, 0, 0, 0)$ *t*-marking

$h(t)$ is the time shown by the clock of t since the last enabling of t

State

The pair $z = (m, h)$ is called a **state** in a TPN $\mathcal{Z}$, iff:

- m is a p-marking in $\mathcal{Z}$.
- h is a t-marking in $\mathcal{Z}$.

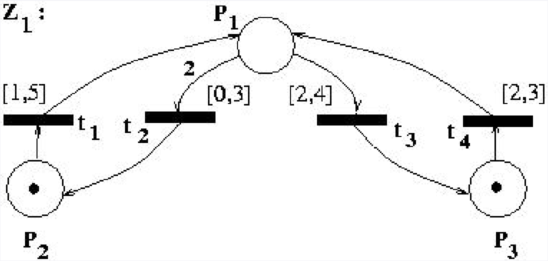
Dynamics: firing rules

Let $\mathcal{Z}$ be a TPN and let $z = (m, h)$, $z' = (m', h')$ be two states.
$\mathcal{Z}$ changes from state $z = (m, h)$ into the state $z' = (m', h')$ by:

firing a transition / \ **time elapsing**

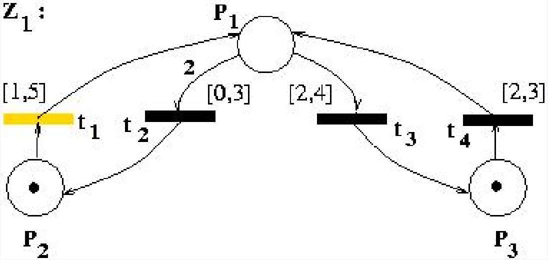
Notation: $z \xrightarrow{t} z'$ $z \xrightarrow{\tau} z'$

An example



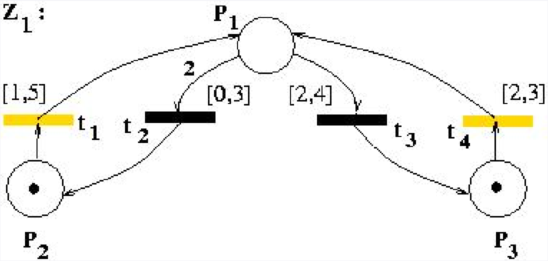
$$\left(m_0, \begin{pmatrix} 0 \\ \sharp \\ \sharp \\ 0 \end{pmatrix}\right)$$

An example



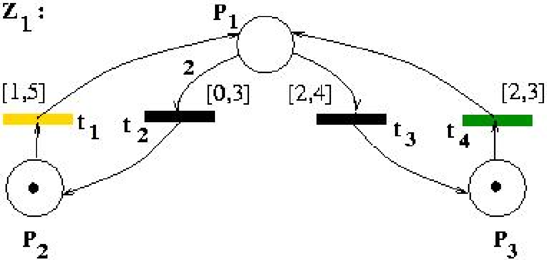
$$(m_0, \begin{pmatrix} 0 \\ \sharp \\ \sharp \\ 0 \end{pmatrix}) \xrightarrow{1.3} (m_1, \begin{pmatrix} 1.3 \\ \sharp \\ \sharp \\ 1.3 \end{pmatrix})$$

An example



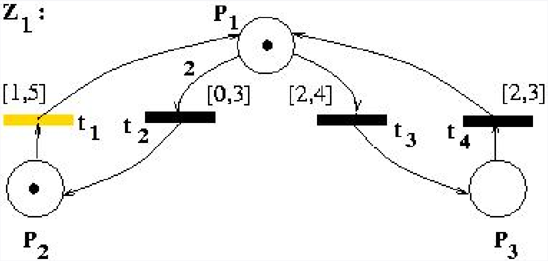
$$z_0 \xrightarrow{1.3} \left(m_1, \begin{pmatrix} 1.3 \\ \sharp \\ \sharp \\ 1.3 \end{pmatrix}\right) \xrightarrow{1.0} \left(m_2, \begin{pmatrix} 2.3 \\ \sharp \\ \sharp \\ 2.3 \end{pmatrix}\right)$$

An example



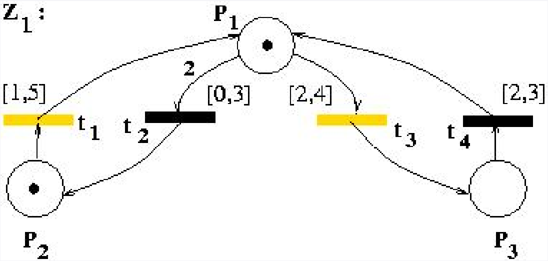
$$z_0 \xrightarrow{1.3} \xrightarrow{1.0} \left(m_2, \begin{pmatrix} 2.3 \\ \sharp \\ \sharp \\ 2.3 \end{pmatrix}\right) \xrightarrow{t_4}$$

An example



$$z_0 \xrightarrow{1.3} \xrightarrow{1.0} \left(m_2, \begin{pmatrix} 2.3 \\ \sharp \\ \sharp \\ 2.3 \end{pmatrix}\right) \xrightarrow{t_4} \left(m_3, \begin{pmatrix} 2.3 \\ \sharp \\ 0.0 \\ \sharp \end{pmatrix}\right)$$

An example



$$z_0 \xrightarrow{1.3} \xrightarrow{1.0} \xrightarrow{t_4} \left(m_3, \begin{pmatrix} 2.3 \\ \sharp \\ 0.0 \\ \sharp \end{pmatrix}\right) \xrightarrow{2.0} \left(m_4, \begin{pmatrix} 4.3 \\ \sharp \\ 2.0 \\ \sharp \end{pmatrix}\right)$$

Time Petri Nets and Turing Machines

Remark:

The power of the Time Petri Nets is equal to the power of the Turing Machines.

Idea:

- Simulating an arbitrary Counter Machine with a Time Petri Net
- Counter Machines and Turing Machines have the same power.

Counter Machine

consists of

1. counters $K_1, \ldots, K_n$,
2. a numbered program, comprising 4 different commands: start, halt, INC, DEC
   l : *command* ...

Counter Machine

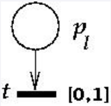
consists of

1. counters $K_1, \ldots, K_n$,
2. a numbered program, comprising 4 different commands: start, halt, INC, DEC
   l : *command* . . .

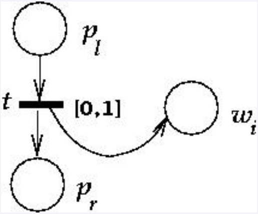
Modelling with TPN:

1. each counter K_i is modelled with a place w_i
2. each program number l is modelled with a place p_l
3. s. next 3 slides

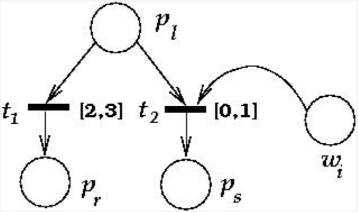
Modelling Counter Machines with Time Petri Nets

Notation of the numbered command	Model of the numbered command as a TPN
0 : *start* : l	
l : *halt*	

Modelling Counter Machines with Time Petri Nets

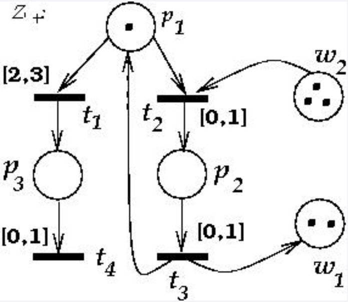
Notation of the numbered command	Model of the numbered command as a TPN
l : *INC(i)* : r	

Modelling Counter Machines with Time Petri Nets

Notation of the numbered command	Model of the numbered command as a TPN
l : *DEC(i)* : r : s	

Example

$$f(x_1, x_2) = x_1 + x_2$$

Counter Machine-program	TPN-model
0 : start : 1 *1: DEC (2) : 3 : 2* *2 : INC (1) : 1* *3: halt*	

A formal definition of a classical Petri Net and of a Time Petri Net can be found in the Appendix.

Definitions:

- **transition sequence:** $\sigma = t_1 \cdots t_n$
- **run:** $\sigma(\tau) = \tau_0\, t_1\, \tau_1\, \cdots\, \tau_{n-1}\, t_n\, \tau_n, \qquad \tau_i \in \mathbb{R}_0^+$
- **feasible run:** $z_0 \xrightarrow{\tau_0} z_0^* \xrightarrow{t_1} z_1 \xrightarrow{\tau_1} z_1^* \cdots \xrightarrow{t_n} z_n \xrightarrow{\tau_n} z_n^*$
- **feasible transition sequence :** σ is feasible if there ex. a feasible run $\sigma(\tau)$

Reachable state, Reachable marking, State space

Definitions:

- z is a **reachable state** in $\mathcal{Z}$ if there ex. a feasible run $\sigma(\tau)$ and $z_0 \xrightarrow{\sigma(\tau)} z$
- m is a **reachable *p*-marking** in $\mathcal{Z}$ if there ex. a reachable state z in $\mathcal{Z}$ with $z = (m, h)$
- The set of all reachable states in $\mathcal{Z}$ is the **state space** of $\mathcal{Z}$ (denoted: $StSp(\mathcal{Z})$).

Qualitative Properties

- static properties:

- dynamic properties:

Qualitative Properties

- static properties: being/having
  - homogenous
  - ordinary
  - free choice
  - extended simple
  - conservative
  - deadlocks, etc.
- dynamic properties: being/having
  - bounded
  - live
  - reachable marking/state, etc.

Qualitative Properties

- static properties: being/having
  - homogenous
  - ordinary
  - free choice
  - extended simple
  - conservative
  - deadlocks, etc.

  decidable **without knowledge** of the state space!
- dynamic properties: being/having
  - bounded
  - live
  - reachable marking/state, etc.

  decidable, if at all (TPN are equiv. to TM!),
  with implicit/explicit knowledge of the state space

Quantitative Properties

Each time proposition, like calculating

- (min/max) time length of path
- path between two states/markings with min/max time length, etc.

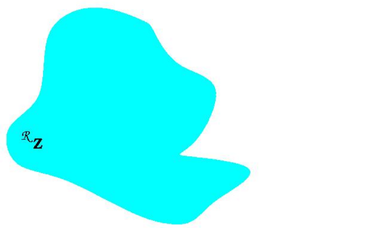
Quantitative Properties

Each time proposition, like calculating

- (min/max) time length of path
- path between two states/markings with min/max time length, etc.

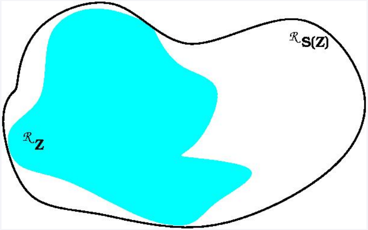
Decidable (if at all) **with implicit/explicit knowledge** of the state space

Some Problems



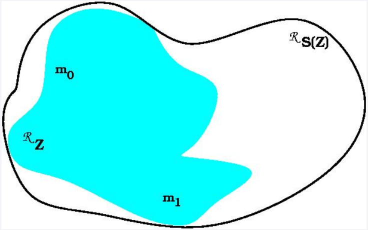
$$StSp(\mathcal{Z}) = \{z \mid \text{ ex. a feasible run } \sigma(\tau) \text{ in } \mathcal{Z} \text{ and } z_0 \xrightarrow{\sigma(\tau)} z\}$$

Some Problems



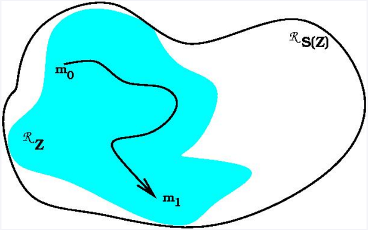
$StSp(\mathcal{Z}) = \{z \mid$ ex. a feasible run $\sigma(\tau)$ in $\mathcal{Z}$ and $z_0 \xrightarrow{\sigma(\tau)} z\}$

Some Problems



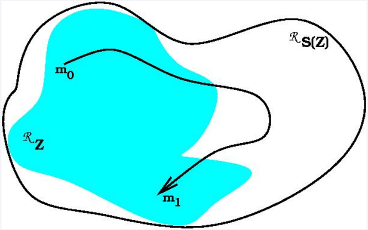
$$StSp(\mathcal{Z}) = \{z \mid \text{ ex. a feasible run } \sigma(\tau) \text{ in } \mathcal{Z} \text{ and } z_0 \xrightarrow{\sigma(\tau)} z\}$$

Some Problems



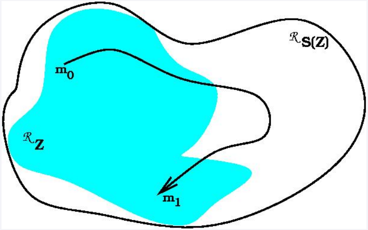
$StSp(\mathcal{Z}) = \{z \mid$ ex. a feasible run $\sigma(\tau)$ in $\mathcal{Z}$ and $z_0 \xrightarrow{\sigma(\tau)} z\}$

Some Problems



$$StSp(\mathcal{Z}) = \{z \mid \text{ ex. a feasible run } \sigma(\tau) \text{ in } \mathcal{Z} \text{ and } z_0 \xrightarrow{\sigma(\tau)} z\}$$

Some Problems

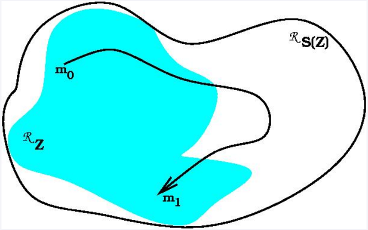


$StSp(\mathcal{Z}) = \{z \mid$ ex. a feasible run $\sigma(\tau)$ in $\mathcal{Z}$ and $z_0 \xrightarrow{\sigma(\tau)} z\}$

Idea of a solution:

$$z_0 \xrightarrow{5.1} z_0' \quad \xrightarrow{t_1} z_1 \quad \xrightarrow{1.0} z_1' \quad \ldots \xrightarrow{t_n} z_n \quad \xrightarrow{2.3} z_n'$$

Some Problems



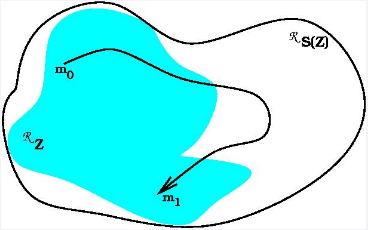
$$StSp(\mathcal{Z}) = \{z \mid \text{ ex. a feasible run } \sigma(\tau) \text{ in } \mathcal{Z} \text{ and } z_0 \xrightarrow{\sigma(\tau)} z\}$$

