# Cognitive Robotics 

## Motion (part 1)

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Rijeka 2018

## Outline

## Introduction

Kinematics of Poses
Kinematics of Drive Systems
Trajectories
Motion Planning
Motion Control
Motions of Legged Robots
Optimization/Learning of Motions
Biologically Inspired Motions

## Motion

## Motion:

Change of position(s) by certain actions/skills, e.g. for locomotion or manipulation.

Great variety of natural and technical systems

Formal description by mechanics
(force, mass, displacement, velocity, acceleration)

Problems in Robotics:
How can motions be realized and controlled (hardware, software)

## Locomotion (Ground, Air, Water,Space...)



## Cars



## 

## First Autonomous State Limousine


"MadeInGermany"
Autonomos Labs (R.Rojas, FU Berlin)
https://www.youtube.com/watch?v=nX-le6JSU5g

How many degrees of freedom? Which poses can be reached?

## Legged Robots, Special Designs

Hirose Robotics Lab, Tokyo


## Boston Dynamics



## Humanoid Robots



How many degrees of freedom? Which poses can be reached?



Myon
(Dr. Manfred Hild, Neurorobotics Lab Humboldt University)

## Flying Robots

Cognitive Robotics Lab
Prof. Verena Hafner, HU


## Bionics

## TU Berlin (Ingo Rechenberg)

http://lautaro.bionik.tu-berlin.de/institut/s2foshow/

## Manipulators

## KUKA Roboter GmbH



## Manipulators



## Manipulators



## Manipulators



## Strawberry Harvesting Robot by Robotic Harvesting LLC

## Wheels, Chains, ...



## Joints

- Active: control with motors, pulleys, ...
- Problem: loading of gear axes
- Passive: Adaptation

Maintaining rest position by drives, gravity, friction, preload,...


## Joints of Nao from Aldebaran



## Nao in Simulation




YawPitch


22 active DOF (motors):

- 2 head
- 4 per arm
- 5 per leg
- 2 hip

Burkhard


## 1 DOF Joints in Technique

With 1 degree of freedom (DOF):

- rotation joint
- torsion joint
- revolver joint

- Linear joint (Translation joint., prismatic joint)

Several DOF by combinations


## Stanford Manipulator

P: Prismatic joint
R: Rotation joint


## Puma (Programmable Universal Manipulation Arm)



## Degrees of Freedom (DOF)

DOF is the

- minimal number $m$ of parameters $p_{1}, \ldots, p_{m}$ for complete description
equivalently:
- maximal number $m$ of independent parameters $p_{1}, \ldots, p_{m}$


## Degrees of Freedom (DOF)

DOF of poses
(= parameters for complete description in work space):

- point on plan $\mathrm{p}=(\mathrm{x}, \mathrm{y}), 2$ DOF (2 position)
- car on plane: $p=(x, y, \theta), \quad 3$ DOF (2 position, 1 orientation)
- airplane: $\mathrm{p}=(\mathrm{x}, \mathrm{y}, \mathrm{z}, \phi, \Psi, \theta), 6$ DOF (3 position, 3 orientation)

DOF of control parameters (in control/configuration space): independently movable parts (joints, wheels/axes, ...)

DOF of control may be active (actuated) or passiv

## Degrees of Freedom (DOF)

## 20 active DOF (motors) <br> - 3 per leg <br> - 3 head <br> - 2 tail <br> - 1 mouth <br> - 1 per ear



## Degrees of Freedom (DOF)



## Degrees of Freedom (DOF)

Reachable poses depend on morphology and environment


Constraints $\mathrm{C}\left(\mathrm{p}_{1}, \ldots \mathrm{p}_{\mathrm{m}}\right)=0$ for parameters may reduce DOF


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## Kinematics of Poses

Kinematics (forward kinematics):

- What is the pose? Inverse kinematics (reverse kinematics):
- How to set the pose?

Simplification in Kinematics:
Neglect mass and force

## Work Space and Configuration Space

Work space: „Relevant" environment of the robot or some part. Pose $\mathrm{p}=\left(\mathrm{p}_{1}, \ldots, \mathrm{p}_{\mathrm{m}}\right)$ :
Position/orientation of the robot or some part in work space (e.g. the pose of an end effector, of a camera etc.).
$\mathrm{m}=$ DOF of the pose in Workspace
End effector of an industrial robot:
$\mathrm{p}=(\mathrm{x}, \mathrm{y}, \mathrm{z}, \phi, \Psi, \theta)$

6 DOF:
ÿ 3 position,
ÿ 3 orientation


## Work Space and Configuration Space

Configuration space:
Configuration $\mathrm{q}=\left(\mathrm{q}_{1}, \ldots, \mathrm{q}_{\mathrm{n}}\right)$ : parameters of joints etc. "generalized coordinates", „control parameters"
$\mathrm{n}=\mathrm{DOF}$ in configuration space