Idea of a solution:

$$z_0 \xrightarrow{5.1} z'_{0,\tau} \xrightarrow{t_1} z_{1,\tau} \xrightarrow{1.0} z'_{1,\tau} \ldots \xrightarrow{t_n} z_{n,\tau} \xrightarrow{2.3} z'_{n,\tau}$$

$$\tau = 5.1 \;\; 1.0 \;\; \ldots \;\; 2.3$$

Some Problems



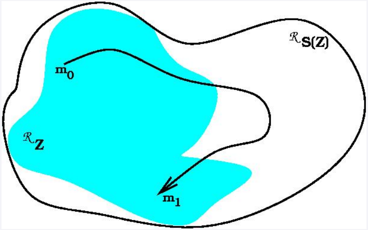
$StSp(\mathcal{Z}) = \{z \mid$ ex. a feasible run $\sigma(\tau)$ in $\mathcal{Z}$ and $z_0 \xrightarrow{\sigma(\tau)} z\}$

Idea of a solution:

$$z_0 \xrightarrow{5.1} z_0' \quad \xrightarrow{t_1} z_{1,\tau} \xrightarrow{1.0} z_{1,\tau}' \ldots \xrightarrow{t_n} z_{n,\tau} \xrightarrow{2.3} z_{n,\tau}'$$

$$z_0 \xrightarrow{x_0} z_{0,x}' \xrightarrow{t_1} z_{1,x} \xrightarrow{x_1} z_{1,x}' \ldots \xrightarrow{t_n} z_{n,x} \xrightarrow{x_n} z_{n,x}'$$

Some Problems



$$StSp(\mathcal{Z}) = \{z \mid \text{ ex. a feasible run } \sigma(\tau) \text{ in } \mathcal{Z} \text{ and } z_0 \xrightarrow{\sigma(\tau)} z\}$$

Idea of a solution:

$$z_0 \xrightarrow{5.1} z_0' \quad \xrightarrow{t_1} z_{1,\tau} \xrightarrow{1.0} z_{1,\tau}' \ldots \xrightarrow{t_n} z_{n,\tau} \xrightarrow{2.3} z_{n,\tau}'$$

$$z_0 \xrightarrow{x_0} z_{0,x}' \xrightarrow{t_1} z_{1,x} \xrightarrow{x_1} z_{1,x}' \ldots \xrightarrow{t_n} z_{n,x} \xrightarrow{x_n} z_{n,x}'$$

parametric run: $\sigma(x) = x_0\, t_1\, x_1\, \ldots\, x_{n-1}\, t_n\, x_n$
(+ some conditions for all x_i)

Parametric Run, Parametric State

Let $\mathcal{Z} = \big(P, T, F, V, m_0, I\big)$ be a TPN and $\sigma = t_1 \cdots t_n$ be a transition sequence in $\mathcal{Z}$.

$(\sigma(x), B_\sigma)$ is a **parametric run** of σ and (z_σ, B_σ) is a **parametric state** in $\mathcal{Z}$ with $z_\sigma = (m_\sigma, h_\sigma)$, if

- $m_0 \xrightarrow{\sigma} m_\sigma$
- $h_\sigma(t)$ is a sum of variables, (h_σ is a parametrical $t-$marking)
- B_σ is a set of conditions (a system of inequalities)

Parametric Run, Parametric State

Let $\mathcal{Z} = (P, T, F, V, m_0, I)$ be a TPN and $\sigma = t_1 \cdots t_n$ be a transition sequence in $\mathcal{Z}$.

$(\sigma(x), B_\sigma)$ is a **parametric run** of σ and (z_σ, B_σ) is a **parametric state** in $\mathcal{Z}$ with $z_\sigma = (m_\sigma, h_\sigma)$, if

- $m_0 \xrightarrow{\sigma} m_\sigma$
- $h_\sigma(t)$ is a sum of variables, (h_σ is a parametrical $t-$marking)
- B_σ is a set of conditions (a system of inequalities)

Obviously

- $z_0 \xrightarrow{\sigma(x)} (z_\sigma, B_\sigma)$,

Parametric Run, Parametric State

Let $\mathcal{Z} = (P, T, F, V, m_0, I)$ be a TPN and $\sigma = t_1 \cdots t_n$ be a transition sequence in $\mathcal{Z}$.

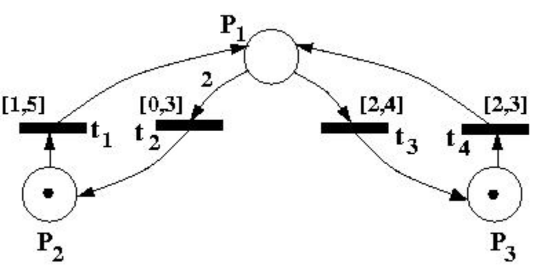
$(\sigma(x), B_\sigma)$ is a **parametric run** of σ and (z_σ, B_σ) is a **parametric state** in $\mathcal{Z}$ with $z_\sigma = (m_\sigma, h_\sigma)$, if

- $m_0 \xrightarrow{\sigma} m_\sigma$
- $h_\sigma(t)$ is a sum of variables, (h_σ is a parametrical $t-$marking)
- B_σ is a set of conditions (a system of inequalities)

Obviously

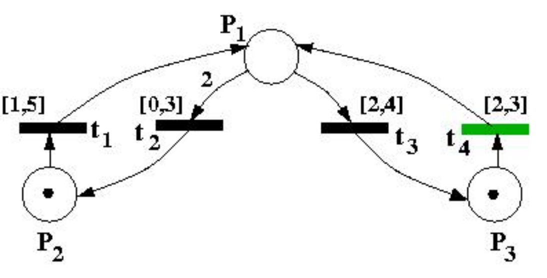
- $z_0 \xrightarrow{\sigma(x)} (z_\sigma, B_\sigma)$,
- $StSp(\mathcal{Z}) = \bigcup_{\sigma(x)} \{z_{\sigma(x)} | x \text{ satisfies } B_\sigma\}$.

Example



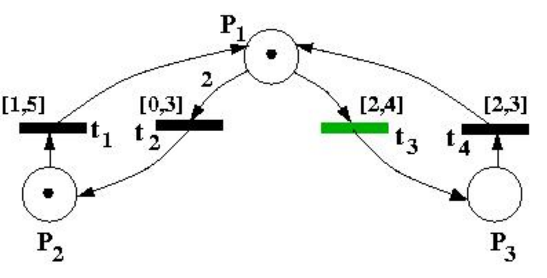
$$\sigma = t_4 \ \ t_3$$

Example



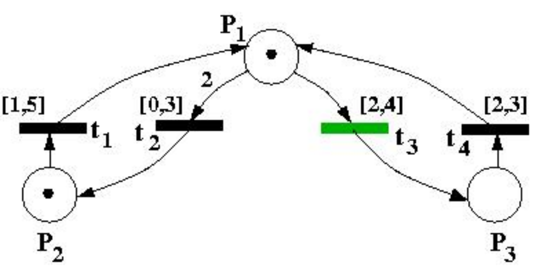
$\sigma = t_4 \ \ t_3 \qquad : \qquad x_1$

Example



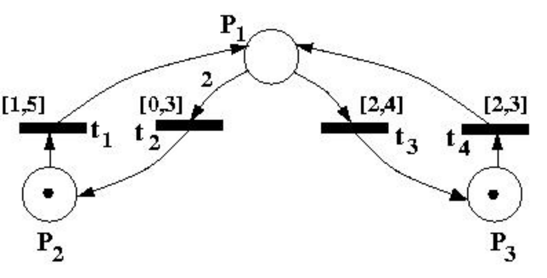
$\sigma = t_4 \;\; t_3 \qquad : \qquad x_1 \; t_4$

Example



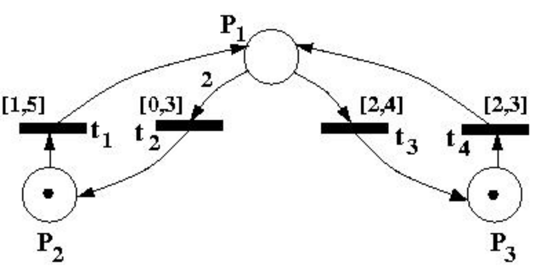
$$\sigma = t_4 \; t_3 \qquad : \qquad x_1 \; t_4 \; x_2$$

Example



$\sigma = t_4 \;\; t_3 \qquad : \qquad x_1 \; t_4 \; x_2 \; t_3 \; x_3$

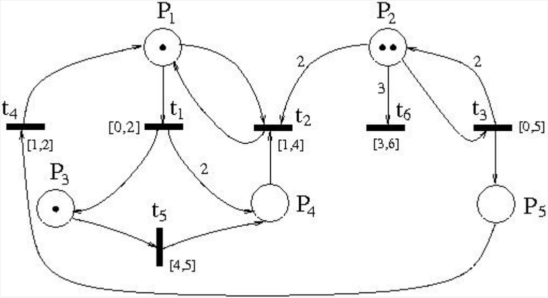
Example



$$\sigma = t_4\, t_3 \qquad \rightsquigarrow \qquad (z_\sigma, B_\sigma) =$$

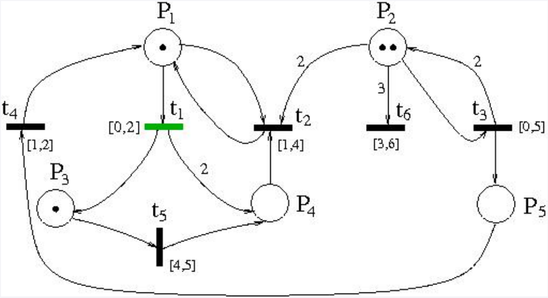
$$\Bigg(\underbrace{\left(\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} x_1 + x_2 + x_3 \\ \sharp \\ \sharp \\ x_3 \end{pmatrix} \right)}_{z_\sigma}, \underbrace{\left\{ \begin{array}{ll} 2 \le x_1 \le 3, & x_1 + x_2 \le 5 \\ 2 \le x_2 \le 4, & x_1 + x_2 + x_3 \le 5 \\ 0 \le x_3 \le 3 & \end{array} \right\}}_{B_\sigma} \Bigg).$$

Runs



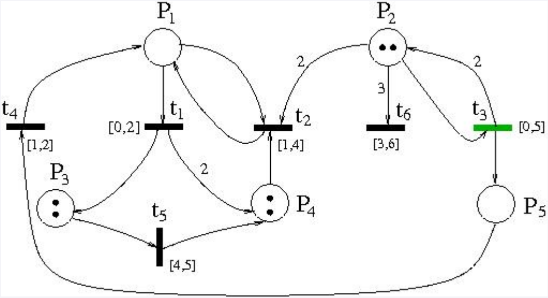
$$\sigma = t_1 \ t_3 \ t_4 \ t_2 \ t_3$$

Runs



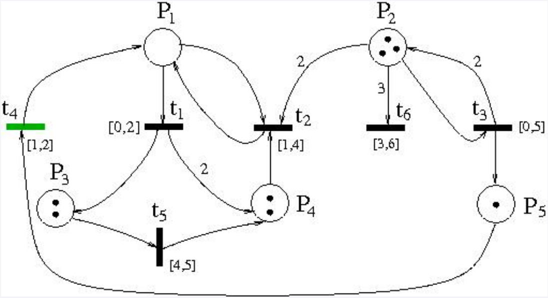
$$\sigma = t_1 \; t_3 \; t_4 \; t_2 \; t_3$$

Runs



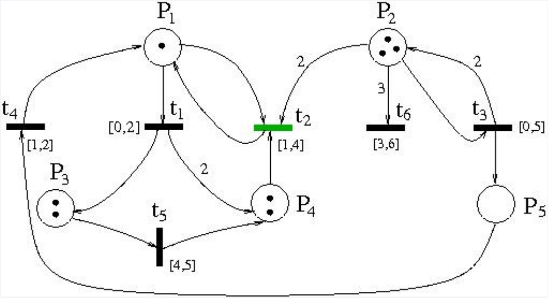
$$\sigma = t_1 \; t_3 \; t_4 \; t_2 \; t_3$$

Runs



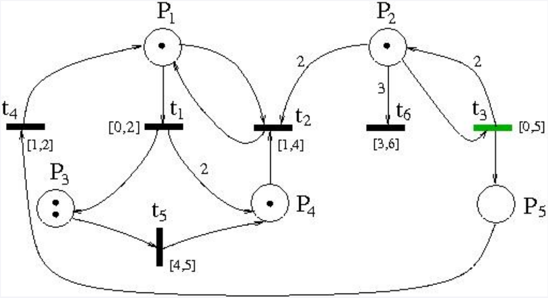
$$\sigma = t_1 \ t_3 \ t_4 \ t_2 \ t_3$$

Runs



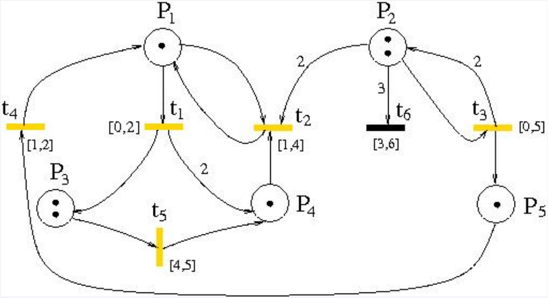
$$\sigma = t_1\ t_3\ t_4\ t_2\ t_3$$

Runs



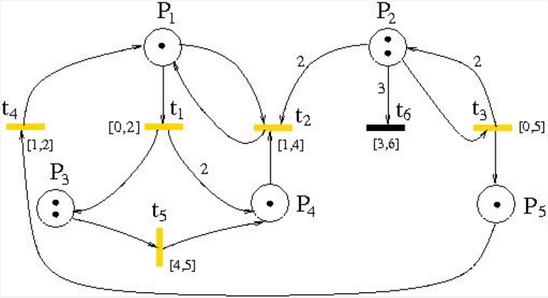
$$\sigma = t_1 \; t_3 \; t_4 \; t_2 \; t_3$$