## Work Space and Configuration Space



## Work Space and Configuration Space



## Constraints

## In Work Space by morphology and environment



In Configuration Space by related unreachable region


Motion planning in Configuration Space


## Work Space and Configuration Space

Kinematics:

$$
p=f(q)
$$

Determine pose from configuration

- Configuration determines pose uniquely

Inverse Kinematics:

$$
q=f^{-1}(p)
$$

Find a configuration for requested pose

- Pose might be realized by different configurations



## Example „Planar Leg"

Work space $x, y$


## Configuration space $\theta_{1}, \theta_{2}$



## Example „Planar Leg"

Achievable points are limited by joint angles $\theta_{1}, \theta_{2}$, limb lengths $I_{1}, I_{2}$, spacial constraints


## Example „Planar Leg"

Kinematics:
-Rotation by $\Theta_{1}$
-Translation by $\mathrm{I}_{1}$
-Rotation by $\Theta_{2}$
-Translation by $\mathrm{I}_{2}$


$$
\left[\begin{array}{l}
x \\
y
\end{array}\right]=l_{1}\left[\begin{array}{c}
\cos \left(\theta_{1}\right) \\
\sin \left(\theta_{1}\right)
\end{array}\right]+l_{2}\left[\begin{array}{c}
\cos \left(\theta_{1}+\theta_{2}\right) \\
\sin \left(\theta_{1}+\theta_{2}\right)
\end{array}\right]
$$

## Example „Planar Leg"

 Inverse Kinematics:(by cosine rule)

$$
\cos \left(\theta_{2}\right)=\frac{x^{2}+y^{2}-l_{1}^{2}-l_{2}^{2}}{2 l_{1} l_{2}}
$$

$\cos \left(\Theta_{1}\right)$ computable by the formula for forward kinematics


## Kinematics: Calculate $p=f(q)$

Joints: Rotations

Limbs: Translations


## Kinematics: Coordinate Transformation

$$
q=\left(q_{1}, \ldots, q_{5}\right)
$$



From local coordinates to world coordinates

## Kinematics: Coordinate Transformation



Coordinate transformation by a sequence of intrinsic rotations and translations along the cinematic chain.

The ordering must be preserved.

## Homogenous Coordinates for 3D

4-dimensional vector
( $x / w, y / w, z / w, w$ ) with arbitrary $w \neq 0$ represents ( $x, y, z$ )
We will use ( $x, y, z, 1$ ) i.e. $w=1$

The 4-dimensional matrix H can describe Rotation $R$ followed by Translation $T$

## Homogenous Coordinates for 3D

Sequence of transformations along cinematic chain can be described by matrix multiplications

$$
\mathrm{M}=\mathrm{H}_{1} \cdot \mathrm{H}_{2} \cdot \mathrm{H}_{3} \cdot \ldots \mathrm{H}_{\mathrm{n}}
$$

## - Kinematics

| $X=M \cdot x$ |
| :---: |
| $\left[\begin{array}{l}X \\ Y \\ Z \\ 1\end{array}\right]=M \cdot\left[\begin{array}{l}x \\ y \\ Z \\ 1\end{array}\right]$ |

by computing $X$ from $X=M \cdot x$ for given $M, x$

- Inverse Kinematics
by finding $M=H_{1} \cdot \mathrm{H}_{2} \cdot \mathrm{H}_{3} \cdot \ldots \mathrm{H}_{n}$ for given $X, x$
(but usually by other calculations resp. approximations)


Body part

| Name | Parent | Translation | Mass | Geometry | Name | Anchor | Axis | Min | Max |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| torso |  |  | 1.2171 | $\begin{aligned} & \text { Box } \\ & 0.100,0.100,0.180 \end{aligned}$ |  |  |  |  |  |
| neck | torso | 0, 0, 0.090 | 0.05 | Cylinder <br> L: 0.080 R: 0.015 | HJ1 | 0, 0, 0 | 0,0,1 | -120 | 120 |
| head | neck | 0, 0, 0.065 | 0.35 | $\begin{aligned} & \text { Sphere } \\ & 0.065 \end{aligned}$ | HJ2 | 0, 0, -0.005 | 1,0,0 | -45 | 45 |
| shoulder | torso | $\begin{aligned} & 0.098,0,0.075(r) \\ & -0.098,0,0.075(\mathrm{l}) \end{aligned}$ | 0.07 | Sphere $0.010$ | AJ1 | 0, 0, 0 | 1,0,0 | -120 | 120 |
| upperarm | shoulder | $\begin{aligned} & 0.010,0.020,0(r) \\ & -0.010,0.020,0(1) \end{aligned}$ | 0.15 | $\begin{aligned} & \text { Box } \\ & 0.07,0.08,0.06 \end{aligned}$ | AJ2 | -Translation | 0,0,1 | $\begin{aligned} & -95(r) \\ & -1(l) \end{aligned}$ | $\begin{aligned} & 1(r) \\ & 95(\mathrm{l}) \end{aligned}$ |
| elbow | upperarm | -0.010, 0.070, 0.009(r) 0.010, 0.070, 0.009(I) | 0.035 | Sphere $0.010$ | AJ3 | 0, 0, 0 | 0,1,0 | -120 | 120 |
| lowerarm | elbow | 0, 0.050, 0 | 0.2 | $\begin{aligned} & \text { Box } \\ & 0.050,0.110,0.050 \end{aligned}$ | AJ4 | -Translation | 0,0,1 | $\begin{aligned} & -1(r) \\ & -90(l) \end{aligned}$ | $\begin{aligned} & 90(r) \\ & 1(I) \end{aligned}$ |
| hip1 | torso | $\begin{aligned} & 0.055,-0.010,-0.115(r) \\ & -0.055,-0.010,-0.115(\mathrm{I}) \end{aligned}$ | 0.09 | Sphere $0.010$ | LJ1 | 0, 0, 0 | $\begin{aligned} & -0.7071,0,0.7071(r) \\ & -0.7071,0,-0.7071(l) \end{aligned}$ | -90 | 1 |
| hip2 | hip1 | 0, 0, 0 | 0.125 | Sphere $0.010$ | LJ2 | 0, 0, 0 | 0,1,0 | $\begin{aligned} & -45(r) \\ & -25(\mathrm{l}) \end{aligned}$ | $\begin{aligned} & 25(r) \\ & 45(l) \end{aligned}$ |
| thigh | hip2 | 0, 0.010, -0.040 | 0.275 | $\begin{aligned} & \text { Box } \\ & 0.070,0.070,0.140 \end{aligned}$ | LJ3 | -Translation | 1,0,0 | -25 | 100 |
| shank | thigh | 0, 0.005, -0.125 | 0.225 | $\begin{aligned} & \text { Box } \\ & 0.080,0.070,0.110 \end{aligned}$ | LJ4 | 0,-0.010, 0.045 | 1,0,0 | -130 | 1 |
| ankle | shank | 0, -0.010, -0.055 | 0.125 | Sphere $0.010$ | LJ5 | 0, 0, 0 | 1,0,0 | -45 | 75 |
| foot | ankle | 0, 0.030, -0.040 | 0.2 | $\begin{aligned} & \text { Box } \\ & 0.080,0.160,0.020 \end{aligned}$ | LJ6 | -Translation | 0,1,0 | $\begin{aligned} & -25(r) \\ & -45(l) \end{aligned}$ | $\begin{aligned} & 45(r) \\ & 25(I) \end{aligned}$ |

- Name is the body part name of Nao
- Parent is the parent of the body
- Translation is the offset relative to its parent (in meter)
- Mass is the mass of this body (in kilogram)
- Geometry is the size of its geometry representation (in meter)
- Name is the joint name installed on this body
- Anchor is the offset of the joint anchor relative to the body that installed on in meter
- Axis is the joint axis relative to the body that installed on ( $x, y$, $z-$ orientation of the axis) - Min is the min angle that the joint can reach (in degrees)
- Max is the max angle that the joint can reach (in degrees)
- Note: All values are relative to the torso coordinate system! (which faces the y-axis)


## Example: Kinematics AIBO

Diploma thesis Uwe Düffert


World coordinates in the shoulder.
What are the coordinates ( $x, y, z$ ) of the left forefoot?
Calculation:
by transformation of the foot coordinates to shoulder coordinates

## Example: Kinematics AIBO

## Diploma thesis Uwe Düffert



Transformation of the foot coordinates to shoulder coordinates: 1. Translation lower leg: shift towards negative $z$ axis $\left(l_{2}\right)$.
2. Rotation knee: rotate clockwise around $y$-axis $\left(\theta_{3}\right)$.
3. Translation upper leg: shift towards negative $z$ axis $\left(l_{1}\right)$.
4. Rotation shoulder 2: rotate counter-clockw. around $x$-axis $\left(\theta_{2}\right)$.
5. Rotation shoulder 1: rotate clockwise around $y$-axis $\left(\theta_{1}\right)$.
$\operatorname{Rot}\left(-\theta_{1}\right) \operatorname{Rot}\left(\theta_{2}\right) \operatorname{Trans}\left(\mathrm{I}_{1}\right) \operatorname{Rot}\left(-\theta_{3}\right) \operatorname{Trans}\left(\mathrm{I}_{2}\right)$

## Example: Kinematics AIBO

## $\operatorname{Rot}\left(-\theta_{1}\right) \operatorname{Rot}\left(\theta_{2}\right) \operatorname{Trans}\left(I_{1}\right) \operatorname{Rot}\left(-\theta_{3}\right) \operatorname{Trans}\left(I_{2}\right)$