Runs



$$\sigma = t_1 \ t_3 \ t_4 \ t_2 \ t_3$$

Runs

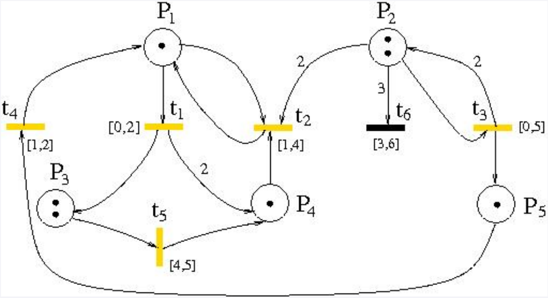


$$\sigma = t_1\ t_3\ t_4\ t_2\ t_3$$

$$\sigma(\tau) := z_0 \xrightarrow{0.7} \xrightarrow{t_1} \xrightarrow{0.0} \xrightarrow{t_3} \xrightarrow{0.4} \xrightarrow{t_4} \xrightarrow{1.2} \xrightarrow{t_2} \xrightarrow{0.5} \xrightarrow{t_3} \xrightarrow{1.4} z$$

$$\tau = 0.7\ 0.0\ 0.4\ 1.2\ 0.5\ 1.4$$

Runs

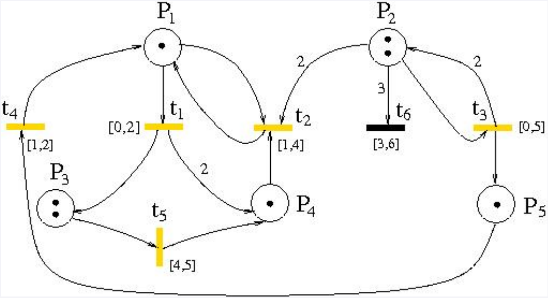


$$\sigma = t_1\ t_3\ t_4\ t_2\ t_3$$

$$\sigma(\tau) := z_0 \xrightarrow{\mathbf{0.7}} \xrightarrow{t_1} \xrightarrow{\mathbf{0.0}} \xrightarrow{t_3} \xrightarrow{\mathbf{0.4}} \xrightarrow{t_4} \xrightarrow{\mathbf{1.2}} \xrightarrow{t_2} \xrightarrow{\mathbf{0.5}} \xrightarrow{t_3} \xrightarrow{\mathbf{1.4}} z$$

$$\tau = 0.7\ 0.0\ 0.4\ 1.2\ 0.5\ 1.4$$

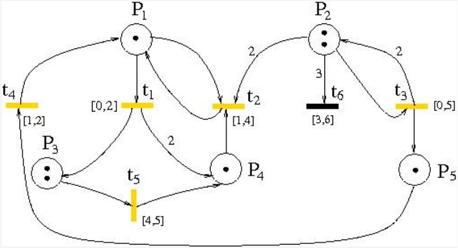
Example



$$\sigma = t_1\ t_3\ t_4\ t_2\ t_3$$

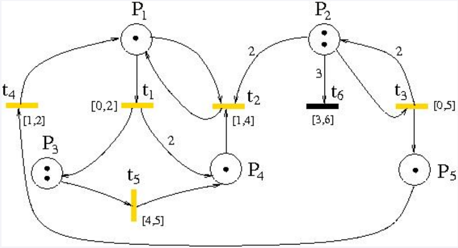
$$m_\sigma = (1, 2, 2, 1, 1)$$

Example - Continiation



$$h_\sigma = \begin{pmatrix} x_4 + x_5 \\ x_5 \\ x_5 \\ x_5 \\ x_0 + x_1 + x_2 + x_3 + x_4 + x_5 \\ \sharp \end{pmatrix}$$ and

Example - Continuation



$$B_\sigma = \left\{ \begin{array}{lll} 0 \le x_0, & x_0 \le 2, & x_0 + x_1 + x_2 \le 5 \\ 0 \le x_1, & x_2 \le 2, & x_2 + x_3 \le 5 \\ 1 \le x_2, & x_3 \le 2, & x_0 + x_1 + x_2 + x_3 \le 5 \\ 1 \le x_3, & x_4 \le 2, & x_0 + x_1 + x_2 + x_3 + x_4 \le 5 \\ 0 \le x_4, & x_5 \le 2, & x_0 + x_1 + x_2 + x_3 + x_4 + x_5 \le 5 \\ 0 \le x_5, & x_0 + x_1 \le 5 & x_4 + x_5 \le 2 \end{array} \right\}.$$

Example - Continuation

The run $\sigma(\tau)$ with

$$z_0 \xrightarrow{\mathbf{0.7}} \xrightarrow{t_1} \xrightarrow{\mathbf{0.0}} \xrightarrow{t_3} \xrightarrow{\mathbf{0.4}} \xrightarrow{t_4} \xrightarrow{\mathbf{1.2}} \xrightarrow{t_2} \xrightarrow{\mathbf{0.5}} \xrightarrow{t_3} \xrightarrow{\mathbf{1.4}} \left(m_\sigma, \begin{pmatrix} 1.9 \\ 1.4 \\ 1.4 \\ 1.4 \\ 4.2 \\ \sharp \end{pmatrix}\right)$$

is feasible.

Example - Continuation

$$\underbrace{\left(m_\sigma, \begin{pmatrix} \mathbf{1.9} \\ \mathbf{1.4} \\ \mathbf{1.4} \\ \mathbf{1.4} \\ \mathbf{4.2} \\ \sharp \end{pmatrix}\right)}_{z_0 \xrightarrow{\sigma(\tau)} z}$$

Example - Continuation

$$\underbrace{(m_\sigma, \begin{pmatrix} \mathbf{1.0} \\ \mathbf{1.0} \\ \mathbf{1.0} \\ \mathbf{1.0} \\ \mathbf{4.0} \\ \sharp \end{pmatrix})}_{z_0 \xrightarrow{\sigma(?)} \lfloor z \rfloor} \qquad \underbrace{(m_\sigma, \begin{pmatrix} \mathbf{1.9} \\ \mathbf{1.4} \\ \mathbf{1.4} \\ \mathbf{1.4} \\ \mathbf{4.2} \\ \sharp \end{pmatrix})}_{z_0 \xrightarrow{\sigma(\tau)} z}$$

Example - Continuation

$$\underbrace{\left(m_\sigma, \begin{pmatrix} \mathbf{1.0} \\ \mathbf{1.0} \\ \mathbf{1.0} \\ \mathbf{1.0} \\ \mathbf{4.0} \\ \sharp \end{pmatrix}\right)}_{z_0 \xrightarrow{\sigma(?)} \lfloor z \rfloor} \qquad \underbrace{\left(m_\sigma, \begin{pmatrix} \mathbf{1.9} \\ \mathbf{1.4} \\ \mathbf{1.4} \\ \mathbf{1.4} \\ \mathbf{4.2} \\ \sharp \end{pmatrix}\right)}_{z_0 \xrightarrow{\sigma(\tau)} z} \qquad \underbrace{\left(m_\sigma, \begin{pmatrix} \mathbf{2.0} \\ \mathbf{2.0} \\ \mathbf{2.0} \\ \mathbf{2.0} \\ \mathbf{5.0} \\ \sharp \end{pmatrix}\right)}_{z_0 \xrightarrow{\sigma(?)} \lceil z \rceil}$$

Example - Continuation

The runs

$$\sigma(\tau_1^*) := z_0 \xrightarrow{\mathbf{1}} \xrightarrow{t_1} \xrightarrow{\mathbf{0}} \xrightarrow{t_3} \xrightarrow{\mathbf{1}} \xrightarrow{t_4} \xrightarrow{\mathbf{1}} \xrightarrow{t_2} \xrightarrow{\mathbf{0}} \xrightarrow{t_3} \xrightarrow{\mathbf{1}} \lfloor z \rfloor$$

and

$$\sigma(\tau_2^*) := z_0 \xrightarrow{\mathbf{1}} \xrightarrow{t_1} \xrightarrow{\mathbf{0}} \xrightarrow{t_3} \xrightarrow{\mathbf{0}} \xrightarrow{t_4} \xrightarrow{\mathbf{2}} \xrightarrow{t_2} \xrightarrow{\mathbf{0}} \xrightarrow{t_3} \xrightarrow{\mathbf{2}} \lceil z \rceil$$

are also feasible in $\mathcal{Z}$.

Example - Continuation

The runs

$$\sigma(\tau_1^*) := z_0 \xrightarrow{\mathbf{1}} \xrightarrow{t_1} \xrightarrow{\mathbf{0}} \xrightarrow{t_3} \xrightarrow{\mathbf{1}} \xrightarrow{t_4} \xrightarrow{\mathbf{1}} \xrightarrow{t_2} \xrightarrow{\mathbf{0}} \xrightarrow{t_3} \xrightarrow{\mathbf{1}} \lfloor z \rfloor$$

$$\sigma(\tau) = z_0 \xrightarrow{\mathbf{0.7}} \xrightarrow{t_1} \xrightarrow{\mathbf{0.0}} \xrightarrow{t_3} \xrightarrow{\mathbf{0.4}} \xrightarrow{t_4} \xrightarrow{\mathbf{1.2}} \xrightarrow{t_2} \xrightarrow{\mathbf{0.5}} \xrightarrow{t_3} \xrightarrow{\mathbf{1.4}} z$$

$$\sigma(\tau_2^*) := z_0 \xrightarrow{\mathbf{1}} \xrightarrow{t_1} \xrightarrow{\mathbf{0}} \xrightarrow{t_3} \xrightarrow{\mathbf{0}} \xrightarrow{t_4} \xrightarrow{\mathbf{2}} \xrightarrow{t_2} \xrightarrow{\mathbf{0}} \xrightarrow{t_3} \xrightarrow{\mathbf{2}} \lceil z \rceil$$

are also feasible in $\mathcal{Z}$.

Main Property

Theorem 1:

Let $\mathcal{Z}$ be a TPN and $\sigma = t_1 \cdots t_n)$ be a feasible transition sequence in $\mathcal{Z}$ with a feasable run $\sigma(\tau)$ of σ $\left(\tau = \tau_0 \ldots \tau_n\right)$ i.e.

$$z_0 \xrightarrow{\tau_0} \xrightarrow{t_1} \cdots \xrightarrow{t_n} \xrightarrow{\tau_n} z_n = (m_n, h_n),$$

and all $\tau_i \in \mathbb{R}_0^+$.

Then, there exists a further feasible run $\sigma(\tau^*)$, $\tau^* = \tau_0^* \ldots \tau_n^*$ of σ with

$$z_0 \xrightarrow{\tau_0^*} \xrightarrow{t_1} \cdots \xrightarrow{t_n} \xrightarrow{\tau_n^*} z_n^* = (m_n^*, h_n^*).$$

such that

Main Property

Theorem 1 – Continuation:

$$z_0 \xrightarrow{\tau_0} \xrightarrow{t_1} \cdots \xrightarrow{t_n} \xrightarrow{\tau_n} z_n = (m_n, h_n),\ \tau_i \in \mathbb{R}_0^+.$$

$$z_0 \xrightarrow{\tau_0^*} \xrightarrow{t_1} \cdots \xrightarrow{t_n} \xrightarrow{\tau_n^*} z_n^* = (m_n^*, h_n^*)$$

1. For each $i, 0 \le i \le n$ the time τ_i^* is a natural number.
2. For each enabled transition t at marking $m_n (= m_n^*)$ it holds:
   1. $h_n^*(t) = \lfloor h_n(t) \rfloor$.
   2. $\sum_{i=1}^{n} \tau_i^* = \lfloor \sum_{i=1}^{n} \tau_i \rfloor$
3. For each transition $t \in T$ it holds:
   t is ready to fire in z_n iff t is also ready to fire in $\lfloor z_n \rfloor$.

Main Property

Theorem 1 – Continuation:

$$z_0 \xrightarrow{\tau_0} \xrightarrow{t_1} \cdots \xrightarrow{t_n} \xrightarrow{\tau_n} z_n = (m_n, h_n),\ \tau_i \in \mathbb{R}_0^+.$$

$$z_0 \xrightarrow{\tau_0^*} \xrightarrow{t_1} \cdots \xrightarrow{t_n} \xrightarrow{\tau_n^*} z_n^* = (m_n^*, h_n^*)\ ,\ \tau_i^* \in \mathbb{N}.$$

1. For each $i, 0 \leq i \leq n$ the time τ_i^* is a natural number.
2. For each enabled transition t at marking $m_n (= m_n^*)$ it holds:
   1. $h_n^*(t) = \lfloor h_n(t) \rfloor$.
   2. $\sum_{i=1}^{n} \tau_i^* = \lfloor \sum_{i=1}^{n} \tau_i \rfloor$
3. For each transition $t \in T$ it holds:
   t is ready to fire in z_n iff t is also ready to fire in $\lfloor z_n \rfloor$.

Main Property

Theorem 2:

Let $\mathcal{Z}$ be a TPN and $\sigma = t_1 \ \cdots \ t_n)$ be a feasible transition sequence in $\mathcal{Z}$, with feasable run $\sigma(\tau)$ of σ $\left(\tau = \tau_0 \ldots \tau_n\right)$ i.e.

$$z_0 \xrightarrow{\tau_0} \xrightarrow{t_1} \cdots \xrightarrow{t_n} \xrightarrow{\tau_n} z_n = (m_n, h_n),$$

and all $\tau_i \in \mathbb{R}_0^+$. Then, there exists a further feasible run $\sigma(\tau^*)$ of σ with

$$z_0 \xrightarrow{\tau_0^*} \xrightarrow{t_1} \cdots \xrightarrow{t_n} \xrightarrow{\tau_n^*} z_n^* = (m_n^*, h_n^*).$$

such that

Main Property

Theorem 2 – Continuation:

1. For each $i, 0 \leq i \leq n$ the time τ_i^* is a natural number.
2. For each enabled transition t at marking $m_n (= m_n^*)$ it holds:
   1. $h_n(t)^* = \lceil h_n(t) \rceil$.
   2. $\sum_{i=1}^{n} \tau_i^* = \lceil \sum_{i=1}^{n} \tau_i \rceil$
3. For each transition $t \in T$ holds:
   t is ready to fire in z_n iff t is also ready to fire in $\lceil z_n \rceil$.

Some Conclusions

- Each feasible transitions sequence σ in $\mathcal{Z}$ can be realized with an **integer** run.
- Each reachable *p*-marking in $\mathcal{Z}$ can be reached using **integer** runs only.
- If z is reachable in $\mathcal{Z}$, then $\lfloor z \rfloor$ and $\lceil z \rceil$ are reachable in $\mathcal{Z}$ as well.
- The length of the shortest and longest time path (if this is finite) between two arbitrary *p*-markings are natural numbers.

A run $\sigma(\tau) = \tau_0 \ t_1 \ \tau_1 \ldots t_n \ \tau_n$ is an **integer** one, if $\tau_i \in \mathbb{N}$ for each $i = 0 \ldots n$.

Integer States

A state $z = (m, h)$ is an **integer** one, if $h(t) \in \mathbb{N}$ for each in m enabled transition t.

Theorem 3:

Let $\mathcal{Z}$ be a finite TPN, i.e. $lft(t) \neq \infty$ for all $t \in T$.
The set of all reachable integer states in $\mathcal{Z}$ is finite
if and only if
the set of all reachable $p-$markings in $\mathcal{Z}$ is finite.

Integer States

A state $z = (m, h)$ is an **integer** one, if $h(t) \in \mathbb{N}$ for each in m enabled transition t.

Theorem 3:

Let $\mathcal{Z}$ be a finite TPN, i.e. $lft(t) \neq \infty$ for all $t \in T$.
The set of all reachable integer states in $\mathcal{Z}$ is finite
if and only if
the set of all reachable $p-$markings in $\mathcal{Z}$ is finite.

Remark:

Theorem 3 can be generalized for all TPNs (applying a further reduction of the state space).

Modified Rule

Let $\mathcal{Z}$ be an arbitrary TPN. The state change **by time elapsing** can be slightly **modified** for each transition t with $lft(t) = \infty$, because to fire such a transition t

- it is important if t is old enough to fire or not, i.e. if t has been enabled last for $eft(t)$ (or more) time units or t is younger.
- Thus, the time $h(t)$ increases **until** $eft(t)$. After that, the clock of t remains in this position (although the time is elapsing), unless t becomes disabled.

Essential States

Theorem 4:

In an arbitrary TPN a *p*-marking is reachable using the non-modified definition iff it is reachable using the modified one.

Essential States

Theorem 4:

In an arbitrary TPN a *p*-marking is reachable using the non-modified definition iff it is reachable using the modified one.

All reachable integer states in an arbitrary TPN, obtained by using the modified definition, are called the **essential states** of this net.

Essential States

Theorem 4:

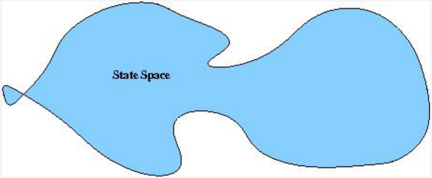
In an arbitrary TPN a *p*-marking is reachable using the non-modified definition iff it is reachable using the modified one.

All reachable integer states in an arbitrary TPN, obtained by using the modified definition, are called the **essential states** of this net.

Theorem 5:

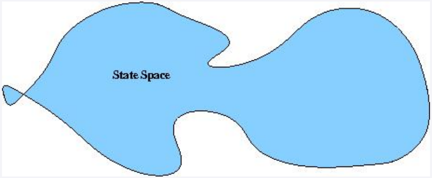
An arbitrary TPN is bounded iff the set of its essential states is finite.

Discrete Reduction of the State Space

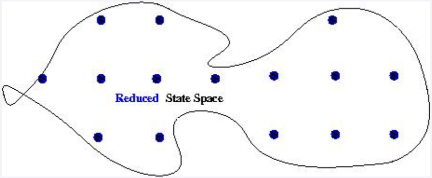


The set of all reachable states

Discrete Reduction of the State Space

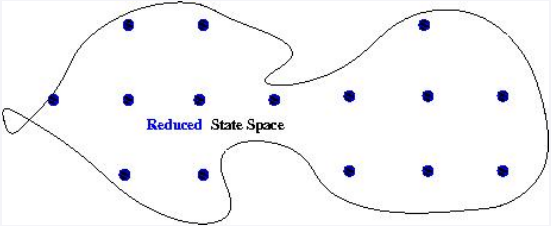


The set of all reachable states

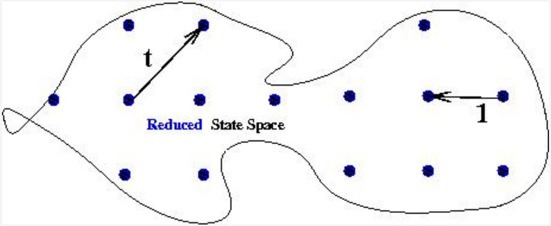


The set of all essential states

(Reduced) Reachability Graph

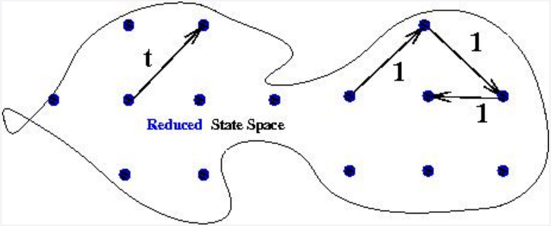


(Reduced) Reachability Graph



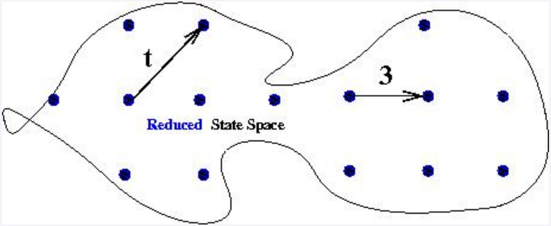
The reachability graph is a weighted directed graph, including the time explicit.

(Reduced) Reachability Graph



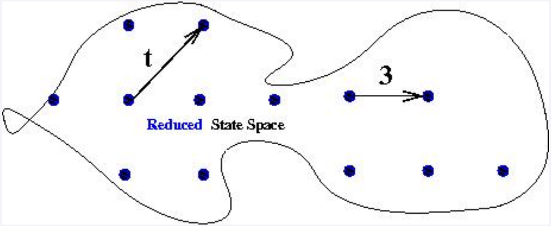
The reachability graph is a weighted directed graph, including the time explicit.

(Reduced) Reachability Graph



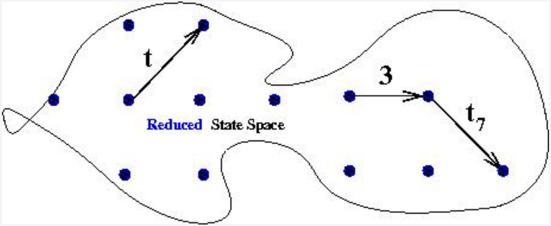
The reachability graph is a weighted directed graph, including the time explicit.

(Reduced) Reachability Graph



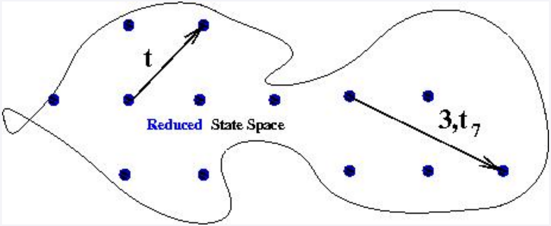
The reachability graph is a weighted directed graph, including the time explicit.

(Reduced) Reachability Graph



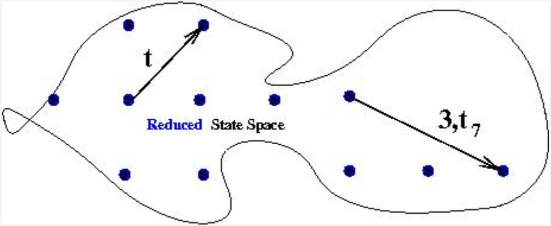
The reachability graph is a weighted directed graph, including the time explicit.

(Reduced) Reachability Graph



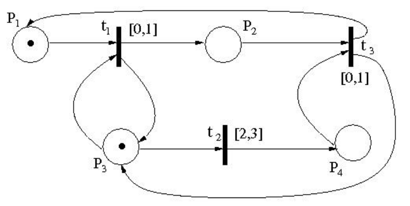
The reachability graph is a weighted directed graph, including the time explicit.

(Reduced) Reachability Graph

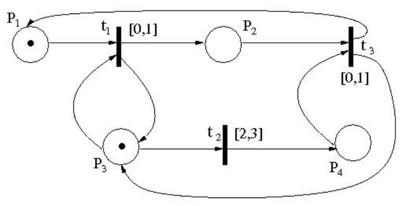


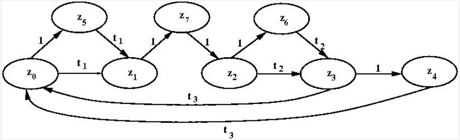
The reachability graph is a weighted directed graph, including the time explicit.

Example: A finite TPN and its reachability graph

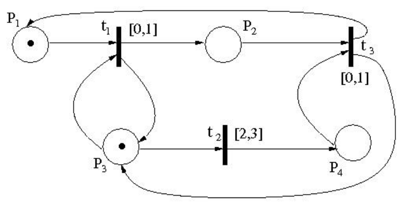


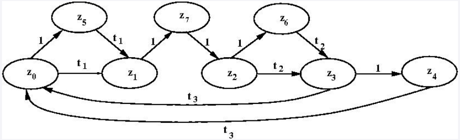
Example: A finite TPN and its reachability graph

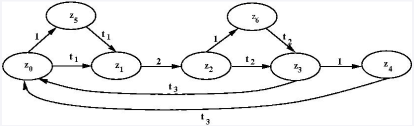




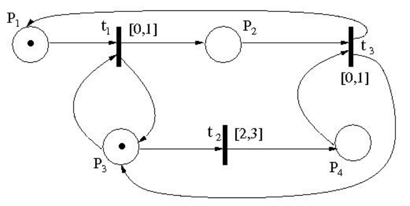
Example: A finite TPN and its reachability graph

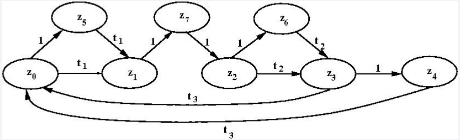


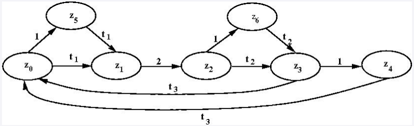


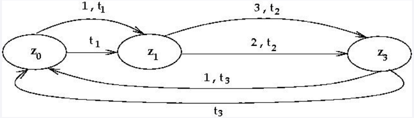


Example: A finite TPN and its reachability graph

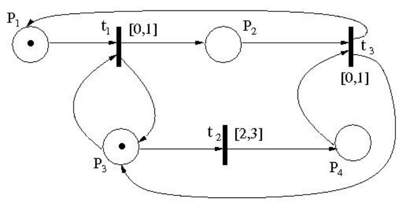


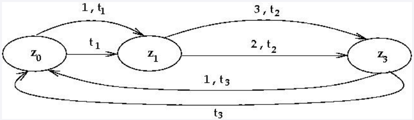




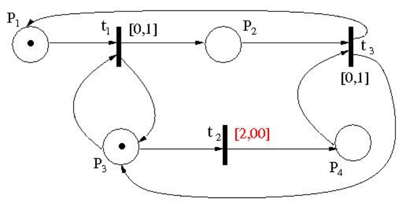


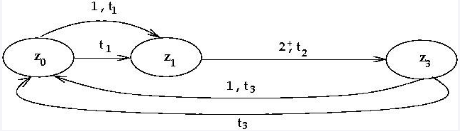
Example: A finite TPN and its reachability graph





Example: A non-finite TPN and its reachability graph





Boundedness: TPN vs. Skeleton

A TPN $\mathcal{Z}$ is bounded if the set of all its reachable *p*-markings is finite.

Theorem 6:

Let $\mathcal{Z}$ be a TPN and $S(\mathcal{Z})$ its skeleton. Than it holds:

- If $S(\mathcal{Z})$ is bounded then $\mathcal{Z}$ is bounded as well.
- If $\mathcal{Z}$ is bounded, then $S(\mathcal{Z})$ can be bounded or unbounded, i.e. the vice versa is not true.

Reachability in finite TPN

Theorem:

Let the skeleton $S(\mathcal{Z})$ of the TPN $\mathcal{Z}$ be bounded. Than it holds:

- The reachability of each *p*-marking in $\mathcal{Z}$ is decidable.
- The reachability of each rational state $z = (m, h)$ (i.e. $h(t)$ is a rational number for each enabled transition t by m) is decidable.

Reachability: TPN vs. Skeleton

Theorem:

Let $\mathcal{Z}$ be a TPN, $S(\mathcal{Z})$ its skeleton and $eft(t) = 0$ for all transitions t in $\mathcal{Z}$. Than a p-marking m is reachable in $\mathcal{Z}$ iff m is reachable in $S(\mathcal{Z})$.

Theorem:

Let $\mathcal{Z}$ be a TPN, $S(\mathcal{Z})$ its skeleton and $lft(t) = \infty$ for all transitions t in $\mathcal{Z}$. Than a p-marking m is reachable in $\mathcal{Z}$ iff m is reachable in $S(\mathcal{Z})$.

Let $\mathcal{Z}$ be a TPN and z a state in $\mathcal{Z}$. If one of the states $\lfloor z \rfloor$ or $\lceil z \rceil$ are not reachable in $\mathcal{Z}$ then z is not reachable in $\mathcal{Z}$ as well.

Liveness: Definitions

Let $\mathcal{Z}$ be a TPN, t be a transition in $\mathcal{Z}$ and z, z' two states in $\mathcal{Z}$.

- t is called **live in** $\mathcal{Z}$, if
  $\forall z \, \exists z' \, (\, z_0 \xrightarrow{*} z \xrightarrow{*} z' \xrightarrow{t} \,)$
- t is called **dead in** $\mathcal{Z}$, if
  $\forall z \, (\, z_0 \xrightarrow{*} z \not\xrightarrow{t} \,)$
- $\mathcal{Z}$ is called **live or dead,** resp., if all transitions in $\mathcal{Z}$ are live or dead , resp.

Liveness: Definitions

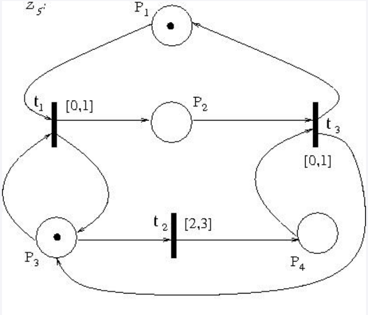
Let $\mathcal{Z}$ be a TPN, t be a transition in $\mathcal{Z}$ and z, z' two states in $\mathcal{Z}$.

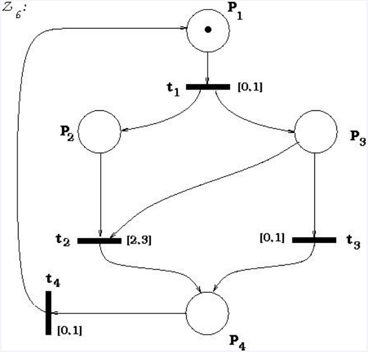
- t is called **live in** $\mathcal{Z}$, if
  $\forall z \, \exists z' \, (z_0 \xrightarrow{*} z \xrightarrow{*} z' \xrightarrow{t})$
- t is called **dead in** $\mathcal{Z}$, if
  $\forall z \, (z_0 \xrightarrow{*} z \stackrel{t}{\not\longrightarrow})$
- $\mathcal{Z}$ is called **live or dead,** resp., if all transitions in $\mathcal{Z}$ are live or dead , resp.

Remark:

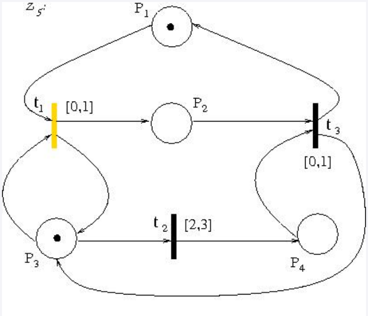
There is not a correlation between the liveness behaviors of a TPN and its skeleton.