$$
\begin{aligned}
\left(\begin{array}{l}
x \\
y \\
z \\
1
\end{array}\right) & =\operatorname{Rot}_{y}\left(-\theta_{1}\right) \cdot \operatorname{Rot}_{x}\left(\theta_{2}\right) \cdot \operatorname{Trans}\left(\begin{array}{c}
0 \\
0 \\
-l_{1}
\end{array}\right) \cdot \operatorname{Rot}_{y}\left(-\theta_{3}\right) \cdot \operatorname{Trans}\left(\begin{array}{c}
0 \\
0 \\
-l_{2}
\end{array}\right) \cdot\left(\begin{array}{l}
0 \\
0 \\
0 \\
1
\end{array}\right) \\
& =\left(\begin{array}{cccc}
\cos \left(\theta_{1}\right) & 0 & -\sin \left(\theta_{1}\right) & 0 \\
0 & 1 & 0 & 0 \\
\sin \left(\theta_{1}\right) & 0 & \cos \left(\theta_{1}\right) & 0 \\
0 & 0 & 0 & 1
\end{array}\right)\left(\begin{array}{ccc}
1 & 0 & 0 \\
0 & \cos \left(\theta_{2}\right) & -\sin \left(\theta_{2}\right) \\
0 \\
0 & \sin \left(\theta_{2}\right) & \cos \left(\theta_{2}\right) \\
0 & 0 & 0 \\
0 & 0 & 1
\end{array}\right) \cdot \\
& \left(\begin{array}{cccc}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & -l_{1} \\
0 & 0 & 0 & 1
\end{array}\right)\left(\begin{array}{ccc}
\cos \left(\theta_{3}\right) & 0 & -\sin \left(\theta_{3}\right) \\
0 \\
0 & 1 & 0 \\
\sin \left(\theta_{3}\right) & 0 & \cos \left(\theta_{3}\right) \\
0 & 0 & 0 \\
0 & 1
\end{array}\right)\left(\begin{array}{cccc}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & -l_{2} \\
0 & 0 & 0 & 1
\end{array}\right)\left(\begin{array}{l}
0 \\
0 \\
0 \\
1
\end{array}\right) \\
& =\left(\begin{array}{c}
l_{2} \cos \left(\theta_{1}\right) \sin \left(\theta_{3}\right)+l_{2} \sin \left(\theta_{1}\right) \cos \left(\theta_{2}\right) \cos \left(\theta_{3}\right)+l_{1} \sin \left(\theta_{1}\right) \cos \left(\theta_{2}\right) \\
l_{1} \sin \left(\theta_{2}\right)+l_{2} \sin \left(\theta_{2}\right) \cos \left(\theta_{3}\right) \\
l_{2} \sin \left(\theta_{1}\right) \sin \left(\theta_{3}\right)-l_{2} \cos \left(\theta_{1}\right) \cos \left(\theta_{2}\right) \cos \left(\theta_{3}\right)-l_{1} \cos \left(\theta_{1}\right) \cos \left(\theta_{2}\right) \\
1
\end{array}\right)
\end{aligned}
$$

## Example:

Calculate $\theta_{1}, \theta_{2}, \theta_{3}$ for feet position ( $x, y, z$ )

## Inverse <br> Kinematics AIBO

$\theta_{3}$ (between I1,I2) by Cosine rule.
Preferably bending forward:

Positive solution.


$$
\begin{aligned}
\cos \left(\pi-\theta_{3}\right) & =\frac{l_{1}^{2}+l_{2}^{2}-\left(x^{2}+y^{2}+z^{2}\right)}{2 l_{1} l_{2}} \\
\theta_{3} & =\pi \pm \arccos \left(\frac{l_{1}^{2}+l_{2}^{2}-\left(x^{2}+y^{2}+z^{2}\right)}{2 l_{1} l_{2}}\right) \\
& =\mp \arccos \left(\frac{\left(x^{2}+y^{2}+z^{2}\right)-l_{1}^{2}-l_{2}^{2}}{2 l_{1} l_{2}}\right)
\end{aligned}
$$

## Example: Inverse Kinematics AIBO

## $\theta_{2}$ by definition of Sine, where $\left|\theta_{2}\right|<=\pi / 2$ by anatomy

$$
\begin{aligned}
y & =\sin \left(\theta_{2}\right) \cdot\left(l_{1}+l_{2} \cdot \cos \left(\theta_{3}\right)\right) \\
\theta_{2} & =\arcsin \left(\frac{y}{l_{1}+l_{2} \cos \left(\theta_{3}\right)}\right)
\end{aligned}
$$



## Example: Inverse Kinematics AIBO

$$
\begin{aligned}
a & =l_{2} \sin \left(\theta_{3}\right) \\
b & =\left(l_{1}+l_{2} \cos \left(\theta_{3}\right)\right) \cos \left(\theta_{2}\right) \\
d & =\sqrt{a^{2}+b^{2}} \\
\beta & =\arctan (b, a) \\
a & =d \cos (\beta) \\
b & =d \sin (\beta)
\end{aligned}
$$