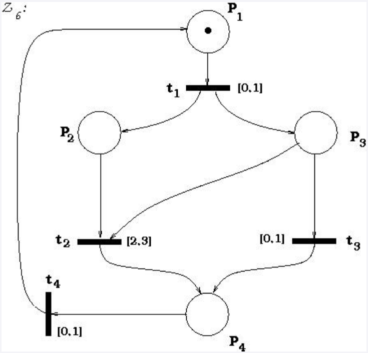
Liveness: TPN vs. Skeleton



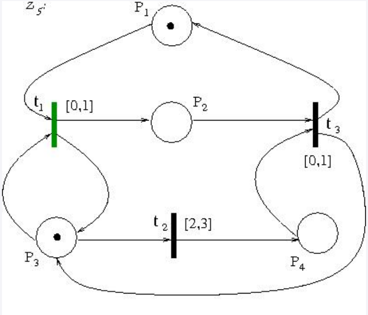


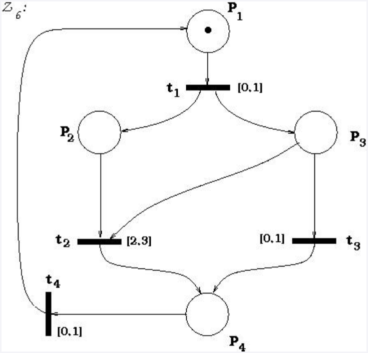
Liveness: TPN vs. Skeleton



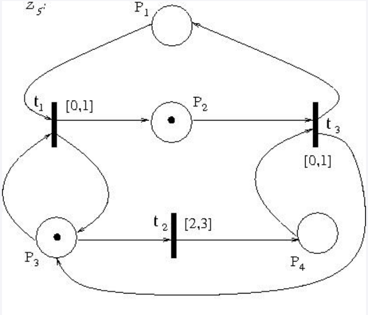


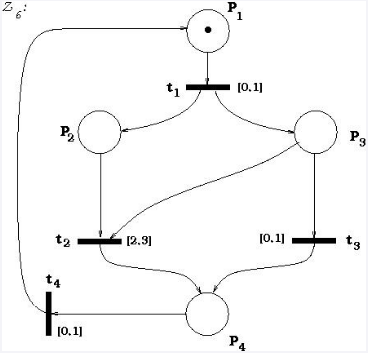
Liveness: TPN vs. Skeleton



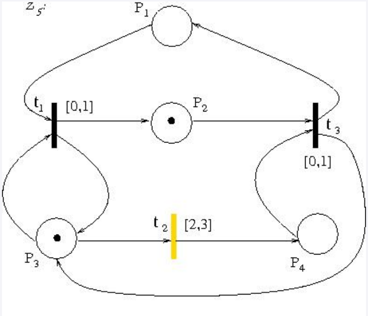


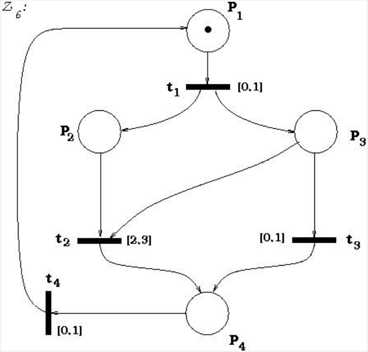
Liveness: TPN vs. Skeleton



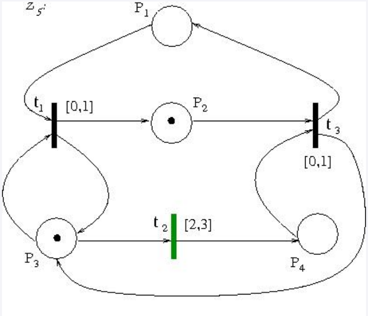


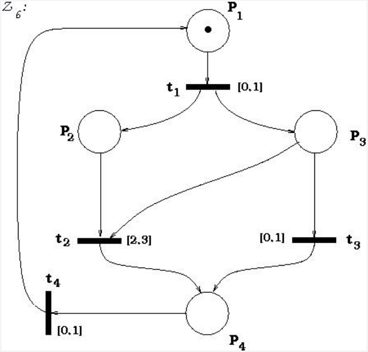
Liveness: TPN vs. Skeleton



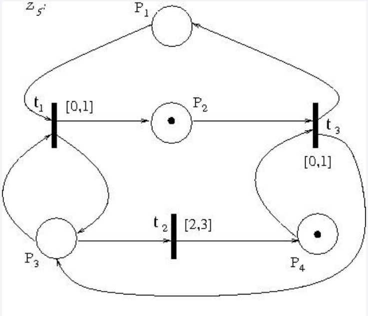


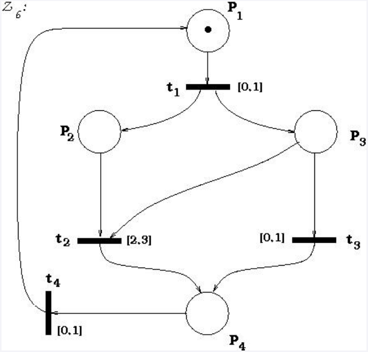
Liveness: TPN vs. Skeleton



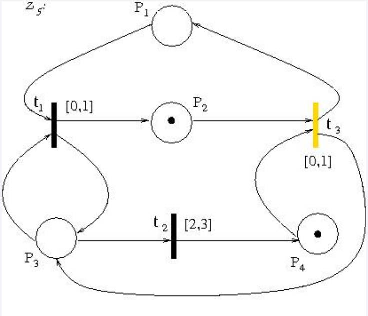


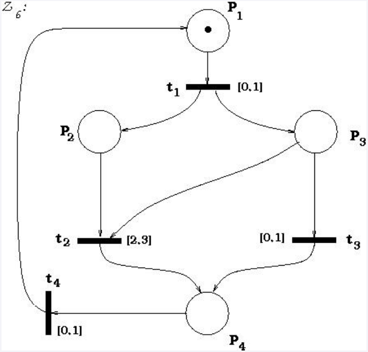
Liveness: TPN vs. Skeleton



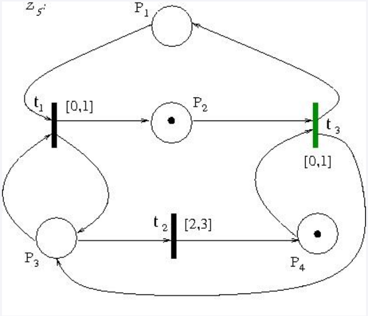


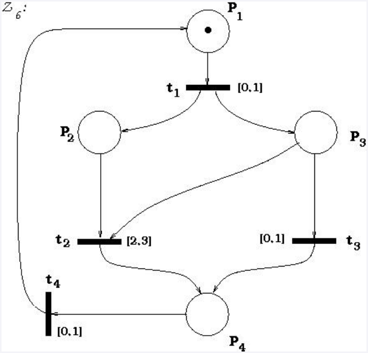
Liveness: TPN vs. Skeleton



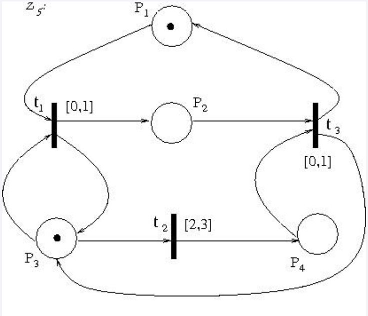


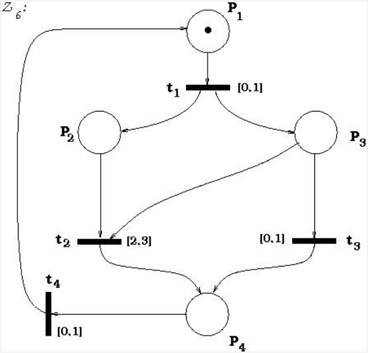
Liveness: TPN vs. Skeleton



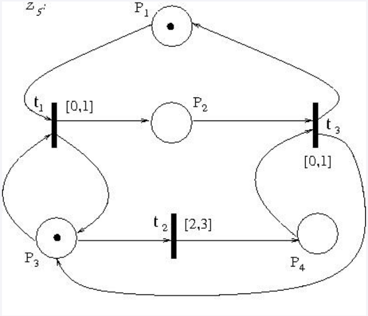


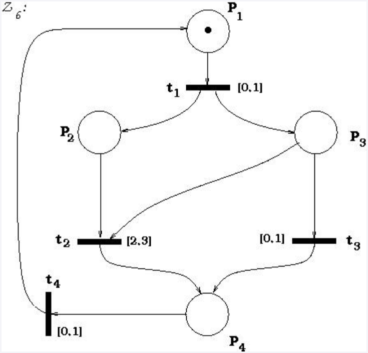
Liveness: TPN vs. Skeleton





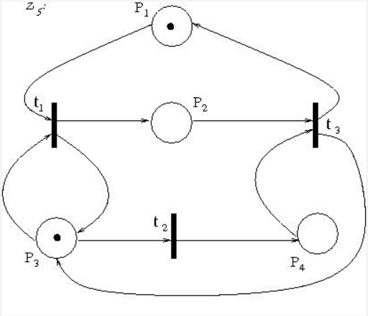
Liveness: TPN vs. Skeleton

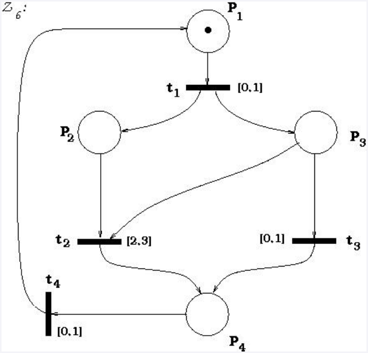




$\mathcal{Z}_5$ is live

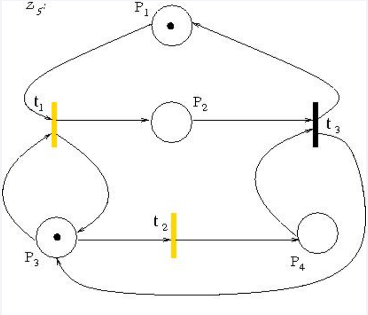
Liveness: TPN vs. Skeleton

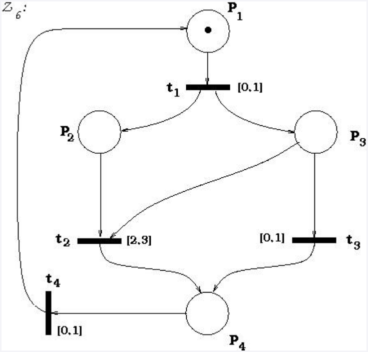




$\mathcal{Z}_5$ is live

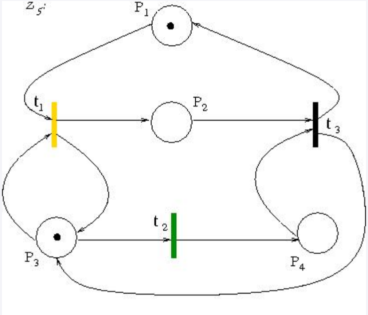
Liveness: TPN vs. Skeleton

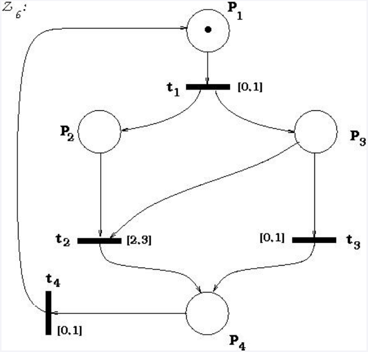




$\mathcal{Z}_5$ is live

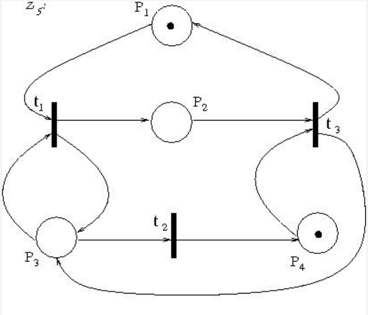
Liveness: TPN vs. Skeleton

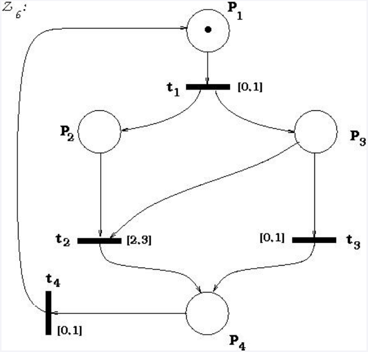




$\mathcal{Z}_5$ is live

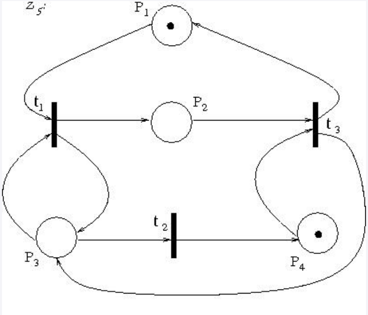
Liveness: TPN vs. Skeleton

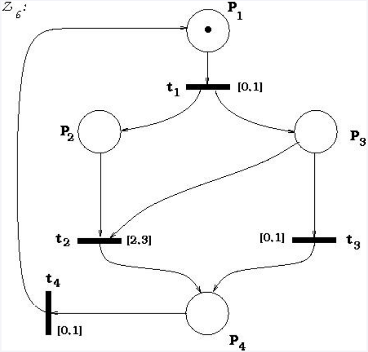




$\mathcal{Z}_5$ is live

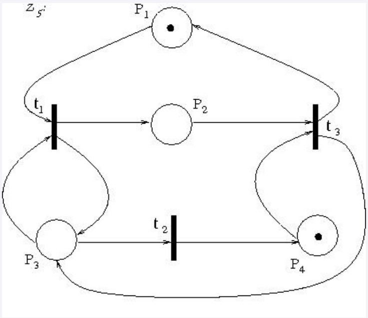
Liveness: TPN vs. Skeleton

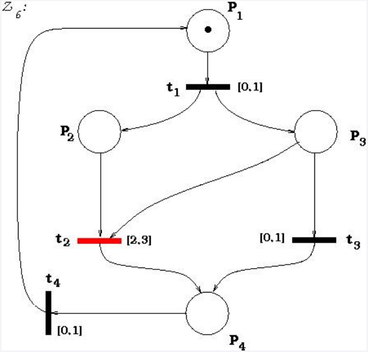




$\mathcal{Z}_5$ is live
$S(\mathcal{Z}_5)$ is not live

Liveness: TPN vs. Skeleton

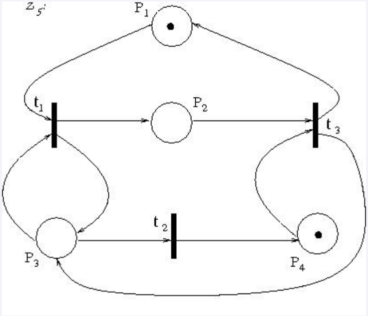


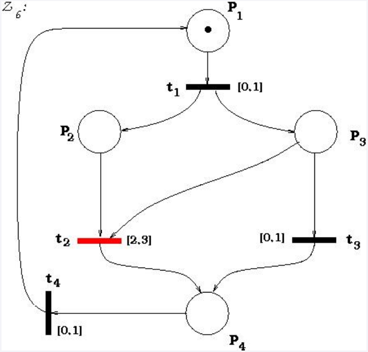


$\mathcal{Z}_5$ is live

$S(\mathcal{Z}_5)$ is not live

Liveness: TPN vs. Skeleton

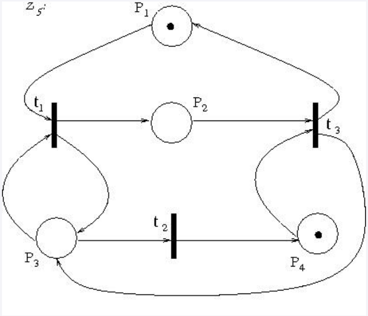


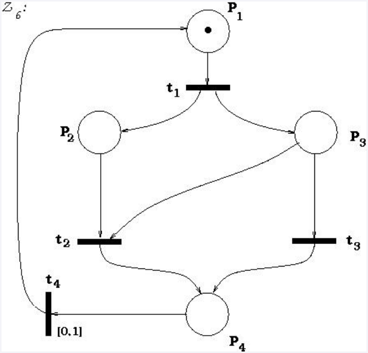


$\mathcal{Z}_5$ is live
$S(\mathcal{Z}_5)$ is not live

$\mathcal{Z}_6$ is not live

Liveness: TPN vs. Skeleton





$\mathcal{Z}_5$ is live
$S(\mathcal{Z}_5)$ is not live

$\mathcal{Z}_6$ is not live
$S(\mathcal{Z}_6)$ is live

Liveness: TPN vs. Skeleton

Theorem:

Let $\mathcal{Z}$ be a TPN, $S(\mathcal{Z})$ its skeleton and $\mathit{eft}(t) = 0$ for all transitions t in $\mathcal{Z}$. Than $\mathcal{Z}$ is live iff $S(\mathcal{Z})$ is live.

Theorem:

Let $\mathcal{Z}$ be a TPN, $S(\mathcal{Z})$ its skeleton and $\mathit{lft}(t) = \infty$ for all transitions t in $\mathcal{Z}$. Than $\mathcal{Z}$ is live iff $S(\mathcal{Z})$ is live.

Liveness: TPN vs. Skeleton

Theorem:

Let $\mathcal{Z}$ be a TPN , $S(\mathcal{Z})$ its skeleton such that

- $S(\mathcal{Z})$ is a Free-Choice-Net,
- $S(\mathcal{Z})$ is homogeneous,

and it holds:

- $\mathcal{M}in(p) \leq \mathcal{M}ax(p)$ for each place p in $\mathcal{Z}$ and
- $lft(t) > 0$ for each transition t in $\mathcal{Z}$.

Than $\mathcal{Z}$ is live iff $S(\mathcal{Z})$ is live.

Some Decidable Problems without using RG

Problem 1:

Input:

- A transition sequence σ in an arbitrary TPN $\mathcal{Z}$.

Output:

1. Is σ a firing sequence in $\mathcal{Z}$?
2. A feasible run $\sigma(\tau)$ of σ, if the answer to (1) is *yes*.

Some Decidable Problems without using RG

Problem 2:

Input:

- A firing sequence σ in an arbitrary TPN $\mathcal{Z}$.

Output:

1. A minimal run of σ.
2. A maximal run of σ, if it exists.

Some Decidable Problems without using RG

Problem 3:

Input:

- A TPN $\mathcal{Z}$ with an *only* partially defined interval function I.
- A transition sequence σ and a number $\lambda \in \mathbb{R}_0^+$.

Output:

1. Is it possible to complete I to a total function such that σ is a firing sequence in $\mathcal{Z}$ and $I(\sigma(\tau)) \leq \lambda$?
2. A completed, totally defined function I, if the answer to (1) is *yes*.
3. Is it possible to complete I to a total function such that σ is a firing sequence in $\mathcal{Z}$ and $I(\sigma(\tau)) \geq \lambda$?
4. A completed, total defined function I, if the answer to (3) is *yes*.

Some Decidable Problems without using RG

Problem 4:

Input:

- A TPN $\mathcal{Z}$ with an *only* partially defined interval function I.
- A transition sequence $\sigma_1 = \sigma t_1$, where σ is a transition sequence and t_1 is a transition in $\mathcal{Z}$.
- A transition sequence $\sigma_2 = \sigma t_2$, where t_2 is a transition in $\mathcal{Z}$ such that $t_1 \neq t_2$.

Output:

1. Is it possible to complete I to a total function such that σ_1 is a firing sequence in $\mathcal{Z}$ and σ_2 is *not* a firing sequence $\mathcal{Z}$?
2. A completed, totally defined function I, if the answer to (1) is *yes*.

Some Decidable Problems with using RG

Problem 5:

Input:

- Two integer states z_1 and z_2, reachable in a TPN $\mathcal{Z}$.

Output:

1. Is $z_2 \in RS_{\mathcal{Z}}(z_1)$?
2. The minimal time distance from z_1 to z_2 as well as the corresponding minimal run, if the answer to (1) is *yes*.

Some Decidable Problems with using RG

Problem 6:

Input:

- Two integer states z_1 and z_2, reachable in an arbitrary TPN $\mathcal{Z}$.

Output:

1. Is $z_2 \in RS_{\mathcal{Z}}(z_1)$?
2. The maximal time distance from z_1 to z_2 as well as the corresponding maximal run, if the answer to (1) is *yes*.

Some Decidable Problems with using RG

Problem 7:

Input:

- Two p-markings m_1 and m_2, reachable in an arbitrary TPN $\mathcal{Z}$.

Output:

1. Is $m_2 \in R_{\mathcal{Z}}(m_1)$?
2. The minimal time distance from m_1 to m_2 as well as the corresponding minimal run, if the answer to (1) is *yes*.

Some Decidable Problems with using RG

Problem 8:

Input:

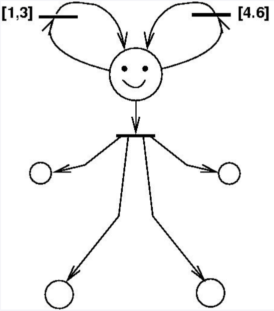
- Two p-markings m_1 and m_2, reachable in an arbitrary TPN $\mathcal{Z}$.

Output:

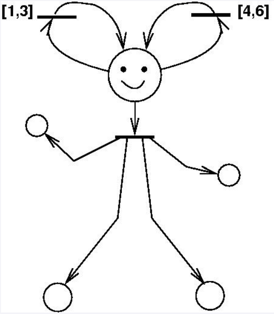
1. Is $m_2 \in R_{\mathcal{Z}}(m_1)$?
2. The maximal time distance from m_1 to m_2 as well as the corresponding maximal run, if the answer to (1) is *yes*.

- Equivalence of parametric states
- Implementation of the quantitative analysis for unbounded TPN

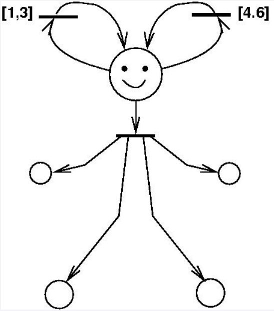
Coffee!



Coffee!



Coffee!



Petri Net

Definition (unmarked Petri Net)

The structure $\mathcal{N} = (P, T, F, V)$ is an unmarked **Petri Net (PN)**, iff

- P, T and F are finite sets,
  $P \cap T = \emptyset, \quad P \cup T \neq \emptyset$,
  $F \subseteq (P \times T) \cup (T \times P)$ and $dom(F) \cup cod(F) = P \cup T$,
- $V : F \longrightarrow \mathbb{N}^+$ (weights of edges).

$$\left.\begin{array}{l}\text{P}-\text{set of places}\\ \text{T}-\text{set of transitions}\end{array}\right\}\text{set of vertices(nodes)}$$

F – set of edges (arcs)

Petri Net

Definition (marked Petri net)

The structure $\mathcal{N}_0 = (\mathcal{N}, m_0)$ is a marked **Petri Net (PN)**, iff

- $\mathcal{N}$ is an unmarked PN,
- $m_0 : P \longrightarrow \mathbb{N}$ (initial marking).

Petri Net

Definition (t^-, t^+)

Let t be a transition in a PN $\mathcal{N}$. t induces the markings t^- and t^+, defined as follows:

$$t^-(p) = \begin{cases} V(p,t) & \text{,iff} \quad (p,t) \in F \\ 0 & \text{,iff} \quad (p,t) \notin F \end{cases}$$

$$t^+(p) = \begin{cases} V(t,p) & \text{,iff} \quad (t,p) \in F \\ 0 & \text{,iff} \quad (t,p) \notin F \end{cases}$$

Petri Net

Definition (firing a transition)

A transition t in a PN $\mathcal{N}$ is **enabled (may fire)** at a marking m iff

$t^- \leq m$ (e.g. $t^-(p) \leq m(p)$ for every place $p \in P$).

When an enabled transition t at a marking m fires, this yields a new marking m' given by

$$m'(p) := m(p) - t^-(p) + t^+(p)$$

(denoted by $m \xrightarrow{t} m'$).

Time Petri Net

Definition (Time Petri net)

The structure $\mathcal{Z} = (\mathcal{N}_o, I)$ is called a **Time Petri net (TPN)** iff:

- $S(\mathcal{Z}) := \mathcal{N}_o$ is a PN (skeleton of $\mathcal{Z}$)
- $I : T \longrightarrow \mathbb{Q}_0^+ \times (\mathbb{Q}_0^+ \cup \{\infty\})$ and $I_1(t) \leq I_2(t)$ for each $t \in T$, where $I(t) = (I_1(t), I_2(t))$.

I – Interval-function
$I_1(t) =: eft(t)$
$I_2(t) =: lft(t)$
w.o.l.g.: $I : T \longrightarrow \mathbb{N} \times (\mathbb{N} \cup \{\infty\})$

state

Definition (state)

Let $\mathcal{Z} = (P, T, F, V, m_o, I)$ be a TPN and $h : T \longrightarrow \mathbb{R}_0^+ \cup \{\#\}$.

$z = (m, h)$ is called a **state** in $\mathcal{Z}$ iff:

- m is a p-marking in $\mathcal{Z}$.
- h is a t-marking in $\mathcal{Z}$.

$h(t)$ is the time shown by the clock of t since the last enabling of t

Definition (state changing: by time elapsing)

Let $\mathcal{Z} = (P, T, F, V, m_o, I)$ be a TPN, $\hat{t}$ be a transition in T and $z = (m, h)$, $z' = (m', h')$ be two states. Then the state $z = (m, h)$ is **changed** into the state $z' = (m', h')$ **by the time elapsing** $\tau \in \mathbb{R}_0^+$, denoted by $z \xrightarrow{\tau} z'$, iff

1. $m' = m$ and
2. $\forall t \; (\; t \in T \wedge h(t) \neq \# \longrightarrow h(t) + \tau \leq lft(t) \;)$ i.e. the time elapsing τ is possible, and
3. $\forall t \; (\; t \in T \longrightarrow h'(t) := \begin{cases} h(t) + \tau & \text{iff} \quad t^- \leq m' \\ \# & \text{iff} \quad t^- \not\leq m' \end{cases} \;)$.

Definition (state changing: by firing a transition)

Let $\mathcal{Z} = (P, T, F, V, m_o, I)$ be a TPN, $\hat{t}$ be a transition in T and $z = (m, h)$, $z' = (m', h')$ be two states. Then the state $z = (m, h)$ is **changed** into the state $z' = (m', h')$ **by firing the transition** $\hat{t}$, denoted by $z \xrightarrow{\hat{t}} z'$, iff

1. $\hat{t}^- \leq m,$ i.e. $\hat{t}$ is enabled in z
2. $eft(\hat{t}) \leq h(\hat{t}),$ i.e. $\hat{t}$ is old enough in z,
3. $m' = m + \Delta\hat{t}$
4. $$h'(t) := \begin{cases} \# & \text{iff} \quad t^- \not\leq m' \\ h(t) & \text{iff} \quad t^- \leq m \wedge t^- \leq m' \wedge \\ & \qquad {}^\bullet t \cap {}^\bullet\hat{t} = \emptyset \wedge t \neq \hat{t} \\ 0 & \text{otherwise} \end{cases}$$ for each $t \in T$.

Definition (state changing: by firing a transition)

Let $\mathcal{Z} = (P, T, F, V, m_o, I)$ be a TPN, $\hat{t}$ be a transition in T and $z = (m, h)$, $z' = (m', h')$ be two states. Then the state $z = (m, h)$ is **changed** into the state $z' = (m', h')$ **by firing the transition** $\hat{t}$, denoted by $z \xrightarrow{\hat{t}} z'$, iff

1. $\hat{t}^- \leq m$, i.e. $\hat{t}$ is enabled in z
2. $eft(\hat{t}) \leq h(\hat{t})$, i.e. $\hat{t}$ is old enough in z,
3. $m' = m + \Delta\hat{t}$
4. $$h'(t) := \begin{cases} \# & \text{iff} \quad t^- \not\leq m' \\ h(t) & \text{iff} \quad t^- \leq m \wedge \\ & \qquad {}^\bullet t \cap {}^\bullet \hat{t} = \emptyset \wedge t \neq \hat{t} \\ 0 & \text{otherwise} \end{cases}$$ for each $t \in T$.

Definition (state changing: by firing a transition)

Let $\mathcal{Z} = (P, T, F, V, m_o, I)$ be a TPN, $\hat{t}$ be a transition in T and $z = (m, h)$, $z' = (m', h')$ be two states. Then the state $z = (m, h)$ is **changed** into the state $z' = (m', h')$ **by firing the transition** $\hat{t}$, denoted by $z \xrightarrow{\hat{t}} z'$, iff

1. $\hat{t}^- \leq m$, i.e. $\hat{t}$ is enabled in z
2. $eft(\hat{t}) \leq h(\hat{t})$, i.e. $\hat{t}$ is old enough in z,
3. $m' = m + \Delta\hat{t}$
4. $$h'(t) := \begin{cases} \# & \text{iff} \quad t^- \not\leq m' \\ h(t) & \text{iff} \quad t^- \leq m \wedge \\ & \qquad {}^\bullet t \cap {}^\bullet\hat{t} = \emptyset \wedge t \neq \hat{t} \\ 0 & \text{otherwise} \end{cases}$$ for each $t \in T$.