## Example: Inverse Kinematics AIBO

$$
\begin{aligned}
& x=l_{2} \cos \left(\theta_{1}\right) \sin \left(\theta_{3}\right)+l_{2} \sin \left(\theta_{1}\right) \cos \left(\theta_{2}\right) \cos \left(\theta_{3}\right)+l_{1} \sin \left(\theta_{1}\right) \cos \left(\theta_{2}\right) \\
&=a \cos \left(\theta_{1}\right)+b \sin \left(\theta_{1}\right) \\
&=d \cos \left(\theta_{1}\right) \cos (\beta)+d \sin \left(\theta_{1}\right) \sin (\beta) \\
&=d \cos \left(\theta_{1}+\beta\right) \\
& z=d \sin \left(\theta_{1}+\beta\right) \\
& \theta_{1}+\beta=\arctan (z, x) \\
& \theta_{1}=\arctan (z, x)-\beta
\end{aligned}
$$

## Example: Inverse

 Kinematics AIBOCalculate $\theta_{1}, \theta_{2}, \theta_{3}$ for feet position ( $\mathrm{x}, \mathrm{y}, \mathrm{z}$ )

b)

$\theta_{3}=\arccos \left(\frac{x^{2}+y^{2}+z^{2}-l_{1}^{2}-l_{2}^{2}}{2 l_{1} l_{2}}\right)$
$\theta_{2}=\arcsin \left(\frac{y}{l_{1}+l_{2} \cos \left(\theta_{3}\right)}\right)$
$\theta_{1}=\arctan (z, x)-\arctan \left(\left(l_{1}+l_{2} \cos \left(\theta_{3}\right)\right) \cos \left(\theta_{2}\right), l_{2} \sin \left(\theta_{3}\right)\right)$

## Special Benefits in Calculations

- Rotations in a plane (around joint axis)
- Select "simple" solutions
- Select "simple" relationships
- Use arctan (better: atan2) instead of arcsin or arccos ( because of large error propagation near $-1 /+1$ )

arctan

arcsin


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## Kinematics of Drive Systems

Kinematics (forward kinematics):

- Where does it move to? Inverse kinematics (reverse kinematics):
- How can it get there?

Simplification:
Neglect mass and force

## Kinematics of Drive Systems

- Driven wheels or chains
- Further wheels as stabilizers or for odometry
- Controlable wheels

Idealizing assumptions:

- Wheels run straight (perpendicular to the axis)
- Forward movement per complete rotation: $2 \pi r$ for radius $r$
- Forward movement per rotation about $\omega$ : $\omega$ r for radius $r$


## Drives for Vehicles on a Plane

## Work space:

## Pose ( $x, y, \theta$ ) with 3 DOF

$$
\theta=0 \text { in x-direction }
$$


$\mathrm{V}(\mathrm{t})=\left(\mathrm{V}_{\mathrm{x}}(\mathrm{t}), \mathrm{V}_{\mathrm{y}}(\mathrm{t})\right)$ and $\omega(\mathrm{t})$ are control parameters for motion. They depend on position and speeds of driving wheels.

## Kinematics/Inverse Kinematics

Kinematics: Calculate motion from control.
Change from pose $(0,0,0)$ to $(x(t), y(t), \theta(t))$ by speed $V(t)=\left(V_{x}(t), V_{y}(t)\right)$ in direction $\omega(t)$

$$
\begin{aligned}
& x(t)=\int_{0}^{t} V_{x}(t) d t=\int_{0}^{t} V(t) \cos [\theta(t)] d t \\
& y(t)=\int_{0}^{t} V_{y}(t) d t=\int_{0}^{t} V(t) \sin [\theta(t)] d t \\
& \theta(t)=\int_{0}^{t} \omega(t) d t
\end{aligned}
$$

## Inverse Kinematics:

Which control $\boldsymbol{V}$ and $\omega$ is needed for desired motion?
Options depend on kind of drive.

## Drives for Vehicles on a Plane

## Configuration space:

Options for control:

- Speeds of the driving wheels
- Directions of the wheels / axes

Limitations by constraints
e.g.

- connections between wheels

- Dependency between direction and speed of wheels


## ICC = instantaneous center of curvature

ICC defined as intersection point of all axes

Constraints for smooth motion:

- ICC exists

- Consistent speed of driving wheels

Otherwise:

Images from
Borenstein et.al.:
Where am I?

- Robot loses traction
- Robot slides, unpredictable motion


## ICC = instantaneous center of curvature

Robot moves on a circle around ICC.
(Straight move for parallel axes: ICC infinitely far.)

ICC can be changed by


- steering of axes/wheels
- different speeds of driving wheels


## Kinematics by ICC

Position of ICC for robot at pose $(x, y, \theta)$ :

$$
I C C=[x-R \sin (\theta), y+R \cos (\theta)]
$$

Pose of Robot after time $\delta \mathrm{t}$ while robot rotates $\omega \delta$ t around ICC:

$$
\left[\begin{array}{l}
x^{\prime} \\
y^{\prime} \\
\theta^{\prime}
\end{array}\right]=\left[\begin{array}{ccc}
\cos (\omega \delta t) & -\sin (\omega \delta t) & 0 \\
\sin (\omega \delta t) & \cos (\omega \delta t) & 0 \\
0 & 0 & 1
\end{array}\right]\left[\begin{array}{c}
x-I C C_{x} \\
y-I C C_{y} \\
\theta
\end{array}\right]+\left[\begin{array}{c}
I C C_{x} \\
I C C_{y} \\
\omega \delta t
\end{array}\right]
$$