Modified Rule for Time Elapsing

Definition

Let $\mathcal{Z} = (P, T, F, V, m_o, I)$ be a TPN, $\hat{t}$ be a transition in T and $z = (m, h)$, $z' = (m', h')$ be two states. Then the state $z = (m, h)$ is **changed** into the state $z' = (m', h')$ **by the time elapsing** $\tau \in \mathbb{R}_0^+$, denoted by $z \xrightarrow{\tau} z'$, iff

1. $m' = m$ and
2. $\forall t\ (\ t \in T \wedge h(t) \neq \# \longrightarrow h(t) + \tau \leq lft(t)\ )$ i.e. the time elapsing τ is possible, and
3. $\forall t\ (\ t \in T \longrightarrow h'(t) := \begin{cases} \# & \text{iff } t^- \not\leq m' \\ h(t) & \text{iff } t^- \leq m' \wedge lft(t) = \infty \\ & \quad \wedge h(t) \geq eft(t) \\ h(t) + \tau & \text{else} \end{cases}\ )$.

$$z_0 \xrightarrow{0.7} \xrightarrow{t_1} \xrightarrow{0.0} \xrightarrow{t_3} \xrightarrow{0.4} \xrightarrow{t_4} \xrightarrow{1.2} \xrightarrow{t_2} \xrightarrow{0.5} \xrightarrow{t_3} \xrightarrow{1.4} (m_\sigma, (1.9, 1.4, 1.4, 1.4, 4.2, \sharp))$$

I	x_0	x_1	x_2	x_3	x_4	x_5	$h_\sigma(t_1)$	$h_\sigma(t_2)$	$h_\sigma(t_5)$
$\hat{\beta} = \beta_0$	0.7	0.0	0.4	1.2	0.5	1.4	1.9	1.4	4.2

$h_\sigma(t_1) = x_4 + x_5$, $\quad h_\sigma(t_2) = h_\sigma(t_3) = h_\sigma(t_4) = x_5$

$h_\sigma(t_5) = x_1 + x_2 + x_3 + x_4 + x_5$

$$z_0 \xrightarrow{0.7} \xrightarrow{t_1} \xrightarrow{0.0} \xrightarrow{t_3} \xrightarrow{0.4} \xrightarrow{t_4} \xrightarrow{1.2} \xrightarrow{t_2} \xrightarrow{0.5} \xrightarrow{t_3} \xrightarrow{1.4} (m_\sigma, (1.9, 1.4, 1.4, 1.4, 4.2, \sharp))$$

I			x_0	x_1	x_2	x_3	x_4	x_5	$h_\sigma(t_1)$	$h_\sigma(t_2)$	$h_\sigma(t_5)$
$\hat{\beta}$	$=$	β_0	0.7	0.0	0.4	1.2	0.5	1.4	1.9	1.4	4.2
		β_1	0.7	0.0	0.4	1.2	0.5				

$h_\sigma(t_1) = x_4 + x_5,$ $\quad h_\sigma(t_2) = h_\sigma(t_3) = h_\sigma(t_4) = x_5$

$h_\sigma(t_5) = x_1 + x_2 + x_3 + x_4 + x_5$

$$z_0 \xrightarrow{0.7} \xrightarrow{t_1} \xrightarrow{0.0} \xrightarrow{t_3} \xrightarrow{0.4} \xrightarrow{t_4} \xrightarrow{1.2} \xrightarrow{t_2} \xrightarrow{0.5} \xrightarrow{t_3} \xrightarrow{1} (m_\sigma, (1.9, 1.4, 1.4, 1.4, 4.2, \sharp))$$

I		x_0	x_1	x_2	x_3	x_4	x_5	$h_\sigma(t_1)$	$h_\sigma(t_2)$	$h_\sigma(t_5)$
$\hat{\beta} =$	β_0	0.7	0.0	0.4	1.2	0.5	1.4	1.9	1.4	4.2
	β_1	0.7	0.0	0.4	1.2	0.5	**1**			

$h_\sigma(t_1) = x_4 + x_5$, $\quad h_\sigma(t_2) = h_\sigma(t_3) = h_\sigma(t_4) = x_5$

$h_\sigma(t_5) = x_1 + x_2 + x_3 + x_4 + x_5$

$$z_0 \xrightarrow{0.7} \xrightarrow{t_1} \xrightarrow{0.0} \xrightarrow{t_3} \xrightarrow{0.4} \xrightarrow{t_4} \xrightarrow{1.2} \xrightarrow{t_2} \xrightarrow{0.5} \xrightarrow{t_3} \xrightarrow{1} (m_\sigma, (1.5, 1.0, 1.0, 1.0, 3.8, \sharp))$$

I			x_0	x_1	x_2	x_3	x_4	x_5	$h_\sigma(t_1)$	$h_\sigma(t_2)$	$h_\sigma(t_5)$
$\hat{\beta}$	$=$	β_0	0.7	0.0	0.4	1.2	0.5	1.4	1.9	1.4	4.2
		β_1	0.7	0.0	0.4	1.2	0.5	**1**	1.5	**1.0**	3.8

$h_\sigma(t_1) = x_4 + x_5,$ $h_\sigma(t_2) = h_\sigma(t_3) = h_\sigma(t_4) = x_5$

$h_\sigma(t_5) = x_1 + x_2 + x_3 + x_4 + x_5$

$$z_0 \xrightarrow{0.7} \xrightarrow{t_1} \xrightarrow{0.0} \xrightarrow{t_3} \xrightarrow{0.4} \xrightarrow{t_4} \xrightarrow{1.2} \xrightarrow{t_2} \xrightarrow{0.5} \xrightarrow{t_3} \xrightarrow{1} (m_\sigma, (1.5, 1.0, 1.0, 1.0, 3.8, \sharp))$$

I	x_0	x_1	x_2	x_3	x_4	x_5	$h_\sigma(t_1)$	$h_\sigma(t_2)$	$h_\sigma(t_5)$
$\hat{\beta} = \beta_0$	0.7	0.0	0.4	1.2	0.5	1.4	1.9	1.4	4.2
β_1	0.7	0.0	0.4	1.2	0.5	**1**	1.5	**1.0**	3.8
β_2	0.7	0.0	0.4	1.2		1			

$h_\sigma(t_1) = x_4 + x_5$, $\quad h_\sigma(t_2) = h_\sigma(t_3) = h_\sigma(t_4) = x_5$

$h_\sigma(t_5) = x_1 + x_2 + x_3 + x_4 + x_5$

$$z_0 \xrightarrow{0.7} \xrightarrow{t_1} \xrightarrow{0.0} \xrightarrow{t_3} \xrightarrow{0.4} \xrightarrow{t_4} \xrightarrow{1.2} \xrightarrow{t_2} \xrightarrow{0} \xrightarrow{t_3} \xrightarrow{1} (m_\sigma, (1.5, 1.0, 1.0, 1.0, 3.8, \sharp))$$

I	x_0	x_1	x_2	x_3	x_4	x_5	$h_\sigma(t_1)$	$h_\sigma(t_2)$	$h_\sigma(t_5)$
$\hat{\beta} = \beta_0$	0.7	0.0	0.4	1.2	0.5	1.4	1.9	1.4	4.2
β_1	0.7	0.0	0.4	1.2	0.5	**1**	1.5	**1.0**	3.8
β_2	0.7	0.0	0.4	1.2	**0**	1			

$h_\sigma(t_1) = x_4 + x_5$, $\quad h_\sigma(t_2) = h_\sigma(t_3) = h_\sigma(t_4) = x_5$

$h_\sigma(t_5) = x_1 + x_2 + x_3 + x_4 + x_5$

$$z_0 \xrightarrow{0.7} \xrightarrow{t_1} \xrightarrow{0.0} \xrightarrow{t_3} \xrightarrow{0.4} \xrightarrow{t_4} \xrightarrow{1.2} \xrightarrow{t_2} \xrightarrow{0} \xrightarrow{t_3} \xrightarrow{1} (m_\sigma, (1.0, 1.0, 1.0, 1.0, 3.3, \sharp))$$

I	x_0	x_1	x_2	x_3	x_4	x_5	$h_\sigma(t_1)$	$h_\sigma(t_2)$	$h_\sigma(t_5)$
$\hat{\beta} = \beta_0$	0.7	0.0	0.4	1.2	0.5	1.4	1.9	1.4	4.2
β_1	0.7	0.0	0.4	1.2	0.5	**1**	1.5	**1.0**	3.8
β_2	0.7	0.0	0.4	1.2	**0**	1	**1.0**		3.3

$h_\sigma(t_1) = x_4 + x_5,$ $\quad h_\sigma(t_2) = h_\sigma(t_3) = h_\sigma(t_4) = x_5$

$h_\sigma(t_5) = x_1 + x_2 + x_3 + x_4 + x_5$

$$z_0 \xrightarrow{0.7} \xrightarrow{t_1} \xrightarrow{0.0} \xrightarrow{t_3} \xrightarrow{0.4} \xrightarrow{t_4} \xrightarrow{1.2} \xrightarrow{t_2} \xrightarrow{0} \xrightarrow{t_3} \xrightarrow{1} (m_\sigma, (1.0, 1.0, 1.0, 1.0, 3.3, \sharp))$$

I	x_0	x_1	x_2	x_3	x_4	x_5	$h_\sigma(t_1)$	$h_\sigma(t_2)$	$h_\sigma(t_5)$
$\hat{\beta} = \beta_0$	0.7	0.0	0.4	1.2	0.5	1.4	1.9	1.4	4.2
β_1	0.7	0.0	0.4	1.2	0.5	**1**	1.5	**1.0**	3.8
β_2	0.7	0.0	0.4	1.2	**0**	1	**1.0**		3.3
β_3	0.7	0.0	0.4		0	1			

$h_\sigma(t_1) = x_4 + x_5$, $\quad h_\sigma(t_2) = h_\sigma(t_3) = h_\sigma(t_4) = x_5$

$h_\sigma(t_5) = x_1 + x_2 + x_3 + x_4 + x_5$

$$z_0 \xrightarrow{0.7} \xrightarrow{t_1} \xrightarrow{0.0} \xrightarrow{t_3} \xrightarrow{0.4} \xrightarrow{t_4} \xrightarrow{1} \xrightarrow{t_2} \xrightarrow{0} \xrightarrow{t_3} \xrightarrow{1} (m_\sigma, (1.0, 1.0, 1.0, 1.0, 3.3, \sharp))$$

I			x_0	x_1	x_2	x_3	x_4	x_5	$h_\sigma(t_1)$	$h_\sigma(t_2)$	$h_\sigma(t_5)$
$\hat{\beta}$	=	β_0	0.7	0.0	0.4	1.2	0.5	1.4	1.9	1.4	4.2
		β_1	0.7	0.0	0.4	1.2	0.5	**1**	1.5	**1.0**	3.8
		β_2	0.7	0.0	0.4	1.2	**0**	1	**1.0**		3.3
		β_3	0.7	0.0	0.4	**1**	0	1			

$h_\sigma(t_1) = x_4 + x_5,$ $\quad h_\sigma(t_2) = h_\sigma(t_3) = h_\sigma(t_4) = x_5$

$h_\sigma(t_5) = x_1 + x_2 + x_3 + x_4 + x_5$

$$z_0 \xrightarrow{0.7} \xrightarrow{t_1} \xrightarrow{0.0} \xrightarrow{t_3} \xrightarrow{0.4} \xrightarrow{t_4} \xrightarrow{1} \xrightarrow{t_2} \xrightarrow{0} \xrightarrow{t_3} \xrightarrow{1} (m_\sigma, (1.0, 1.0, 1.0, 1.0, 3.1, \sharp))$$

I	x_0	x_1	x_2	x_3	x_4	x_5	$h_\sigma(t_1)$	$h_\sigma(t_2)$	$h_\sigma(t_5)$
$\hat{\beta} = \beta_0$	0.7	0.0	0.4	1.2	0.5	1.4	1.9	1.4	4.2
β_1	0.7	0.0	0.4	1.2	0.5	**1**	1.5	**1.0**	3.8
β_2	0.7	0.0	0.4	1.2	**0**	1	**1.0**		3.3
β_3	0.7	0.0	0.4	**1**	0	1			3.1

$h_\sigma(t_1) = x_4 + x_5$, $\qquad h_\sigma(t_2) = h_\sigma(t_3) = h_\sigma(t_4) = x_5$

$h_\sigma(t_5) = x_1 + x_2 + x_3 + x_4 + x_5$

$$z_0 \xrightarrow{0.7} \xrightarrow{t_1} \xrightarrow{0.0} \xrightarrow{t_3} \xrightarrow{0.4} \xrightarrow{t_4} \xrightarrow{1} \xrightarrow{t_2} \xrightarrow{0} \xrightarrow{t_3} \xrightarrow{1} (m_\sigma, (1.0, 1.0, 1.0, 1.0, 3.1, \sharp))$$

I	x_0	x_1	x_2	x_3	x_4	x_5	$h_\sigma(t_1)$	$h_\sigma(t_2)$	$h_\sigma(t_5)$
$\hat{\beta} = \beta_0$	0.7	0.0	0.4	1.2	0.5	1.4	1.9	1.4	4.2
β_1	0.7	0.0	0.4	1.2	0.5	**1**	1.5	**1.0**	3.8
β_2	0.7	0.0	0.4	1.2	**0**	1	**1.0**		3.3
β_3	0.7	0.0	0.4	**1**	0	1			3.1
β_4	0.7	0.0		1	0	1			

$h_\sigma(t_1) = x_4 + x_5$, $h_\sigma(t_2) = h_\sigma(t_3) = h_\sigma(t_4) = x_5$

$h_\sigma(t_5) = x_1 + x_2 + x_3 + x_4 + x_5$

$$z_0 \xrightarrow{0.7} \xrightarrow{t_1} \xrightarrow{0.0} \xrightarrow{t_3} \color{red}{\xrightarrow{1}} \xrightarrow{t_4} \xrightarrow{1} \xrightarrow{t_2} \xrightarrow{0} \xrightarrow{t_3} \xrightarrow{1} (m_\sigma, (1.0, 1.0, 1.0, 1.0, 3.1, \sharp))$$

I			x_0	x_1	x_2	x_3	x_4	x_5	$h_\sigma(t_1)$	$h_\sigma(t_2)$	$h_\sigma(t_5)$
$\hat{\beta}$	$=$	β_0	0.7	0.0	0.4	1.2	0.5	1.4	1.9	1.4	4.2
		β_1	0.7	0.0	0.4	1.2	0.5	**1**	1.5	**1.0**	3.8
		β_2	0.7	0.0	0.4	1.2	**0**	1	**1.0**		3.3
		β_3	0.7	0.0	0.4	**1**	0	1			3.1
		β_4	0.7	0.0	**1**	1	0	1			

$h_\sigma(t_1) = x_4 + x_5,$ $\quad h_\sigma(t_2) = h_\sigma(t_3) = h_\sigma(t_4) = x_5$

$h_\sigma(t_5) = x_1 + x_2 + x_3 + x_4 + x_5$

$$z_0 \xrightarrow{0.7} \xrightarrow{t_1} \xrightarrow{0.0} \xrightarrow{t_3} \xrightarrow{1} \xrightarrow{t_4} \xrightarrow{1} \xrightarrow{t_2} \xrightarrow{0} \xrightarrow{t_3} \xrightarrow{1} (m_\sigma, (1.0, 1.0, 1.0, 1.0, 3.7, \sharp))$$

I			x_0	x_1	x_2	x_3	x_4	x_5	$h_\sigma(t_1)$	$h_\sigma(t_2)$	$h_\sigma(t_5)$
$\hat{\beta}$	=	β_0	0.7	0.0	0.4	1.2	0.5	1.4	1.9	1.4	4.2
		β_1	0.7	0.0	0.4	1.2	0.5	**1**	1.5	**1.0**	3.8
		β_2	0.7	0.0	0.4	1.2	**0**	1	**1.0**		3.3
		β_3	0.7	0.0	0.4	**1**	0	1			3.1
		β_4	0.7	0.0	**1**	1	0	1			3.7

$h_\sigma(t_1) = x_4 + x_5$, $h_\sigma(t_2) = h_\sigma(t_3) = h_\sigma(t_4) = x_5$

$h_\sigma(t_5) = x_1 + x_2 + x_3 + x_4 + x_5$

$$z_0 \xrightarrow{0.7} \xrightarrow{t_1} \xrightarrow{0.0} \xrightarrow{t_3} \xrightarrow{1} \xrightarrow{t_4} \xrightarrow{1} \xrightarrow{t_2} \xrightarrow{0} \xrightarrow{t_3} \xrightarrow{1} (m_\sigma, (1.0, 1.0, 1.0, 1.0, 3.7, \sharp))$$

I	x_0	x_1	x_2	x_3	x_4	x_5	$h_\sigma(t_1)$	$h_\sigma(t_2)$	$h_\sigma(t_5)$
$\hat{\beta} = \beta_0$	0.7	0.0	0.4	1.2	0.5	1.4	1.9	1.4	4.2
β_1	0.7	0.0	0.4	1.2	0.5	**1**	1.5	**1.0**	3.8
β_2	0.7	0.0	0.4	1.2	**0**	1	**1.0**		3.3
β_3	0.7	0.0	0.4	**1**	0	1			3.1
β_4	0.7	0.0	**1**	1	0	1			3.7
β_5	0.7		1	1	0	1			

$h_\sigma(t_1) = x_4 + x_5,$ $\quad h_\sigma(t_2) = h_\sigma(t_3) = h_\sigma(t_4) = x_5$

$h_\sigma(t_5) = x_1 + x_2 + x_3 + x_4 + x_5$

$$z_0 \xrightarrow{0.7} \xrightarrow{t_1} \xrightarrow{0} \xrightarrow{t_3} \xrightarrow{1} \xrightarrow{t_4} \xrightarrow{1} \xrightarrow{t_2} \xrightarrow{0} \xrightarrow{t_3} \xrightarrow{1} (m_\sigma, (1.0, 1.0, 1.0, 1.0, 3.7, \sharp))$$

I	x_0	x_1	x_2	x_3	x_4	x_5	$h_\sigma(t_1)$	$h_\sigma(t_2)$	$h_\sigma(t_5)$
$\hat{\beta} = \beta_0$	0.7	0.0	0.4	1.2	0.5	1.4	1.9	1.4	4.2
β_1	0.7	0.0	0.4	1.2	0.5	**1**	1.5	**1.0**	3.8
β_2	0.7	0.0	0.4	1.2	**0**	1	**1.0**		3.3
β_3	0.7	0.0	0.4	**1**	0	1			3.1
β_4	0.7	0.0	**1**	1	0	1			3.7
β_5	0.7	**0**	1	1	0	1			

$h_\sigma(t_1) = x_4 + x_5$, $\quad h_\sigma(t_2) = h_\sigma(t_3) = h_\sigma(t_4) = x_5$

$h_\sigma(t_5) = x_1 + x_2 + x_3 + x_4 + x_5$

$$z_0 \xrightarrow{0.7} \xrightarrow{t_1} \xrightarrow{0} \xrightarrow{t_3} \xrightarrow{1} \xrightarrow{t_4} \xrightarrow{1} \xrightarrow{t_2} \xrightarrow{0} \xrightarrow{t_3} \xrightarrow{1} (m_\sigma, (1.0, 1.0, 1.0, 1.0, 3.7, \sharp))$$

I	x_0	x_1	x_2	x_3	x_4	x_5	$h_\sigma(t_1)$	$h_\sigma(t_2)$	$h_\sigma(t_5)$
$\hat{\beta} = \beta_0$	0.7	0.0	0.4	1.2	0.5	1.4	1.9	1.4	4.2
β_1	0.7	0.0	0.4	1.2	0.5	**1**	1.5	**1.0**	3.8
β_2	0.7	0.0	0.4	1.2	**0**	1	**1.0**		3.3
β_3	0.7	0.0	0.4	**1**	0	1			3.1
β_4	0.7	0.0	**1**	1	0	1			3.7
β_5	0.7	**0**	1	1	0	1			3.7

$h_\sigma(t_1) = x_4 + x_5$, $\quad h_\sigma(t_2) = h_\sigma(t_3) = h_\sigma(t_4) = x_5$

$h_\sigma(t_5) = x_1 + x_2 + x_3 + x_4 + x_5$

$$z_0 \xrightarrow{0.7} \xrightarrow{t_1} \xrightarrow{0} \xrightarrow{t_3} \xrightarrow{1} \xrightarrow{t_4} \xrightarrow{1} \xrightarrow{t_2} \xrightarrow{0} \xrightarrow{t_3} \xrightarrow{1} (m_\sigma, (1.0, 1.0, 1.0, 1.0, 3.7, \sharp))$$

I			x_0	x_1	x_2	x_3	x_4	x_5	$h_\sigma(t_1)$	$h_\sigma(t_2)$	$h_\sigma(t_5)$
$\hat{\beta}$	$=$	β_0	0.7	0.0	0.4	1.2	0.5	1.4	1.9	1.4	4.2
		β_1	0.7	0.0	0.4	1.2	0.5	**1**	1.5	**1.0**	3.8
		β_2	0.7	0.0	0.4	1.2	**0**	1	**1.0**		3.3
		β_3	0.7	0.0	0.4	**1**	0	1			3.1
		β_4	0.7	0.0	**1**	1	0	1			3.7
		β_5	0.7	**0**	1	1	0	1			3.7
β^*	$=$	β_6		0	1	1	0	1			

$h_\sigma(t_1) = x_4 + x_5,$ $\quad h_\sigma(t_2) = h_\sigma(t_3) = h_\sigma(t_4) = x_5$

$h_\sigma(t_5) = x_1 + x_2 + x_3 + x_4 + x_5$

$$z_0 \xrightarrow{1} \xrightarrow{t_1} \xrightarrow{0} \xrightarrow{t_3} \xrightarrow{1} \xrightarrow{t_4} \xrightarrow{1} \xrightarrow{t_2} \xrightarrow{0} \xrightarrow{t_3} \xrightarrow{1} (m_\sigma, (1.0, 1.0, 1.0, 1.0, 3.7, \sharp))$$

I			x_0	x_1	x_2	x_3	x_4	x_5	$h_\sigma(t_1)$	$h_\sigma(t_2)$	$h_\sigma(t_5)$
$\hat{\beta}$	$=$	β_0	0.7	0.0	0.4	1.2	0.5	1.4	1.9	1.4	4.2
		β_1	0.7	0.0	0.4	1.2	0.5	**1**	1.5	**1.0**	3.8
		β_2	0.7	0.0	0.4	1.2	**0**	1	**1.0**		3.3
		β_3	0.7	0.0	0.4	**1**	0	1			3.1
		β_4	0.7	0.0	**1**	1	0	1			3.7
		β_5	0.7	**0**	1	1	0	1			3.7
β^*	$=$	β_6	**1**	0	1	1	0	1			

$h_\sigma(t_1) = x_4 + x_5$, $\quad h_\sigma(t_2) = h_\sigma(t_3) = h_\sigma(t_4) = x_5$

$h_\sigma(t_5) = x_1 + x_2 + x_3 + x_4 + x_5$

$$z_0 \xrightarrow{1} \xrightarrow{t_1} \xrightarrow{0} \xrightarrow{t_3} \xrightarrow{1} \xrightarrow{t_4} \xrightarrow{1} \xrightarrow{t_2} \xrightarrow{0} \xrightarrow{t_3} \xrightarrow{1} (m_\sigma, (1.0, 1.0, 1.0, 1.0, 4.0, \sharp))$$

I	x_0	x_1	x_2	x_3	x_4	x_5	$h_\sigma(t_1)$	$h_\sigma(t_2)$	$h_\sigma(t_5)$
$\hat{\beta} = \beta_0$	0.7	0.0	0.4	1.2	0.5	1.4	1.9	1.4	4.2
β_1	0.7	0.0	0.4	1.2	0.5	**1**	1.5	**1.0**	3.8
β_2	0.7	0.0	0.4	1.2	**0**	1	**1.0**		3.3
β_3	0.7	0.0	0.4	**1**	0	1			3.1
β_4	0.7	0.0	**1**	1	0	1			3.7
β_5	0.7	**0**	1	1	0	1			3.7
$\beta^* = \beta_6$	**1**	0	1	1	0	1			**4.0**

$h_\sigma(t_1) = x_4 + x_5$, $\quad h_\sigma(t_2) = h_\sigma(t_3) = h_\sigma(t_4) = x_5$

$h_\sigma(t_5) = x_1 + x_2 + x_3 + x_4 + x_5$

$$z_0 \xrightarrow{0.7} \xrightarrow{t_1} \xrightarrow{0.0} \xrightarrow{t_3} \xrightarrow{0.4} \xrightarrow{t_4} \xrightarrow{1.2} \xrightarrow{t_2} \xrightarrow{0.5} \xrightarrow{t_3} \xrightarrow{1.4} (m_\sigma, (1.9, 1.4, 1.4, 1.4, 4.2, \sharp))$$

$\hat{\beta}$ =	x_0	x_1	x_2	x_3	x_4	x_5	$h_\sigma(t_1)$	$h_\sigma(t_2)$	$h_\sigma(t_5)$
β_0	0.7	0.0	0.4	1.2	0.5	1.4	1.9	1.4	4.2
β_1	0.7	0.0	0.4	1.2	0.5	**2**	2.5	**2.0**	4.8
β_2	0.7	0.0	0.4	1.2	**0**	2	**2.0**		4.3
β_3	0.7	0.0	0.4	**2**	0	2			5.1
β_4	0.7	0.0	**0**	2	0	2			4.7
β_5	0.7	**0**	0	2	0	2			4.7
β_6	**1**	0	0	2	0	2			**5.0**

$$z_0 \xrightarrow{1} \xrightarrow{t_1} \xrightarrow{0} \xrightarrow{t_3} \xrightarrow{0} \xrightarrow{t_4} \xrightarrow{2} \xrightarrow{t_2} \xrightarrow{0} \xrightarrow{t_3} \xrightarrow{2} (m_\sigma, (2,\ 2,\ 2,\ 2,\ 5,\ \sharp))$$

$$z_0 \xrightarrow{0.7} \xrightarrow{t_1} \xrightarrow{0.0} \xrightarrow{t_3} \xrightarrow{0.4} \xrightarrow{t_4} \xrightarrow{1.2} \xrightarrow{t_2} \xrightarrow{0.5} \xrightarrow{t_3} \xrightarrow{1.4} \left(m_\sigma, (1.9, 1.4, 1.4, 1.4, 4.2, \sharp)\right)$$

The time length of the run $\sigma(\tau)$ is

$$\hat{\beta}(x_0) + \hat{\beta}(x_1) + \hat{\beta}(x_2) + \hat{\beta}(x_3) + \hat{\beta}(x_4) + \hat{\beta}(x_5) = 4.2$$

$$z_0 \xrightarrow{0.7} \xrightarrow{t_1} \xrightarrow{0.0} \xrightarrow{t_3} \xrightarrow{0.4} \xrightarrow{t_4} \xrightarrow{1.2} \xrightarrow{t_2} \xrightarrow{0.5} \xrightarrow{t_3} \xrightarrow{1.4} (m_\sigma, (1.9, 1.4, 1.4, 1.4, 4.2, \sharp))$$

The time length of the run $\sigma(\tau)$ is

$$\hat{\beta}(x_0) + \hat{\beta}(x_1) + \hat{\beta}(x_2) + \hat{\beta}(x_3) + \hat{\beta}(x_4) + \hat{\beta}(x_5) = 4.2$$

In tableau I: The time length of the run $\sigma(\tau_1^*)$ is 4

$$z_0 \xrightarrow{0.7} \xrightarrow{t_1} \xrightarrow{0.0} \xrightarrow{t_3} \xrightarrow{0.4} \xrightarrow{t_4} \xrightarrow{1.2} \xrightarrow{t_2} \xrightarrow{0.5} \xrightarrow{t_3} \xrightarrow{1.4} (m_\sigma, (1.9, 1.4, 1.4, 1.4, 4.2, \sharp))$$

The time length of the run $\sigma(\tau)$ is

$$\hat{\beta}(x_0) + \hat{\beta}(x_1) + \hat{\beta}(x_2) + \hat{\beta}(x_3) + \hat{\beta}(x_4) + \hat{\beta}(x_5) = 4.2$$

In tableau I: The time length of the run $\sigma(\tau_1^*)$ is 4

In tableau II: The time length of the run $\sigma(\tau_2^*)$ is 5

Definition

Let $\mathcal{Z} = (P, T, F, V, m_0, I)$ be a TPN and p a place in $\mathcal{Z}$. Then

$$\mathcal{M}in(p) := \max\{\ eft(t) \mid t \in T \ \wedge \ t \in p^\bullet \ \}$$

$$\mathcal{M}ax(p) := \min\{\ lft(t) \mid t \in T \ \wedge \ t \in p^\bullet \ \}$$

Definition (maximal time distance between two states)

Let $\mathcal{Z}$ be a TPN and let z_1 and z_2 be two reachable states in $\mathcal{Z}$ with $z_2 \in RS_{\mathcal{Z}}(z_1)$. The maximal time distance $d_{\max}(z_1, z_2)$ from z_1 to z_2 is defined by:

$$d_{\max}(z_1, z_2) := \begin{cases} \infty & \text{, if a cycle or a dead state is reachable starting at } z_1 \text{ before reaching } z_2 \\ \max\limits_{\substack{\sigma, \\ z_1 \xrightarrow{\sigma(\tau)} z_2}} \sum\limits_{i=0}^{l(\tau)} \tau_i & \text{, else.} \end{cases}$$