## Synchrodrive

All wheels in same steerable direction $\omega$ with identical speed. ICC infintely far perpendicular to direction $\omega$

## Control: <br> Speed $v$ and direction $\omega$ <br> of wheel(s)



## Differential Drive

## Driving wheels on 1 axis with different speeds



## Differential Drive

ICC on the axis, position depends on $v_{1}, v_{r}$
$v_{1}=v_{r}$ moves straight on
$v_{1}=-v_{r}$ turns around

## Control: <br> Speeds $\mathrm{v}_{\mathrm{l}}$ and $\mathrm{v}_{\mathrm{r}}$



$$
\begin{gathered}
\omega(R+I / 2)=v_{r} \\
\omega(R-I / 2)=v_{l} \\
R=\frac{l}{2} \frac{\left(v_{l}+v_{r}\right)}{\left(v_{r}-v_{l}\right)} \quad \omega=\frac{v_{r}-v_{i}}{l}
\end{gathered}
$$

## Differential Drive: Kinematics

Change from pose $(0,0,0)$ to $(x(t), y(t), \theta(t))$ by speeds $v_{1}$ and $v_{r}$ of left and right wheel

$$
\begin{aligned}
x(t) & =\frac{1}{2} \int_{0}^{t}\left[v_{r}(t)+v_{l}(t)\right] \cos [\theta(t)] d t \\
y(t) & =\frac{1}{2} \int_{0}^{t}\left[v_{r}(t)+v_{l}(t)\right] \sin [\theta(t)] d t \\
\Theta(t) & \left.=\frac{1}{l} \int_{0}^{t}\left[v_{r}(t)-v_{l}(t)\right]\right) d t
\end{aligned}
$$

## Differential Drive: Inverse Kinematics

Which controls $\mathrm{v}_{\mathrm{l}}(\mathrm{t}), \mathrm{v}_{\mathrm{r}}(\mathrm{t})$ result indesired motion?

$$
\begin{aligned}
x(t) & =\frac{1}{2} \int_{0}^{t}\left[v_{r}(t)+v_{l}(t)\right] \cos [\theta(t)] d t \\
y(t) & =\frac{1}{2} \int_{0}^{t}\left[v_{r}(t)+v_{l}(t)\right] \sin [\theta(t)] d t \\
\Theta(t) & \left.=\frac{1}{l} \int_{0}^{t}\left[v_{r}(t)-v_{l}(t)\right]\right) d t
\end{aligned}
$$

Many different solutions to arrive at a given target.
No motion in direction of the axis (towards ICC).


## Differential Drive: Inverse Kinematics

$$
x(t)=\frac{1}{2} \int_{0}^{t}\left[v_{r}(t)+v_{l}(t)\right] \cos [\theta(t)] d t
$$

Special cases:

$$
y(t)=\frac{1}{2} \int_{0}^{t}\left[v_{r}(t)+v_{l}(t)\right] \sin [\theta(t)] d t
$$

$$
\left.\Theta(t)=\frac{1}{l} \int_{0}^{t}\left[v_{r}(t)-v_{l}(t)\right]\right) d t
$$

$$
\begin{aligned}
& v=v_{1}=-v_{r}: \\
& \left(\begin{array}{l}
x^{\prime} \\
y^{\prime} \\
\theta^{\prime}
\end{array}\right)=\left(\begin{array}{c}
x \\
y \\
\theta+2 v \delta t / l
\end{array}\right) \quad \text { Turn on place } \\
& v=v_{1}=v_{r}: \\
& \left(\begin{array}{l}
x^{\prime} \\
y^{\prime} \\
\theta^{\prime}
\end{array}\right)
\end{aligned}
$$

## AIBO: „Wheel model" (Differential Drive)

Curved motion by different speeds of legs.


## Controlled Wheels

One (or more connected) steerable wheels, other wheels passiv: Bicycle, Tricycle, Wagon etc.
ICC on the axis of passive wheel(s), position depends on $\omega$


## Ackermann-Drive: Automobile

Front wheels are individually steerable


Modell with ICC like for tricycle by a phantom wheel at $\mathrm{P}_{2}$

## Characteristica of Drives:

Rotation on place:

- Differential drive

Mostly 2 control parameters for 3 spatial DOF
ÿ Nonholonomic drives

- Tricycle, Ackerman only for $\omega=90^{\circ}$ (with stability problems)

Differential drive:

- Uneven terrain and sliding results in direction errors for.

Tricycle, Ackerman:

- Complicated maneuvers (parking!)

Ackerman:

- Improved stability by separated (and slanted) front wheels


## Degrees of Freedom (DOF) - continued

DOF is in both work space resp. configuration space the

- minimal number of parameters for complete description equivalently:
- maximal number of independent parameters

Work space: effective DOF

Configuration space: controlable DOF

## Degrees of Freedom (DOF) - continued

Number of effective DOF
i.g. different from number of controlable DOF.

- All poses in work space may be reachable even in case of effective DOF > controlable DOF
(e.g. differential drive)
- effective DOF < controlable DOF is useful in case of obstacles



## Nonholonomic Drive Systems

Nonholonomic Constraints $C\left(p_{1}, \ldots p_{n}, \dot{p}_{1}, \ldots \dot{p}_{n}, t\right)=0$ impose dependencies of paramaters and their derivatives.


> Holonomic Constraints
> $C\left(p_{1}, \ldots, p_{n}, t\right)=0$ impose dependencies of parameters.

Nonholonomic Constraint:
$\tan \theta=v_{y} / v_{x} \quad$ i.e. $\quad v_{x} \sin \theta-v_{y} \cos \theta=0$ $\cos \theta=0$ for $\theta=\pi / 2$ implies $v_{x}=0$ :
No motion in direction of axis (e.g. for differential drive)

## Holonomic Drive Systems

Most drive systems are nonholonomic and have only 2 controllable parameters

Holonomic drives:

- Omnidirectional drive
(Control by separate motors of wheels’,

- Synchrodrive for rotationally symmetric vehicles (2 spatial DOF)
- Synchrodrive with additional body rotation


## Outline

## Introduction

Kinematics of Poses
Kinematics of Drive Systems
Trajectories
Motion Planning
Motion Control
Motions of Legged Robots
Optimization/Learning of Motions
Biologically Inspired Motions

## Trajectories

Trajectory in work space/configuration space:
Sequence of spatial parameters (positions/poses of the robot or its parts) or of control parameters at different times, e.g.

- trajectory of CoM (center of mass)
- trajectory of feets
- trajectory of limb angles


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## Trajectories

## Set of poses $p(t)$ and corresponding configurations $q(t)$ :



Motion planning: Find realistic (and optimal) trajectories. The trajectories in the figures are not realistic.

## Trajectories of Keyframes

Sequence of Keyframes:
Characteristic poses during a motion ("like in a comic"). Originally used in animated movies.

Transition times define speed to reach next pose.
Poses between keyframes must be interpolated.

## Keyframe

Time 1000
HeadPitch HeadYaw 0
RShoulderPitch LShoulderPitch 120
RShoulder RollLShoulderRoll 0
RElbowRoll 90
LElbowRoll -90
RElbowYaw 90
LElbowYaw -90
RHipYawPitch LHipYawPitch 0
RHipPitch LHipPitch -31
RHipRoll LHipRoll 0
RKneePitch LKneePitch 63
RAnklePitch LAnklePitch -31

## Motion Skill: Sequence of Keyframes

300 0-21-62 32-69-59 0-\& FILE walk_forward-flemming-nika.txt 300-5-21-62 46-69-59 0 ( in .../keyframes
$3000-21-6260-69-5908-10-012-110812-0-3-11-110-326959$ $3000-21-7560-69-59086-3627-110812-157-11-97-326959$ $3000-21-8660-69-590842-6913-110812-3023-11-86-326959$ $3000-21-11060-69-590812-0-9-1108-10-012-14-62-326959$ $300-5-21-11046-69-590018-0-9-400-10-017-5-62-466959$ $3000-21-11032-69-590-812-0-3110-8-10-01211-62-606959$ $3000-21-9732-69-590-812-157110-86-362711-75-606959$ $3000-21-8432-69-590-812-3023110-842-691311-84-606959$

Each line starts with the transition time followed by the target angles of joints in a predefined order.
RoboNewbie:
Keyframe sequences are "played" by class keyframeMotion.

## Order of Joints in RoboNewbie Keyframes

NeckYaw $=0$
NeckPitch = 1
LeftShoulderPitch =2
LeftShoulderYaw = 3
LeftArmRoll = 4
LeftArmYaw = 5
LeftHipYawPitch = 6
LeftHipRoll $=7$
LeftHipPitch = 8
LeftKneePitch = 9
LeftFootPitch $=10$

LeftFootRoll = 11
RightHipYawPitch $=12$
RightHipRoll = 13
RightHipPitch = 14
RightKneePitch $=15$
RightFootPitch $=16$
RightFootRoll = 17
RightShoulderPitch $=18$
RightShoulderYaw = 19
RightArmRoll = 20
RightArmYaw $=21$

## Keyframes: MotionNets

Cycles: Repeated motions (e.g. walking) Conditional Branches (e.g. stop motion)

Motion Editor could be extended for Motion Nets



## Keyframes

Simple implementation
Simple design (especially with "teaching")
But motions can not adapt

## Best suited for short sequences (stand-up, kick)

## Usage of Trajectories for Motion Planning

Find a trajectory (path) of the robot or a part of the robot in work space or configuration space which satisfies certain conditions, e.g.

- Motion from start to destination while avoiding obstacles
- Motion of a limb while maintaining stability
- Motion of a manipulator to grasp an object

Side conditions may be time, energy, smoothness, stability, safety...

Appropriate trajectories can be found e.g.
by physical models or by Machine Learning

## Usage of Trajectories for Motion Control

Control the actuators (joint, limbs, ...) such that the robot or a part of the robot follows a given trajectory.

Inverse kinematics
can be used to find the appropriate control parameters.

Shift CoM following a (straight) trajectory implies trajectories of feet, e.g. semi-ellipses or parallelograms. Related joint controls by inverse kinematics.

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## Planning

... is a broad field in AI with many different methods.

Planning can be used for motions and for more complex behaviors (different time horizons).

## Here: Some useful methods for motion planning

Later: Behavior planning

## Motion Planning vs. Control

Robot can

- plan motions (and more complex behavior) before execution
- execution is then performed by appropriate control

Robot Control can be performed as

- Open loop control:


## ("blind" control)

 Preplanned motions performed without sensor feedback.- Closed loop control:

Sensor feedback is used for adaptation of intended motions.
Some planning methods lead directly to controls
(e.g. potential fields).

## Motion Planning vs. Control

Alternative for Planning:

Online motion control by immediate reactions to sensor measurements (e.g. for maintaining balance)

- sensor actor coupling
- behavioral robotics


## ÿ later more

- emergence principle


## Teaching

- Set „characteristic poses" of a motion by hand (at real robot) or by motion editor
- Protocol joint angles of each such pose as keyframe resulting in a sequence of keyframes
- Optimize (transition times, smoothing, ...) e.g. by machine learning



## Motion Editor from Bioloid Manual (2006)



## Motion-Capturing

Imitate demonstrated motions

Markers at important points
Record motions (3D Motion Tracker)
Implement related control (e.g. by analyzing the motion)

## Motion Planning

Optimality of a trajectory may concern

- Length of path
- Time
- Smoothness
- Stability
- Safety
- Energy consumption
- Esthetics
- ...

Planning can be performed
in work space or configuration space using path planning algorithms (e.g. A*)

Images


## Planning in Configuration Space

Special regions in the configuration space for

- obstacles in work space (gray),
- geometry of the robot (black)


Results of planning in configuration space can be directly used as control for motion in work space.


97

## Grid Based Search in Configuration Space

 e.g. using graph search methods like A*

## Skeleton Based Search in Configuration Space

Skeleton: Connects certain points.
Search for path on skeleton.

- as Voronoi-Graph:
points with equal minimal distances to obstacles

- as Visibility Graph:

Nodes at corners of obstacles
Arcs between mutually observable nodes
Problems:

- complex algorithms
- results often in detours


## Random Point Search in Configuration Space

Graph search through random points in free space. Ranking of preferable areas by differently distributed points.


## Potential Field in Configuration Space

„Potentials at the field":

- Target attracts
- Obstacles repel

Can be combined with other search methods.


Can be used as control: Robot follows attractions in the potential field.

## Potential field

Control in a point ( $\mathrm{x}_{0}, \mathrm{y}_{0}$ )
as direction vector $\left[\mathrm{F}_{\mathrm{x}}\left(\mathrm{x}_{0}, \mathrm{y}_{0}\right), \mathrm{F}_{\mathrm{y}}\left(\mathrm{x}_{0}, \mathrm{y}_{0}\right)\right]$ of vector field $\mathrm{F}(\mathrm{x}, \mathrm{y})=\left[\mathrm{F}_{\mathrm{x}}(\mathrm{x}, \mathrm{y}), \mathrm{F}_{\mathrm{y}}(\mathrm{x}, \mathrm{y})\right]$

Special case:
Vector field $\mathbf{F}(x, y)$ is gradient of a potential field $\mathbf{U}(x, y)$

$$
F(x, y)=[\delta \mathbf{U}(x, y) / \delta x, \delta \mathbf{U}(x, y) / \delta y]
$$

For application:

- Potential determined by environment/from sensory information
- Motion follows the gradient


## Potential field

## target: attracting field


$\underset{\mathbf{U}_{\text {goal }}(p)=\alpha \operatorname{dist}(p, \text { goal })^{2}, ~}{\text { e.g. }}$

## obstacles:

repelling fields

e.g.
$\mathbf{U}_{\text {obstacle }}(\mathrm{p})=\beta \operatorname{dist}(\mathrm{p}, \text { obstacle })^{-1}$

## Potential field

## Potential field by superposition (addition):

$$
\begin{aligned}
\mathbf{U}(\mathrm{p}) & =\mathbf{U}_{\text {goal }}(\mathrm{p})+\Sigma \mathbf{U}_{\text {obstacle }}(\mathrm{p}) \\
\mathbf{F} & =-[\delta \mathbf{U} / \delta \mathbf{x}, \delta \mathbf{U} / \delta \mathrm{y}]
\end{aligned}
$$



## Potential field

## Benefits:

- direct usage for control
- local evaluation


## Problems:

- local minima
- Compensation of fields,
- "Trap" by close obstacles
- oscillating movements for
- narrow areas
- high speed


## Potential field




## Potential field

Additional other fields, e.g.

- rotating fields
- random fields can
- specify directions
- break symmetries
- avoid (some) local minima



